MUSIC AND DEEP MEMORY

Speculations in Ancient Mathematics, Tuning, and Tradition
In Memoriam Ernest G. McClain.

Prima la musica e poi le parole

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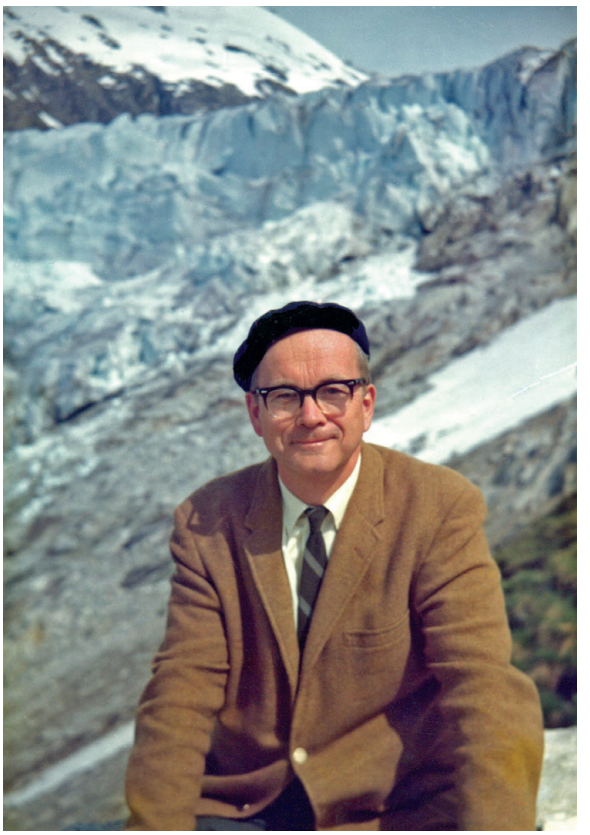
Late 1940s post war, with ruptured duck veteran pin

On the hiking trail, USA, 1970s

Sailing on Long Island South, 1960s



Hiking in the Alps, circa 1970.

Foreword

Ernest McClain was one of the last great spiritual antediluvians. In a century during which scholarship tended overwhelmingly to regard the ancients with either scientistic or deconstructive condescension, McClain simply settled down to patiently learn from them. Eschewing the wishful projection which would cast them in the image of a lost golden age, or a defensive antiquarianism which retreated into museum exhibits, McClain discovered the ancients continuing to speak -- or more precisely, to sing -- in the musical heritage of the whole human race. Listening to this unbroken voice, he eventually attuned himself to it so well he became, himself, the living embodiment of a mode of thought which one might have guessed had gone out of the world.

Beginning in the 1970s, starting with three extraordinarily dense books and continuing in a stream of essays and correspondence that lasted until the day of his death, McClain propounded a thesis, notable equally for its profundity and its simplicity, which read the archaic mythico-speculative inheritance of the West "from the Rg Veda to Plato" and beyond, as a musical cosmology. His work never gained anything like mainstream recognition (a fact which in later years he occasionally noted with bemused resignation), but for a small cadre of researchers, McClain is (as Joscelyn Godwin called him), "one of the most original and ingenious researchers of our time."

McClain's work was concentrated upon recouping the heritage of the ancient world, but he had more proximate sources in classical European culture. He was pointed to his method and to his important conclusions by three great friends. McClain always insisted that his work had been inspired and encouraged, in a manner far from casual, by his relationships with Hugo Kauder, Ernst Levy and Siegmund Levarie. Jewish refugees who had fled just ahead of Hitler's expansionism, they were also representatives of a musical and cultural milieu that had grown up in Europe continuously for centuries -- the heritage of folk music, Church psalmody, carmina burana; of Troubadors, Renaissance polyphony, Baroque innovations; of the evolution and cross-fertilization of Western music from the Classical era through the Romantics to the upheavals of the early 20th century. His close friendship with these men -- composers, performers, and scholars of high order -- was why, as a clarinetist trained in the Western classical repertoire, McClain was also sensitive to a Pythagorean heritage he eventually came to believe derived from the Neolithic age, via Egypt and Sumer -- and yet reaching all the way to us. This is important to emphasize, not simply in the interests of giving due credit, but because McClain frequently puzzled interlocutors by claiming that his work had a serious contemporary significance, and was not meant merely as a divertimento of footnotes.

Each of McClain's books -- The Myth of Invariance, The Pythagorean Plato, and Meditations through the Quran -- is a set of closely-argued excurses through a body of literature as if through an underground mine, looking for the telltale glint of something sparkling in the walls. That sparkle is number, and McClain demonstrated over and over that numbers are not scattered randomly throughout ancient texts. There is a preponderance of multiples of very low primes -- notably 2, 3, and 5; and very often, when a number that cannot be so reduced does occur (say, 37), looking to the context with the small primes in mind will yield a plausible rationale. The books have been noted for the density of their presentation.

("Obscure," "hard to understand," "inaccessible," are terms that come up in the (positive!) reader reviews online.) This challenge to readers is only partly due to the mathematics. More challenging is the fact that once McClain has a numerical trope established, he frequently runs with it, employing it just as the ancients (he held) did: as an extremely abbreviated figure of thought, which could be adapted to many different situations. And yet, he insisted repeatedly, the mathematics involved was itself not difficult. "A child can learn it," he claimed, and he implied moreover that in the era of the pocket calculator, no one, not even the math-averse, had any excuse. (As of this writing, all three of McClain's books are available in pdf from his website, www.ernestmcclain.net , as are numerous essays. The shortest, most accessible, and least tendentious introduction to McClain's basic insights, however, may be the third and fourth chapters of Jay Kappraff's excellent popular mathematics book Beyond Measure.)

Serious engagement with McClain's work cannot help but alter one's apprehension of the whole apparent shape of ancient literature -- not least, the Platonic dialogues. My own experience is probably not too aberrant in this respect. For years I had known that I did not know how to read Plato. The stupid caricature of the body-denier, the philosopher who invented "another world" since "this" one was so changeable and disappointing (and, let's not forget, who "banished the poets"!), had always rang false -- a whipping-philosopher dragged out whenever we needed to blame someone for "essentialism." But although I could "smell" that this travesty was simply wrong, I did not know what to replace it with. There was obviously a tremendous amount going on between the lines in Plato that was going right over my head. No doubt much of this was due to the fact that it was written in 2,300-year-old Greek. And yet, Plato was so obviously concerned to transcend the particular, to reach beyond the limitations of a given setting -- not to deny them, but to refuse to be ruled by them. The limitations of a particular language were real, but they could not be dispositive. There must be a way in -- but where was it?

The Pythagorean Plato pointed out that the way was right where we had always known it was. The door to the Academy famously had on its welcome mat the phrase, Some Geometry Required (loosely translated). "Platonism" was expressly characterized by its coupling to the mathematical truth-condition. But however much commentators might acknowledge this at a kind of high-altitude level, the actual mathematics that occurs in the dialogues is very frequently ignored. (One stark example of this is found in the 1947 translation of the Republic by F.M. Cornford, in which Cornford permitted himself to omit entirely Plato's "extremely obscure" account (at 8.546b) of the so-called ruling or nuptial number, and also to "simplify" the text (at 9.587b) concerning the number of the Tyrant. Even when scholars do not give themselves such free rein, they very often let the mathematics pass by without much comment.)

McClain himself did find the clues in some commentary, including some very old commentary -- above all, Albert von Thimus, to whom he was pointed by Kauder, Levy, and Levarie; but also James Adam, Thomas Taylor, Plutarch, Proclus, Aristotle. Really, though, we might have guessed, for it is obvious once you think of it: Plato's mathematics is musical -- not accidentally, but essentially so. McClain understood the stakes of this interpretation to reach far beyond the exegetical:

> *From Philolaus in the fifth century BC, through Plato and Aristoxenus in the fourth, and down to Ptolemy in the second century AD and Aristides in the third or fourth, Greek acoustical theorists moved confidently between two modes of expression: the absolutely precise and the conveniently approximate. ... There is an urgent need for a review of all these ancient materials, not simply for their intrinsic interest to musicians and historians of science, but for their wider relevance to the philosophical foundations of Western culture.* (*The Pythagorean Plato,* p.162)

Indeed, (though this is perhaps not quite so obvious), this tradition is itself part of a great underground current of musico-mythical cosmology, which McClain worked very hard to unbury, stretching back to the Vedas (and likely before) and forward at least as late as the Quran. The most obvious "fossil record" of this tradition is the recurrence, not just of very specific numbers -- numbers which are usually multiples only of very small primes (mostly not higher than 7) -- in cosmological and visionary contexts, but of various sets of numbers which can be seen to "go together," in a way that indicates that writers knew the provenance of the numbers, or at least that certain numbers called for certain other numbers, even when the surface meaning of the text has nothing overly to do with music -- aside from, say, the mention of a number of harpists or trumpeters attending the celestial court.

All throughout a largely misunderstood (when not ignored) career of four decades, McClain never tired of insisting upon the tremendous import of this project. He himself declined to write philosophy in any but the most occasional or offhand modes -- he was unpacking a prelude to philosophy, he said. (His friend and correspondent the Aristotelian philosopher John Holthouse once opined to me that "Ernest is a philosopher, but would rather die than admit it.") It was, I came to see, not just that the numbers were a sort of scaffolding for a widely various but shared cultural background. The numbers were symptomatic of something else. They were features of a whole way of looking at, and being in, the world -- not an artificially schematized worldview parsed out in multiples of 2, 3, and 5, but a world in which the "metaphor" of cosmic harmony came perfectly naturally, and indeed was no metaphor. (Indeed, the phrase "cosmic harmony" may make us cringe in reaction to Newagey overtones, but did no such thing for the ancients).

In saying this much, I've already gone beyond what McClain himself explicitly argued. Whatever the range of associations he allowed himself, he nonetheless held himself to a strictly empirical program. His numbers were all there on the surface of the text itself, or in a very few cases, easily derivable from those that were. No one ever disputed this. It was the rationale he deduced that earned him occasional rebuke and eventually either polite disregard or sometimes misapprehending fandom. Early on, Gilbert Ryle set the tone. "Plato would never," he informed McClain, "have planted all that musicology for you to find." To which one rejoinder must surely be, well then, how is one to account for the numbers, the very specific numbers, in (for example) Plato's texts? The Tyrant is held, in the Republic, to be exactly 729 times less fortunate than the good ruler. Not "about 700," not 730. There are exactly thirty-seven guardians of the city Magnesia in the Laws, a city which Plato repeatedly insists will be composed of 5,040 citizens.

McClain's conclusion was not that Plato really "supposed that the well-being of the city depended almost as much on the number 5040 as on justice and moderation," (as Jowett remarks). Nor did he believe, as Ryle feared, that Plato had played a kind of nudge-wink game of find-the-tuning-theory with his readers for the fun of a few initiates. It was, rather, that Plato's exposition of justice and moderation found a completely natural expression in terms that privileged this musical and numerical grammar, and did not find it distracting. Far from being some private diversion on the part of Plato, it was an inherited vocabulary shared across a wide spectrum of wisdom texts descending from a common tradition, which lasted in oral culture even until the early strata of the Quranic tradition.

Even among his disciples, there has been significant breadth of opinion about the nature of the nature of the importance of McClain's work, and much of this variation is occasioned by this wide-net approach which drew in a vast range of background, beginning with the Rg Veda (on which his friend Antonio de Nicholas had written a book, Four Dimensional Man, whose importance for his own work -- and for his serious students -- McClain frequently emphasized). Some readers seized upon McClain as grist for anti-modern contentions, trying to recover an ostensibly lost tradition capable of producing "real magic." Some imagined that McClain's numbers would provide something like the resonant frequencies of the soul, a means for opening the crown chakra by just the right solfeggio. Others were intrigued enough by the musical ramifications to build instruments aligned to various tunings derived from McClain's work. And some were content to multiply contexts in which McClain's tonal harmonics could be plausibly applied, but without raising larger questions as to why.

My own interpretation is, I am sure, no less idiosyncratic. Tuning a musical instrument is a continual practical exercise in letting good enough be good enough; in making one adjustment here and then a counter-adjustment there. The great paradox is that this became the flowering seedbed of an effort to understand the whole. Because there are incommensurables built into the theory, the theory becomes a self-referential exercise in showing how theory itself fails to account for the whole, but in a way that weirdly manages to show the whole as needing no accounting. Approximation and precision become the warp and woof of cosmology and indeed of askesis. (And, I will add, Plato is especially significant in this account because he comes at an historical moment when, under the inexorable influence of writing, the complete naturalness of this way of thinking is no longer so evident, but has become itself a problem.)

McClain kept a respectful engagement with all contacts and the proclaimers of all interpretations, never disdaining them; often profiting from their suggestions even while insisting that what he was talking about was not "secret" and never had been, in any para-Masonic sense. It was all out on the surface of the texts; you just had to learn to think like the authors. (Here again, the fact that his method grew out of continuity with the whole tradition of Western music and culture, helps one to gain purchase on this point.) He had warm and deep correspondence with giants like John Bremer and Seyyed Hossein Nasr, and also with young and eager readers who had discovered his books or his website on their own and sometimes had no credentials aside from being intellectually alive and not risk-averse. In the last decade of McClain's life, many of these exchanges occurred under the auspices of the online BIBAL forum, moderated by Duane Christensen, a scholar of the Old and New Testaments and himself an embattled proponent and architect of a structural hermeneutic which read the Hebrew and Christian scriptures alike as very precisely (and numerically) engineered. An extremely accommodating moderator and an enthusiastic partner in dialogue, as well as a tireless pastor (he and his wife Martha carried out a prison ministry for years), Christensen not only fostered a conversation among a number of very diverse interlocutors -- a conversation which was very invigorating for McClain and those who encountered him there -- but also did much himself to advocate for McClain's work, including using it as an important aspect of his theoretical basis in his commentary on the prophet Nahum in his new edition for the Anchor Bible series.

Much encouraged by this late-blooming attention, McClain evinced a palpable optimism, continuing to believe that a breakthrough insight might well surprise him and force revision of everything he'd written. I've never known anyone with more intellectual gumption. On BIBAL, he relished sharing and sparring with friends, throwing out variations on the book of Ezekiel one day, a Sufi poem the next, always ready to make mistakes in public, and insisting both that no one believe him "until you must," and that whatever your own work was, you did it "your way." (I remember a titanic debate between him and David Crookes over the pertinence of gematria, which reminded me at the time of Nabokov and Wilson clashing in the pages of the New York Review of Books: the rest of us went scurrying. Well, I did. McClain said at the time, "Two musicians couldn't agree more perfectly to disagree profoundly! Ain't that usually the way with our breed?" Crookes repaid the compliment when in his remarkable book The Lord Shall Count he thanked McClain "for his instruction, for his encouragement, and above all for his regular bursts of gunfire.")

An invaluable -- and now keenly missed -- friend and mentor, a never-flagging enthusiast of "adventures in ideas" (a Whiteheadian phrase he loved), McClain took with great seriousness the ancients' love of play and their easily-shifting referents. I slowly came to see that he had indeed learned to think like them. The oft-remarked density of his books is a function not of obscurity of his subject-matter, but of the extreme compression with which his mind was accustomed to move, the way he could pack whole clusters of "contradictory meaning" into root-metaphors. To the outsider this is bewildering, and looks like either eye-glazing calculus or word salad. But after spending enough time with him, one came to see that the details, while ready to open up if you did the work (which in every case turned out to be almost as easy as he promised), were actually part of the "precision" that took its accustomed place within approximation's relaxed mode. "Agreement" and "disagreement" are thus themselves in a continuous dialectic with one another, in theory as in practice -- which means that the effort to understand becomes itself an instance of what is to be understood. The musician recapitulates the music. In short, McClain taught us that the law was always already included within grace.

Bryan Carr

Introduction: Music and Deep Memory
Speculations in mathematics, tuning, and tradition

In Memoriam Ernest G. McClain

Ernest McClain's work is an enthusiastic and painstaking excavation of tradition. McClain always maintained that what he was "uncovering" had in an important sense never been hidden; the numbers were an inevitable side-effect and index of the phenomenon of human music, and their symbolic and "mystical" import remained recoverable with due humility and patience. He was, eventually, philosophical about his relative marginalization in Academe, and he knew he had produced work which was a taxonomical challenge to librarians and scholars. Was it history? Music theory? Philosophy? Despite its empirical foundations, McClain's work is speculative insofar as it hypothesizes certain techniques which cannot be verified to have been used anciently -- though it makes sense to think they were used -- and in making use of parallels which are broadly cross-cultural (anciently) and cross-disciplinary (today). McClain was sometimes cavalier about what explanation(s) should be entertained for these parallels. Diffusion? Structural similarities? Archetypal psychology? Mystical insight? What he insisted upon was the need to be arithmetically accurate, musically realistic, hermeneutically responsible, and speculatively adventurous.

"Music" in our title, we take it, is self-evidently apposite. "Deep Memory" pertains to this tradition -- both intentionally and unconsciously maintained and recorded -- which McClain felt (though he could not prove) was continuous from the Yangtze to the Thames and from Gilgamesh to the Quran (his correspondents sometimes pressed it as early as Avebury or Lascaux -- to say nothing of Atlantis! -- and as late as Snorri, or Chaucer, or Bacon). The essays in this volume explore this tradition in some of its historical, mathematical, musical, architectural, and philosophical aspects. These essays are acts of homage and affection, attempts to carry forward in the spirit of living enquiry which McClain exemplified. Their topics are perhaps more various than is typical for a collection of this sort; but they belong together not merely accidentally as associations of a particular scholarly career, but essentially as different dimensions of a coherent research program of extensive relevance. However, it bears mentioning that perhaps not all of them would have met with McClain's unreserved approval. He would have applauded their seriousness and their daring; he might well have taken issue with their method or conclusions. Every essay is testimony to the unreserved encouragement of spirit he offered during his life; his willingness to critique -- and the way he did so -- was part of that generosity.

McClain was deeply concerned with how human beings, in their musical and mathematical ingenuity, devised ways of developing and exploring variation under different degrees of constraint. Such constraints are many: the resistance and different behaviors of materials, the limited capacity of the human ear and voice; the fact that all geometrical depictions are approximations of idealities. Jean Le

Mée's paper, "The Challenge of Abul Wafa," treats one particular set of constraints: the requirement, imposed by mathematical discipline, to construct geometrical figures (in this case, the five Platonic solids) with compass and straightedge alone. This constraint is made more rigid by Abul Wafa by restricting the compass setting to a single width: the so-called "fixed" or "rusty" compass. Le Mée points out not only that this challenge can (with due intrepidity) be met, but gives detailed instructions for meeting it. (It remains an open question whether Le Mée's solution is precisely the one Abul Wafa would have used; it must, in any case, be close.) This very specific problem finds its place in a very wide and lengthy tradition of geometrical and practical inquiry, which forms the background to Leon Crickmore's contribution, "Castlerigg: Stone or Tone Circle?" Crickmore offers an interpretation of the Castlerigg, a neolithic monument situated in the northwest of England and remarkable for its preservation and its beautiful site. Crickmore reads Castlerigg in terms of both astronomical and musicological systems, seeing it not just as significantly oriented to celestial events, but as a large-scale model of the octave. The background Crickmore assumes ranges from the elaborate diagrams from the 18th-century papers of John Byrom, through Greek tuning systems, to Babylonian geometrical problems (for the interpretation of which he makes some specific suggestions); and he concludes with a suggestion that, in "playing" with the correspondences, one may generate new insights, "even contradictory" ones, poetically resonant even if not verifiable. This may seem a tremendous amount to include in a single glance, but the perhaps bewildering range of materials is underlain by an expressly musical rationale. Jay Kappraff's paper "Ancient Harmonic Law" goes into great detail explicating this. Kappraff, who has previously devoted several chapters to McClain in his books *Beyond Measure* and *Connections*, and who collaborated with McClain extensively, is perhaps uniquely qualified to offer such exegesis. In his paper, Kappraff unpacks McClain's close reading of Nichomachus and Boethius. He demonstrates that, however broad the metaphorical applications, McClain's mathematics was always rigorously grounded in concrete and demonstrable musical proportions, with which these ancient authors were familiar and comfortable.

The number seven looms large in many of these considerations, because these proportions generate a scale of seven notes, widely associated in antiquity with the seven classical "planets." Seven is also linked to one of the earliest geometrical problems which is unsolvable with compass and unmarked straightedge alone: the drawing of a heptagon. Sarah Reichart and Vivian Ramalingam show how this challenge was navigated by convenient approximations in several striking monuments of sacred architecture. They provide a rich account of the histories and implicit symbolisms of these sites, striking in their cross-cultural resonance. Their survey covers three buildings, in France, the Netherlands, and Germany (and mentions several other sites of interest along the way), and also treats the extensive symbolism of the number seven rooted in myth, scripture, and numerology. Such architectural (and acoustical) projects are of course not carried out for the sake of overcoming a geometrical Thou Shalt Not; they occur in cultural milieus which provide cosmological and spiritual grounding for those who participate in them. In "Pattern of Settlements 1-9," Petur Halldórsson shows how far back (temporally), and how widely diffused (geographically), such cosmologico-architectural impulse may be found. Halldórsson's approach is rooted in the work of Icelandic scholar Einar Pálsson, but his survey extends beyond the Icelandic context that Pálsson mostly stayed within. Considering sites in Iceland, Denmark, France, Greece, Italy, and Egypt, Halldórsson contends that one may discern in each case a method of plotting human settlements with respect to significant landscape features in such a way as to align with important recurrent astronomical events. This practical dialogue between human artifice, terrestrial environment, and celestial pattern, is of the essence for understanding the application of musical grammar McClain

read. Anne Bulckens' paper "The Metonic Cycle and the Parthenon" argues that a similar (not at all identical) encoding of astronomical and musical proportions was used by the architects and builders of the Athenian Parthenon. Bulckens' work includes an imaginative but plausible method by which the lengths of the year could have been determined with considerable precision, and her architectural claims are spelled out, in many cases, down to fractions of millimeters. Her reconstruction includes a highly ingenious (albeit speculative) system whereby various astronomical time-spans would have been represented not by lengths but by areas; a solution which is, as far as I know, unique in the literature.

As a sample of McClain's own work, "The Proportional System of the Parthenon" exemplifies the way he too experimentally applied this grammar to ancient sacred architecture. This paper, an earlier version of which built upon some of Bulckens' preliminary results, is a collaborative effort with Kappraff, who has also finished the paper and prepared it for publication. Kappraff and McClain show not merely how numerous measurements of the Parthenon are plausibly connected, via musical proportion, to the Vedic fire altars whose construction is described in the Shrauta Sutras. This chapter by Kappraff and McClain is accompanied by an appendix by Richard Heath, which relates their findings to broader disputes in the field of ancient metrology. Heath's larger contribution to this volume is "Ernest McClain's Musicological Interpretation of Ancient Texts," which is an application of McClain's method to the text of Genesis. A researcher whose work has focused upon ancient metrology and astronomy and the remarkable ways in which these intertwine with music to suggest a coherent ancient cosmology, Heath developed his website HarmonicExplorer.com (a tool which happily McClain was able to utilize during the last years of his life), in order to more readily unpack such resonances. Using graphics from this indispensable resource, Heath is able to show with considerable detail how an interplay of symbolic, narrative, and mathematical elements illustrates the way these aspects worked together in the minds of the authors and redactors of Hebrew scripture.

Such textual analysis, this time of Plato, is also the focus John Bremer's paper on "The Opening of Plato's *Polity*." It is an honor to include this contribution by John Bremer, who did not live to see its publication; his scholarship and his care for the real ends of education made him that rarity of rarities, a philosopher in the real sense: a lover of wisdom. (He always modestly shook his head at McClain's compliment that he was "the best Greek scholar I know," and preferred to remember what was said of Thomas Taylor: "that his opponents knew more Greek, but he knew more Plato.") Making the case that Plato gave extreme care to fine-grained micro-engineering of his texts, Bremer attends to the first eight words of this dialogue – "I went down yesterday to the Piraeus with Glaucon son of Ariston." From this hyper-focus, Bremer then gradually opens up the ramifications that arise when one takes seriously the possibility of such attention on the part of Plato as author. Bremer believed that these ramifications extended as far as the counts not just of lines or words, but of syllables, in Plato's texts; but he also always insisted upon remaining rooted in and oriented by the fundamental issues of philosophy: how one should live. (Forget these questions, Bremer said, and we may as well do crossword puzzles.)

Either in the case of the Bible, or in the case of the Platonic corpus, one is often struck by the question of whether such fine-structured engineering is historically or textually plausible; but also, why it would matter. Both Heath and Bremer show its plausibility, and moreover what such structuring could accomplish. But – granted that philosophy could take such pains – why should one attend to such a philosophy? The paper by Bryan Carr, "Ontology Inside-Out," is meant as an exploration and illustration

of what might now be at issue for such a musically-inflected cosmology — not anciently, but today. It asks this by way of a comparison between Aristides Quintilianus, the Neoplatonist musical theorist, and Quentin Meillassoux, a significant contemporary philosopher whose work aims to press as far as possible the implications of the mathematical, as opposed to the musical, reading of the cosmos following from the Copernican-Galilean revolution. The stakes of this philosophical contest, Carr argues, are high, and include the terms under which the Hellenic and the Biblical heritage of the West can fruitfully engage with each other. The following paper, by Babette Babich, can be regarded as one possible set of variations on the playing-out of these ramifications in the contemporary world. Babich's paper, "The Hallelujah Effect," is a kind of "retrospective preface" to her book by the same title, which had its origin in ongoing email correspondence with McClain in the last years of his life. A long and meandering excursion into the conditions of musical culture today, *The Hallelujah Effect* takes its title from the remarkable song "Hallelujah" by Leonard Cohen, one of the most-recorded songs in recent decades, and traces it through several versions, raising along the way far-flung questions about internet culture, advertising and ideology, and philosophy of music (with special attention to Nietzsche and Adorno). Those looking for the immediate connection with some of McClain's own concerns may wish to start with section III of the paper: "On Nietzsche's Greeks and Nietzsche's Beethoven."

So much, then, for allegations that McClain's contentions are implausible, or irrelevant. There remains, however, an oft-met difficulty about whether they are anachronistic. This is the question addressed by Pete Dello in his paper on "McClain's Matrices." Step by step, Dello shows that although it remains an open question whether it was utilized precisely in the form in which McClain presents it, it is plainly not anachronistic when compared with the mathematical and musical achievements of the Sumerians already in the third millennium BC. Dello gently insists that a musical scribe of the era, easily grasping McClain's method and point, would readily have recognized him as one of their own. The question of anachronism also informs the next paper, Richard Dumbrill's "Seven? Yes, but…", an extensive inquiry into the tuning systems that can be decoded – not uncontroversially – from Akkadian and Sumerian sources. Dumbrill gives an account of – and takes a strong position in – the polemics concerning reconstruction of the scale used in this ancient music: how many notes? Where was the root tone? How did the tuning proceed? And were the scales supposed to be ascending or descending? Over a long professional friendship, Dumbrill and McClain argued over the details of various such reconstructions; Dumbrill always insisting that the texts had to have the final word; McClain often shrugging that his mathematics made perfect symbolic sense, and that practical musicians were always entitled to ignore theoretical niceties in any case.

The questions of mediation between antiquity and today, between music and mathematics, between the Hebrew and the Greek (and Egyptian, Sumerian, Chinese!) heritage, and between the very specific and the extremely broad, remain the focus of the next chapter. Howard Barry Schatz argues, in "Through the Eyes of Plato," that McClain's findings may be fruitfully applied to -- or perhaps, are themselves an application of -- a spiritual technique that has its provenance in the foundational Kabbalistic document, the *Sepher Yetzirah*. Pointing out the essential continuity of McClain's work with that of Kauder, Levy, and Levarie, and that of the important and neglected Albert von Thimus, Schatz argues that these insights shed crucial light on comparative religious studies, the musical history of the West, and indeed on the eventual promise of a scientific theory of the whole universe in terms of string theory; most significantly, perhaps, he underscores their import for the contemporary possibilities in ecumenical dialogue and interreligious spiritual discipline.

In keeping not only with our intention to honor a significant scholar, but with the spirit in which his work unpacked the intertwining of the particular and the universal, many of our chapters include personal recollections of Ernest McClain by the authors. We conclude the volume as a whole with a moving memoir by Gerald M. Turchetto, who recounts in vivid terms both the philosophical stakes of McClain's project and the difficulties of understanding it oneself -- let alone of getting it a fair hearing. The (pseudo?-) Platonic *Epinomis* includes this prediction:

> *To the man who pursues his studies in the proper way, all geometric constructions, all systems of numbers, all duly constituted melodic progressions, the single ordered scheme of all celestial revolutions, should disclose themselves, and disclose themselves they will, if, as I say, a man pursues his studies aright with his mind fixed on their single end. As such a man reflects he will receive the revelation of a single bond of natural interconnection between these problems.* [*Epinoimis* 991e>]

This moment of insight bears comparison with account Turchetto gives of his own Eureka-moment during his lesson from McClain with the monochord: "By following his various placements of the triangular fret along the monochord and listening as he did so, the mathematics and the music finally merged in my experience. All the mathematical decisions that drive the music, and all the musical decisions that drive the mathematics, were brought home to me, along with the intense frustration of wanting to make them work together harmoniously without compromising the integrity of either. Good luck! That dilemma is at the core of it all."

These essays are each attempts to wrestle with this dilemma. They all bear the marks of practice in the studio where learning is the fret, speculation the tuning peg, and the string is the human soul itself. Knowing the stakes, the poised tension between spirit and letter, Ernest McClain would surely have argued strenuously with (or against!) any number of contentions herein, even while insisting that his opponents stick to their guns. What is crucial is the rapport, in which argument and agon is but a single phase – albeit an essential one. Offered in a spirit of tribute and homage, these writings are given also as continuation of a great adventure in which each of us is tempered by every other.

Ernest and Augusta on their wedding in 1973

2010

Ernest G. McClain
(August 6, 1918, Massillon, Ohio - April 25, 2014, Washington, DC)

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1-8

THE CHALLENGE OF ABUL WAFA

Jean Le MÉE

Introduction

Abul Wafa's full name was *Abū al-Wafā', Muḥammad ibn Muḥammad ibn Yaḥyā ibn Ismā'īl ibn al-'Abbās al-Būzjānī or Abū al-Wafā Būzhgānī*. He was born on the 10th June 940, and died on the 15th July 998. He was a Persian philosopher, mathematician and astronomer who worked in Baghdad. Among other things, he developed the field of trigonometry. In particular, he introduced the tangent function. He is also credited[1] with 'the feat of drawing all five Platonic solids, (Fig.1a), using only a straightedge and a pair of compasses at a fixed setting.' Such fixed compasses (known as 'rusty' compasses), adds Hersey[2], have been the tools of virtuoso geometrical draftsmanship in many periods.

We would like to show here, without laying claim to virtuosity eleven centuries after, that the deed can easily be accomplished. Indeed it is already outlined in Euclid's *Elements*. We first look at Euclid's method and then propose an approach based on the Maraldi angle.

1. Euclidean Approach

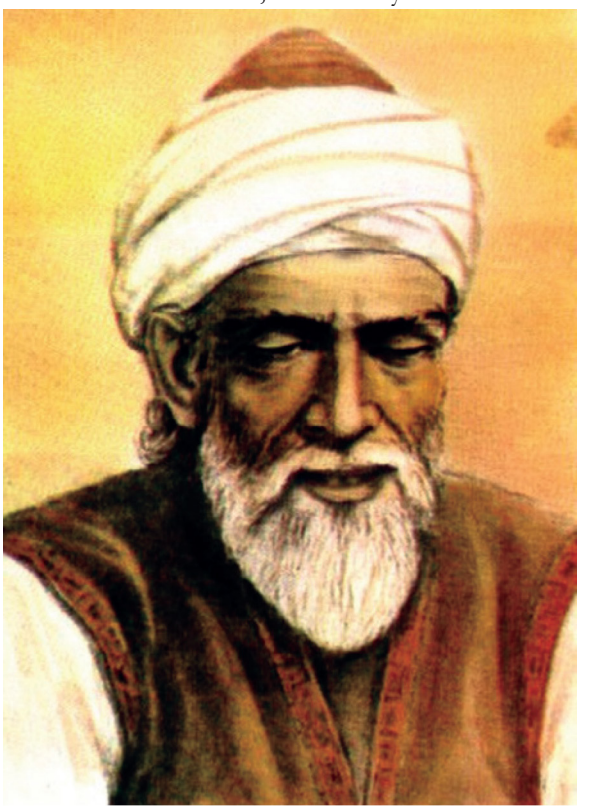
In proposition 18 of his Book XIII, *The Elements*, Euclid had already given a geometric construction for the edges of all five Platonic solids inscribed within a common circumsphere[3].

1 Hersey, G.L., (2000). *Architecture and Geometry in the Age of the Baroque*, Chicago UP, p. 88. Ibid

2 Ibid.

3 March, L., (1998). Architectonics in Humanism: *Essays on Number in Architecture*. Chichester, UK: Academy Ed. John Wiley& Sons Ltd. pp. 88-90.

The construction in this case with a set compass is straightforward except for the dodecahedron. With some additional steps, however, outlined at the end of this section, it can easily be achieved.



Abul Wafa (940-998)

The Euclidean approach limits itself to giving the lengths of the edges of each of the five Solids. The approach that we propose in the next section suggests also an actual construction of the solids.

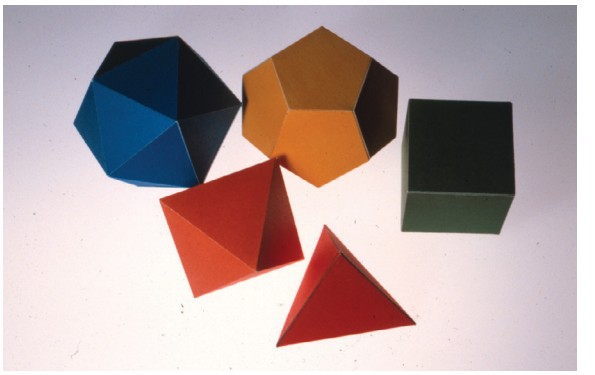


Figure 1a. The Five Platonic Solids.

Referring to figure 2, the circle of center O and diameter AB is a great circle of the sphere

circumscribing the five Platonic solids. ABCD is a square built on AB.

Diagonals DB and AC meet at E. DO and CO cross AC and DB at X and Y respectively. CO cuts circumference of circle O at G. Draw triangle AGB. Through X, E, and Y draw perpendiculars to AB. Perpendicular through Y intersects circumference at F. Draw triangle AFB.

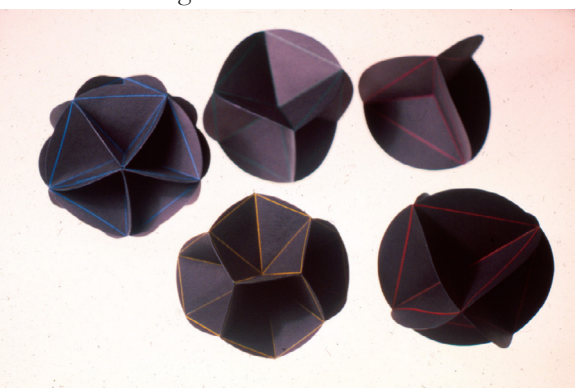


Figure 1b. The Five Platonic Solids and their Maraldi Angle.

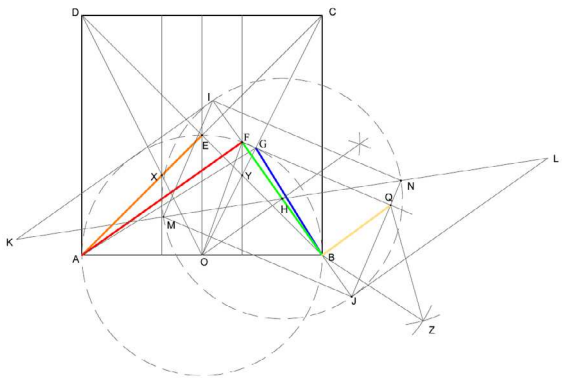


Figure 2. Euclidean Construction of Edges of Platonic solids.

Then the half-square diagonal (AE) is the length of the edge of the octahedron. FB is the length of the edge of the cube; AF is the length of the tetrahedron; and GB, the length of the edge of the icosahedron. Note that all these segments result from operations with a straight edge and a single compass opening. We leave the proof to the reader who may want to refer to Euclid himself with the help of Heath[4], Serlio[5], and Fowler[6].

Examination of the dodecahedron reveals that since its faces are pentagons and the diagonals of the pentagons form a cube of common circumsphere with the dodecahedron, the relation between cube edge and dodecahedron edge is in the Golden Ratio φ. Given the cube edge FB, the problem is to find a segment that would be equal to the short side of the Golden Rectangle where FB is the longer side.

Such a construction, however, can easily be achieved with a single compass setting:

- Extend FB both ways and draw its mediatrix intersecting FB at H.
- Draw circle centered at H with rusty setting.
- Build perpendiculars at the end I and J of diameter containing segment FB. This is most easily done by drawing the tangents to both circles O and H through I and J.
- Extend these perpendiculars by the length of one diameter on alternate sides and join the extremities K and L.
- KL cuts circle of center H at M and N.
- INJM is a Golden Rectangle and therefore IN = φNJ.
- Draw perpendicular to FB at B.
- Draw perpendicular from F to NJ.
- Both meet at Q.
- FQB, similar to IJN, therefore FB/BQ = IN/JN = φ.
- But since FB is edge of cube, then BQ is edge of dodecahedron.

AF, tetrahedron edge subtends angle AOF.
FB, cube " " " " FOB.
AE, octahedron " " " AOE.
GB, icosahedron " " " GOB.

By striking arcs BZ and QZ (with rusty compass setting) one forms angle BZQ.

All these angles are the Maraldi angles defined in the next section, showing the two methods to provide the same information, the first through geometric means alone, the second, through the means of trigonometric ratios.

2. Maraldian Approach

The method proposed here is based on a construction of the Maraldi angle for each of the five Platonic solids. The Maraldi angle, also called internal angle, is the angle formed by any pair of consecutive radii joining the center of the circumscribing sphere to both ends of a given

4 Heath, T. L., (1956). *The Thirteen Books of Euclid's Elements*, 2nd edition. New York: Dover.
5 Hart, V. and Hicks, P., trans. (1996). *Sebastiano Serlio: On Architecture*. New Haven: Yale University Press.
6 Fowler, D., (1999). *The Mathematics of Plato's Academy: A New Reconstruction*. 2nd edition. New York: Oxford University Press (Chapter 5 and Appendix 10.1).

edge, i.e. to the corresponding vertices of the regular solids being circumscribed.

It is easy to demonstrate that the trigonometry of the Maraldi angles for the five Platonic solids is given by the ratios of the first three integers - something that would have pleased Plato and that Abul Wafa might have known. Thus for the cube, C is the internal or Maraldi angle with cos Ci = X/R = 1/3 (Figs. 3, 4, & 5), where 'a' is the edge of the cube. CA=R, the half-diagonal of the cube or the radius of the circumsphere, X=CH is obtained by dropping the perpendicular AH from A on CB. Figure 6 shows the construction of the internal angles for the five Platonic solids.

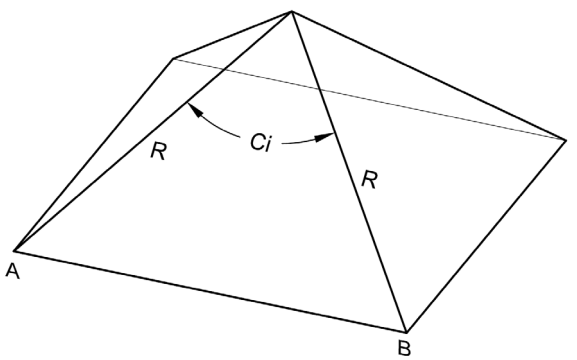


Figure 3. Cube Constitutive Pyramid.

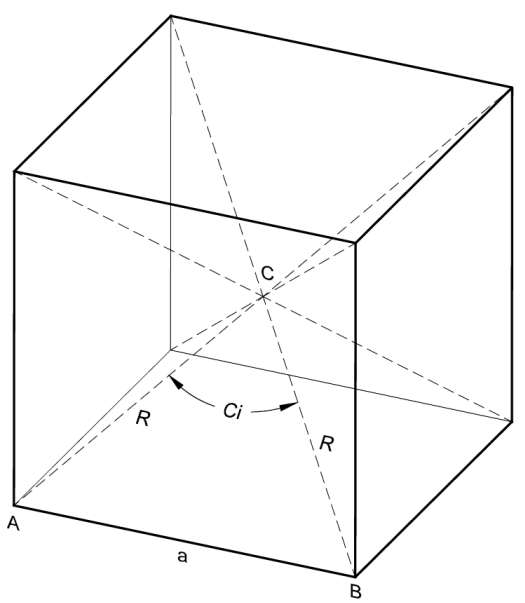


Figure 4. Cube.

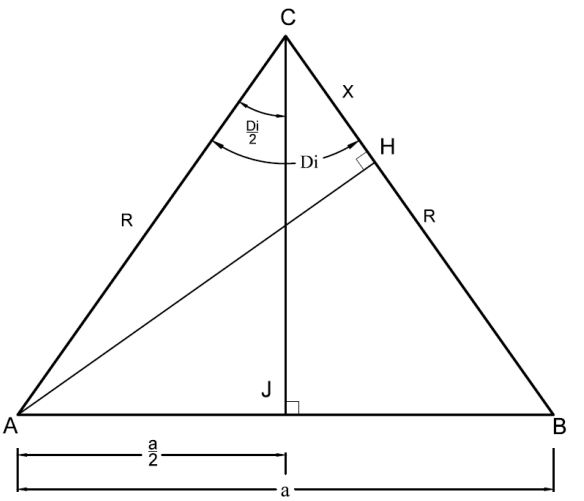


Figure 5. Face of Pyramid.

The cube is seen as being formed of six pyramids with square bases of side 'a' and a common apex. The sloping edges of each pyramid are radii of the circumscribed sphere. Each such pyramid can therefore be constructed a shown on figures 8a and 8b.

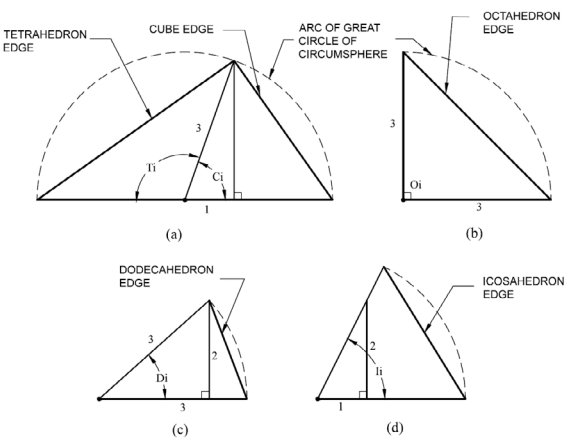


Figure 6.

Similarly, the tetrahedron is constituted of four pyramids with triangular bases and a Maraldi angle with cos Ti = -1/3 (Fig. 7a and 7b).

For the octahedron, we have eight pyramids with triangular bases and a Maraldi angle with sin Oi = 1 (Figs. 9a and 9b).

The dodecahedron has twelve pyramids with pentagonal bases and a Maraldi angle with sin Di = 2/3 (Figs. 10a and 10b).

Finally, the icosahedron will have twenty pyramids with a triangular basis and a Maraldi angle with tan Ii = 2 (Figs. 11a and 11b).

We can therefore construct the Maraldi angle and the edge typical of each of the Platonic solids as shown on figure 12. At this point we have used the adjustable compass to demonstrate the principle. We now show that this figure can be drawn with a 'rusty' compass.

We begin by tracing a line xy with the straightedge (Fig. 13). Setting our compass opening at R, radius of the circumsphere common to all the regular forms, we then proceed by drawing four intersecting circles whose centers O1,O2,O3,O4 are on line xy and the circumferences of their neighbors as shown on figure 13. The three vesicas determine both a square ABCD of side equal to the circle diameter and the median EF of the square to which we add diagonals AC and BD and square EHFG.

Now, draw IJ (it passes through O2) and join AO2 and DO2 cutting GE and GF at K and L respectively. Join KL cutting XY at M. Call center point of square ABCD, O5.

Then

MO5 = 2/3 GO5

or MO5 = 2/3 R

so that if we (arbitrarily) set R=3, MO5 = 2. This is easily established through similarity of triangles and is a standard construction for the harmonic series.

Now, repeat a similar construction along EF, i.e., draw NP cutting EF at Q. Then join G to Q and H to Q, cutting DB and AC at S and T respectively. ST cuts EF at U.

Then O5U = 1/3 O5F

or O5U = 1/3 R

again with R=3, O5U =1.

Now with the same 'rusty' compass opening (Fig. 14), draw circle centered at O5. It is inscribed within square ABCD. Extend ST on both sides so that it cuts circle O5 at V1.

On the left, it passes through L; O5 L extended cuts the circumference at V2 and KL extension will cut it at V3.

Join FV1, FV2, FV3, EV1, EG.

These are the edges respectively of: The cube, the icosahedron, the dodecahedron, the tetrahedron, and the octahedron. Each of them subtends the central angle that is the Maraldi angle of the corresponding polyhedron. The triangle formed by the edges of each of the solids indicated and the radii ending at the extremities of the edges constitute the faces of the pyramids.

To complete the challenge it remains to construct each of the solids. In our case, the problem is to draw the development of each of the pyramids making up each solid. If we desire to construct Platonic solids that can be inscribed in a common sphere, patterns can be cut out to build all the pyramids from the triangles.

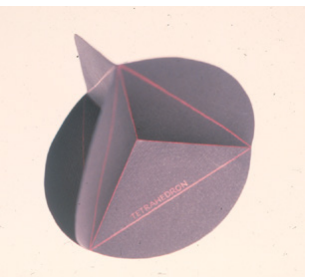
If we want to draw the face of the pyramid at any scale, it is just sufficient to draw the bisector of each of the Maraldi angles (an easy task with a rusty compass) and draw on this a perpendicular at any desired point marking the height of the face of the pyramid.

The Abul Wafa challenge has therefore been met.

Comparing the two methods we see that they are roughly equivalent in terms of number of steps to get the data, namely the length of the edge and the corresponding Malardi angle of each of the five Platonic solids. If the first method seems to imply less circles, it is because the construction of square ABCD or that of the perpendiculars through X, E, Y, F, B have not been included.

For further applications of the Maraldi angles to regular stellated forms as well as to other topics such as Pythagorean triples, gnomic golden rectangle series, the golden and the exponential spirals, the tuning of the monochord and other things, the reader is referred to:

http://www.gatewaycoalition.org/files/millennium_sphere/site/index.html



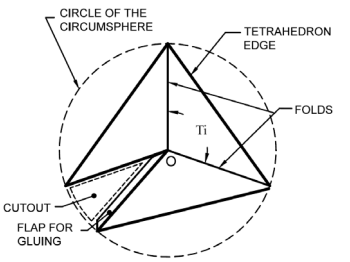
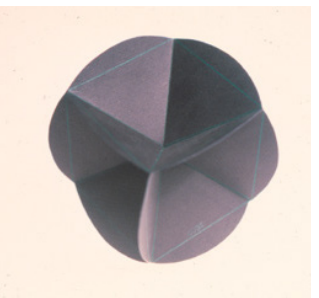


Figure 7a, left; 7b, right.



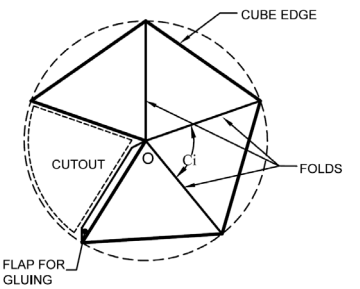
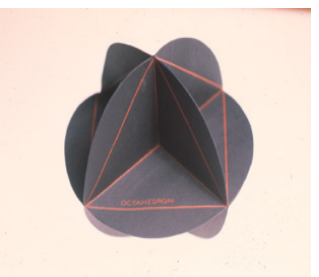


Figure 8a, left; 8b, right.



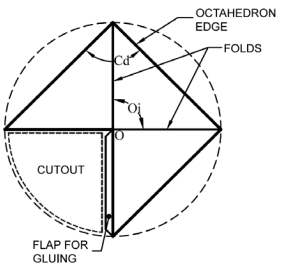
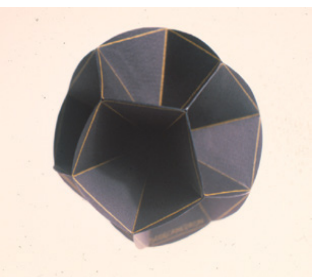


Figure 9a, left; 9b, right.



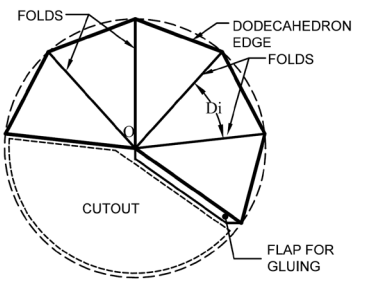
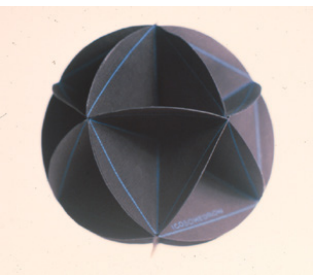


Figure 10a, left; 10b, right.



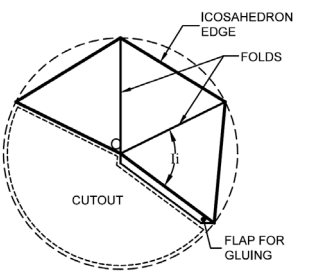


Figure 11a., left;11b, right.

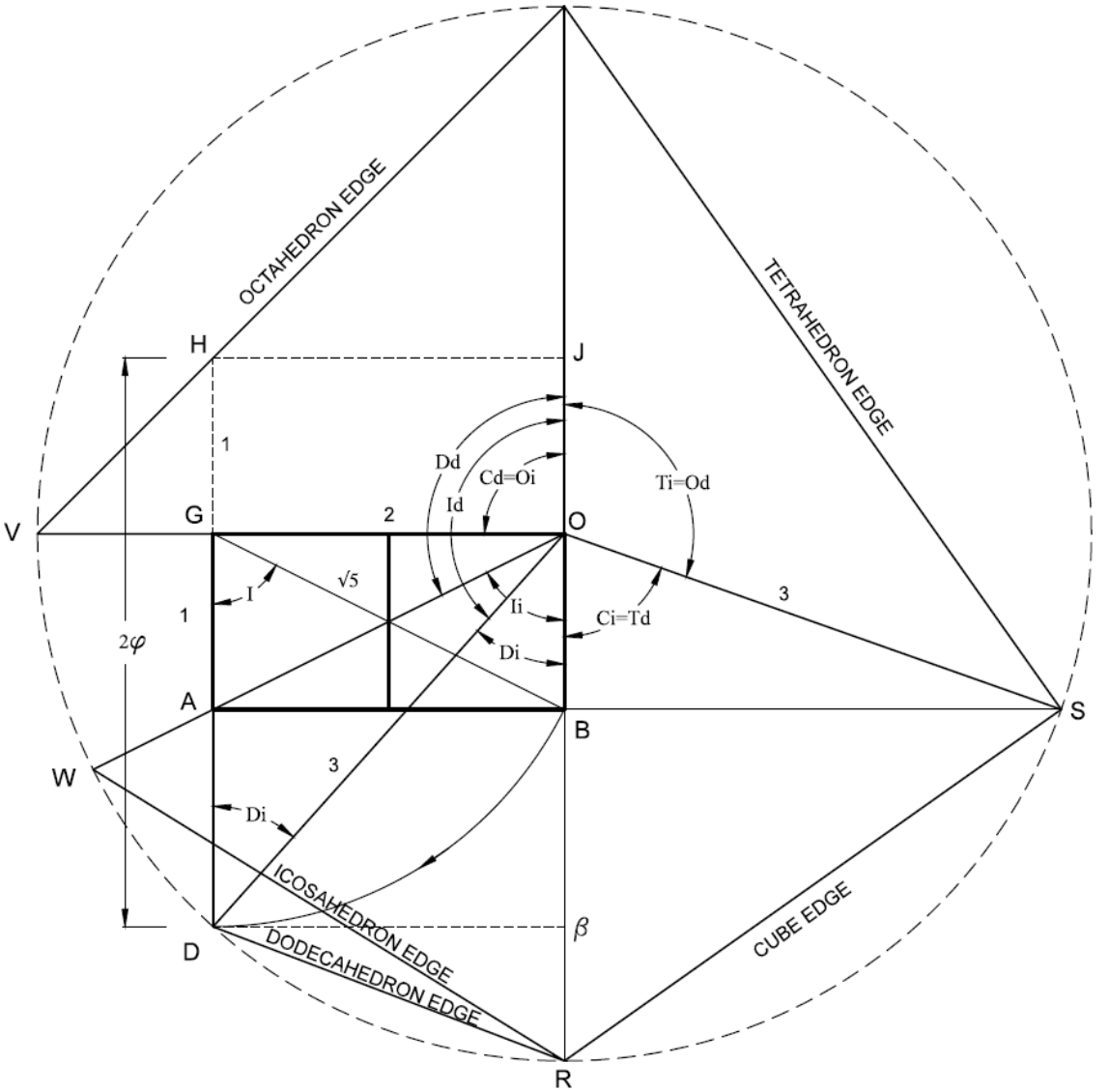


Figure 12. Construction of the Platonic Solid Edges.

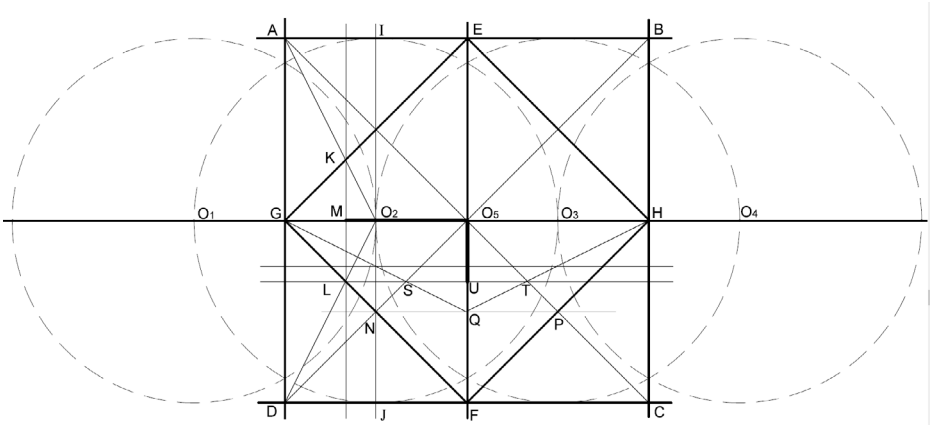


Figure 13. First Step Rusty Compass Construction of Platonic Solid Edges.

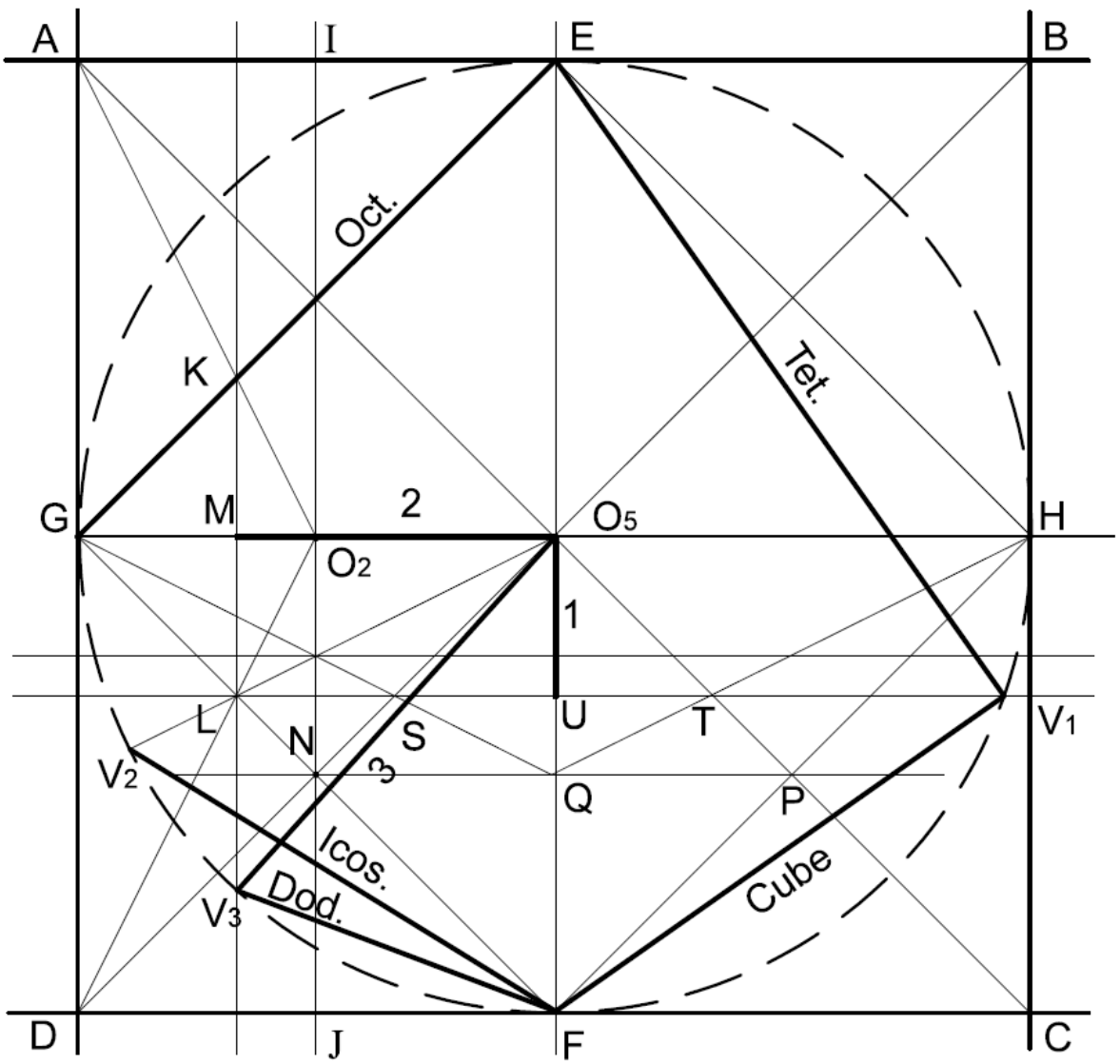


Fig. 14. Completion of the Construction of the Platonic Solid Edges.

In memoriam

Like with many such 'chance' meetings, whose significance becomes clear only much later, I came across Ernest McClain on Broadway, at the then Weiser bookshop, famous for its esoterica. This was in 1974, I believe, when I was working on the translation of some Hymns from the Rig Veda. I was foraging in a dark corner of the shop, looking for some references, when a man who had been floating about the place came to me and asked if I knew where he could find something on the Rig Veda! 'Well, that's precisely what I'm doing here,' I said. There was an instant connection and for the next few years we met frequently over dinner, together with my wife Katharine and his wife Augusta, and had animated conversations related to some aspects of Vedic Philology, mathematics, meter, or philosophy and sundry topics. When, in retirement, he moved to New England, where we visited him once, the exchanges, naturally, became less frequent but in those pre-Internet days, we kept in touch by correspondence. Though I cannot say I followed all his developments on tuning theory as it applied to Plato, the Rig Veda, or the other areas of scholarship he probed, nevertheless, I must say that I was always impressed by the rigor of his analysis, the depth and clarity of his thought on any topic to which he directed his attention.

Above all, however, it is his generosity of spirit, his indefatigable will to explain what he conceived, and the communicative enthusiasm he brought to any exchange with fellow scholars that will remain imprinted in my memory.

His intellectual life has been a brilliant display worthy of a Fourth of July of the mind. All who had the privilege of knowing him may be grateful for it and those who will meet him, in the future, through his writings will glean there examples of a splendid scholarship.

Jean Le Mée.
Emeritus Professor of Engineering
at the Cooper Union for the Advancement
of Science and Art,
New York City.

Acknowledgements

Special thanks to Dr. Marcin Balicki for handling the digitalization of the figures with his usual mathematical rigor and artistic sense and to Astrid Fitzgerald for her exquisite models of the Maraldian view of the Platonic forms (Figures 7a – 11a).

9-17

CASTLERIGG:
Stone or Tone Circle?

Leon CRICKMORE

This paper is intended as a tribute to the musicologist Ernest McClain (1918-2014) and his seminal insights, on the basis of which it will attempt to complement Alexander Thom's astronomical interpretation of the Castlerigg Stone Circle with a subsidiary musical one.

Castlerigg is a Stone Circle in the Lake District National Park, near Keswick, Cumbria, England. Impressively encircled by mountains and fells, it is thought to have been built about 3200 BC.

Figure 1. Castlerigg Stone Circle.

Probably the best available interpretation of the purpose of the Castlerigg Circle is that of Professor Alexander Thom[1], who considered it to have been an ancient astronomical observatory (see figure 2).

This paper celebrates the work of the musicologist Ernest McClain (1918-2014) and the seminal insights contained in his three books: *The Myth of Invariance*; *The Pythagorean Plato*[2], and *Meditations through the Quran*[3]. By applying certain of these insights which relate music, number and mythology, to the Castlerigg Stone Circle, it will attempt to complement Thom's astronomical interpretation with a subsidiary musical one. With regard to the ancient *quadrivium* of arithmetic, music, geometry and astronomy, McClain believed that he had found evidence of the use of similar numbers and mathematical constructs in Sumer, Babylon, India, Egypt, Palestine and Greece. As a result, he postulated the evolution and existence of a single global mathematical, musical, geometric,

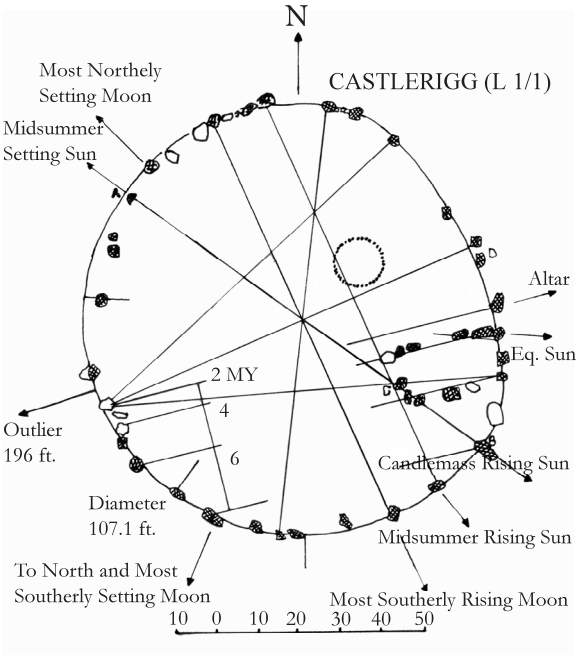


Figure 2. Alexander Thom's archeo-astronomical plan of Castlerigg. After Thom (1967).

astronomical and spiritual tradition. In particular, he has certainly uncovered much of the missing arithmetic underlying the concept of what is usually described as the '*Harmony of the Spheres*'.

At the beginning of *The Myth of Invariance*, McClain points out how the poets of the *Rg Veda* seem obsessed by a concern about 'the exact number of everything they encounter'. Later, he claims that 'the central geometrical image of the *Rg Veda* is the mandala of the "single-wheeled chariot of the Sun", harmonizing moon months with solar years and the signs of the zodiac', quoting, in support, from the *Rg Veda* 1.164.11 & 48:

1 Thom, A. (1967), *Megalithic Sites in Britain*, Fig.12.10, Oxford University Press.

2 As Plato recognised (*Republic*, 530 c-e), astronomy and music have long been viewed as sister disciplines.

3 Since the seventeenth century, however, modern science has gradually 'untuned the heavens' and demythologized our understanding of the universe. Knowledge has thereby tended to be reduced to evidence within the limits of statistical mathematics.

> 'Formed with twelve spokes, by length of
> Time, unweakened, rolls round the
> Heaven this wheel of during order,
> Twelve are the fellies, and the wheel is single.'

In his book *Beyond Measure*[4], the mathematician, Jay Kappraff generously devotes the whole of chapters 3 and 4 to McClain's speculations and the mathematics of music in general. On page 60, he presents a diagram of McClain's 'Mandala of the Single-Wheeled Chariot of the Sun'. For figure 3, this has been superimposed onto a later analysis of the site, supportive of Thom's conclusions, by D. P. Gregg[5]. Since, strictly speaking, Castlerigg is shaped like a flattened egg rather than a circle, the alignments are approximate. Anachronistically, the signs of the zodiac and the pitches of the equal-tempered scale are also included. Kappraff comments:

> 'This hypothetical tonal zodiac shows how a twelve-spoked mandala harmonizes music and astronomy at an abstract geometrical level although in ancient times neither the constellations nor the intervals of the chromatic scale divided the cycle equally'.

Notice how the co-ordinates of each diameter – and these include the important astronomical line between the Midsummer Setting Sun and the Candlemas Rising Sun (line A-B in figure 3) – correspond to musical tritones (augmented fourths/diminished fifths): c-f sharp; b-f etc[6]. Kappraff also points out that, when the twelve points of this diagram are joined in various ways: the sides any equilateral triangle define major thirds (e.g. C-E-G sharp/A flat); the sides of any hexagon define a whole tone scale (e.g. C-D-E-F sharp- G sharp-A sharp/B flat) ; the sides of any square define minor thirds (e.g. C-E flat-G flat/F sharp- A); while a connected twelve-pointed star displays a circle of perfect fifths - a tuning method dating back before recorded history, but still in use by modern piano-tuners today. Yet in the context of the Castlerigg Stone Circle, much more still needs to be said.

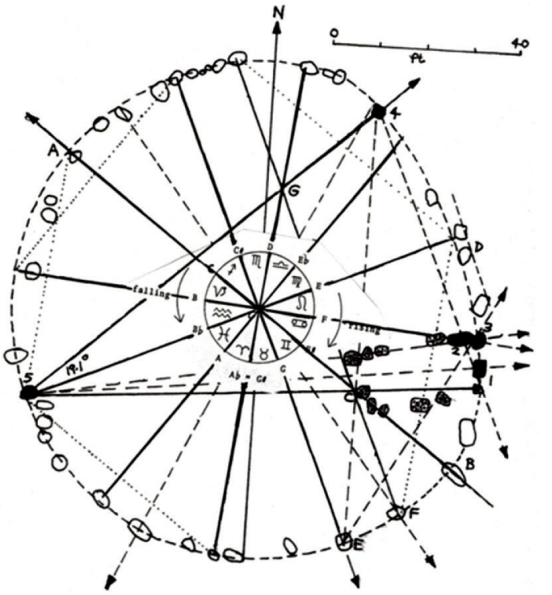


Figure. 3. McClain's mandala superimposed on Professor Gregg's diagram.

Equal temperament is a system of tuning in which the octave is divided into twelve equal semitones. Pitch is calculated as a logarithm of frequency. The whole system comprises a logarithmic spiral[7], which has to be flattened to form McClain's mandala. The ancients, however, had no means of measuring frequency accurately. They had no logarithms. Nevertheless, they skilfully calculated musical pitch by means of ratios of string or pipe length, having observed that two pitches an octave apart are in a ratio of 1:2. But the tritone, at the mid-point of the octave ($\sqrt{2}$ in equal temperament) is an irrational number. As such, it presented a serious problem to the ancients. There is a story that Pythagoras had Hippasus thrown into the sea for making such an alarming truth public. In mediaeval times, the tritone was still known as *diabolus in musica*. However, knowledge of logarithmic mathematics would not have been necessary to construct Castlerigg. John Hill[8] has demonstrated, through actual practice, how stone circles and henges could have been designed by

4 Kappraff, J., (2002) Beyond Measure: a Guided Tour through Nature, Myth and Number, *World Scientific*. See also Ch. 11.7, 'The Stone Circles'.

5 Gregg, D.P., (2013) *Castle Rigg Circle Analysis*, figure A9E in private communication.

6 Notice that there is no stone for E flat. Some commentators believe there were originally forty stones in all.

7 By a curious coincidence, a large spiral has been discovered incised onto one of the stones at Castlerigg. See Beckensall, S., (2002) *Prehistoric Rock Art in Cumbria*, Tempus Publishing, pp. 70-8.

8 Hill, J., Video: Ancient Knowledge – The sacred geometry behind British Stone Circles, (Web reference).

means of rope-folding; and McClain[9] has provided a number of analogous paper-folding exercises, by which a monochord can be tuned in various ways - even including equal temperament.

We must now return to the ancient system of tuning by means of perfect fifths - that is, in accordance with the twelve 'fellies' of 'the single-wheeled chariot of the sun', drawn as a continuous line so as to form a twelve – pointed star. The tuning ratio for a perfect fifth is 2:3. The Egyptians, whose arithmetic was usually carried out by means of reciprocals, known as Egyptian fractions[10], had a special hieroglyph for two-thirds. In *Gilgamesh*, dating from the second millennium BC, Urshanabi (Old Babylonian: *Sursunabi*), the boatman who sailed across the Waters of Death, is described as the 'Servant of Two-Thirds' – a reference to his service to Ea, patron of music, whose symbolic number was 40, which is two-thirds of that of the head of the pantheon, Anu, whose number was 60. Mesopotamian mathematicians, of course, counted sexagesimally.

Hilprecht has published four tables of divisors of 60^4, dating from about 2200BC Commenting on an anomaly in the first line of each of them, he writes:

> 'They all read: "1, 8,640,000 A-AN". The quotient being 2/3 of 12,960,000, we should rather expect 1½ instead of 1 as its divisor, for 12,960,000 divided by 3/2 is = 12,960,000 x 2/3.'

Hilprecht[11] suggests that we should probably regard this anomaly as 'an abbreviated expression well understood by the Babylonians' – especially, one might add, by musicians. For each of the numbers in these four tables is appropriate for use as a tone-number, defining a musical pitch in terms of hypothetical string or pipe length. This is also true of the numbers in the related standard Tables of Reciprocals, to be found so frequently in the eighteenth and nineteenth century scribal schools of Larsa, Ur and Nippur[12].

The tuning system of ancient Greece, attributed to Pythagoras and found explicitly in the writings of Plato is based on numbers in the form 2^p3^q. It produces what modern musicologists call 'Pythagorean tuning'. The Babylonian tuning system, on the other hand, additionally uses the prime number five, and is thus based on numbers in the form $2^p3^q5^r$[13]. It thereby produces 'Just' rather than 'Pythagorean' tuning. Every tone-number has to be an integer – hence the difficulty over the tritone, mentioned earlier. In order to define a tritonal pitch an approximation has to be employed. Tone numbers can be freely multiplied or divided by two (a procedure to which modern musicologists refer as 'octave equivalence'). Such a multiplication or division leaves the 'pitch-class' of any tone unchanged.

There is evidence from the seventh century BC[14] for the existence of a simple rule of thumb to enable tuning by a series of perfect fifths, namely: 'add or subtract one-third' from successive string or pipe lengths. Since in the Babylonian and ancient Greek tuning systems, the addition or subtraction of ratios called for multiplication or division, this rule can also be expressed as 'a tone-number multiplied by 4/3 and 2/3, successively'. To calculate the tone numbers for a series of thirteen perfect fifths, which McClain suggests might be the origin of various dragon and serpent myths, one must start with a reference value of $3^{12} = 531{,}441$ and apply the rule until one reaches $524{,}288 = 2^{19}$ – or, to be more strictly accurate – 262,144, which then has to be doubled in octave equivalence. 531,441: 524,288 constitutes the Pythagorean Comma, the difference in pitch between seven octaves in the ratio 2:1 and 12 perfect fifths in the ratio 3:2. It is therefore the amount by which the thirteenth tone to be generated will be out of tune. This calculation may perhaps be the origin of our

9 McClain, E., (1978) *The Pythagorean Plato*, Appendix IV, 'Introduction to the Monochord', Nicolas-Hays, York Beach, Maine, pp. 169-75.

10 Crickmore, L., 'Egyptian Fractions and the Ancient Science of Harmonics', *ICONEA* 2009-2010, eds. Richard Dumbrill & Irving Finkel, Co-Published by Gorgias Press LCC, pp. 1-8.

11 Hilprecht, H.V., (1906) *The Babylonian Expedition of the University of Pennsylvania*, Series A: Cuneiform Texts, Volume XX, Part 1, published by the Department of Archaeology, University of Pennsylvania, 25.

12 Robson, E., (2002), 'Words and Pictures: New Light on Plimpton 322', *American Mathematical Monthly*, 109: 105-20.

13 For further details see Crickmore, L., (2010) New Light on the Babylonian Tonal System, *ICONEA* 2008, eds. Richard Dumbrill & Irving Finkel, *ICONEA PUBLICATIONS*, London, pp. 11-22.

14 Nakaseko, K., (1957) 'Symbolism in ancient Chinese music theory', *Journal of Music Theory* 1 (2), pp. 147- 80.

modern superstition that thirteen is an unlucky number.

In ancient Greece, musical scales comprised seven octave species or eight-note modes. In Babylon, the corresponding scales were heptachords, the octave-note being analogous to the first day of a new seven-day week. In Egyptian mythology, Thoth was known as 'the Eighth which completes the Seven'. The best available evidence for the Just tuning of the seven tones of each of the seven Babylonian heptachords is probably the remarkable cuneiform text *CBS* 1766, which is headed by a seven-pointed star within two concentric circles[15]. Figure 4 shows the cuneiform tablet *CBS* 1766.

Figure 4. Cuneiform tablet CBS 1766.

CBS 1766 originally photographed by Hilprecht[16] and published in 1906 with the description 'an astronomical tablet from the Temple Library', was saved from oblivion and republished with a mathematical interpretation by Horowitz[17] in 2006. A year later, Waezeggers and Siebes[18], noticing that around the seven-pointed star on this tablet, there stand the names of seven of the nine musical strings listed in both Sumerian and Akkadian in *UET* VII 126, proposed an alternative musical interpretation of the text as a visual aid for the tuning of seven heptachords on a seven-stringed instrument. The names of the seven strings also appear in *CBS* 10996. The present author has subsequently published his own musical and mathematical interpretation of the seven-pointed star (reference in footnote 15). While the date and provenance of *CBS* 1766 still remain uncertain, Joran Friberg, who possesses a deep understanding of the entire Old Babylonian mathematical corpus, has pointed out that its unusual format is similar to that of *Plimpton* 322.

Robson[19] reports that the data on *Plimpton* 322 'is laid out in a landscape-orientated table with the final heading MU.BI.IM ('its name') for the numerical data'. Importantly, she adds that these were 'formal features of administrative tables from Larsa during the period of rigorous standardisation in 1790-80 B.C.E.'.

In the light of all this, Friberg concludes (reference in footnote 15) that '*CBS* 1766, like *Plimpton* 322, in all probability is an Old Babylonian text from Larsa, dating from the period 1790-1780 B.C.'. He also acknowledges that it is 'a text with a mixed topic' – that is, both a description of how to draw a heptagon by means of a continuous line, analogous to the continuous line forming the twelve-pointed star mentioned by Kappraff and corresponding to a cycle of perfect fifths, and also a description of how to tune heptachords, though the tuning he suggests is Pythagorean rather than Just. My musical and mathematical interpretation of *CBS* 1766 is shown in figure 5.

The seven-pointed star, originally within two concentric circles in *CBS* 1766 is shown at the centre in red, though the drawing on the tablet is rougher. On the tablet, the points of the star are numbered from 1-7 and are labelled as follows: 1 the foremost; 2 the next; 3 the third thin; 4 the one made by (the god) Ea; 5 the fifth; 6 the fourth behind (that is, from the other end of the instrument); and 7 the third behind. With two additions, these names also appear in the list of

15 Crickmore, L., (2008) 'A Musical and Mathematical Context for CBS 1766', *Music Theory Spectrum*,Volume 30, no. 2 : 327- 338, and Friberg, J., (2011) 'Seven-Sided Star Figures and Tuning Algorithms in Mesopotamian, Greek and Islamic Texts', *Archiv fur Orientforschung* 52, pp. 121-55.

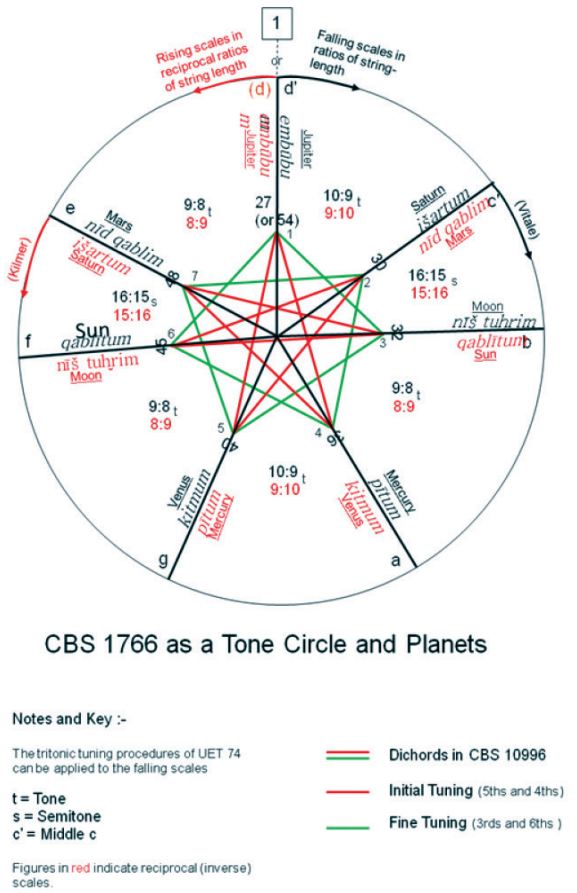
16 Hilprecht, V., (1903)*Explorations in Bible Lands During the 19th Century*, Philadelphia, Molman, A.J.

17 Horowitz, W., (2006) 'A Late Babylonian Tablet with Concentric Circles from the University Museum (CBS 1766)' *Journal of the Ancient Near East Society*, 30, pp. 37-53.

18 Waerzeggers, C., and Siebes, R., (2007), 'An Alternative Interpretation of the Seven-Pointed Star in CBS 1766', *Nouvelles Assyriologiques Breves et Utilitaires* (*N.A.B.U.*) 2, pp. 43-5.

19 Robson, E., (2003) 'Tables and tabular formatting in Sumer, Babylonia, and Assyria, 2500 B.C.E. – 50 C.E.' in Campbell Kelly, M. et al (eds. *The History of Mathematical Tables*, Oxford University Press, pp. 18-47.

nine strings in *UET* VII 126 /*Nabnitu* XXXII a bilingual lexical text – the earliest evidence we have about string/pitch names - in which



CBS 1766 as a Tone Circle and Planets

Figure 5. The cuneiform tablet CBS 1766.

Sumerian, but as 'Ea-created' in Akkadian. *CBS* 1766 also displays two columns of the numbers 1-7 arranged in pairs: (1) 2-6; 6-3; 3-7; 7-4; 4-1; 1-5; 5-2, and (2) 1-7; 5-4; 2-1; 6-5; 3-2; 7-6; 4-3. Its format then includes several empty columns, suggesting that the tablet might have been intended as an exercise to be completed by a student. There is also some further writing on the tablet which has so far proved indecipherable. However, Friberg's diagram (Figure 6.1 in *op.cit.* footnote 15) shows the possible beginning of the heptachordal names *išartum* and *kitmum*, which would support my reconstruction (*op.cit.* footnote 15). Friberg points out that a seven-pointed star figure can be drawn by the use of an uninterrupted chain of straight lines through the pairs of points listed in the first column; while joining each of the paired points in the second column produces a regular heptagon. *CBS* 10996 also contains two columns of the numbers 1-7, as follows: (1) 1-5; 2-6; 3-7; 4-1; 5-2; 6-3;7-4 and (2) 7-5; 1-6; 2-7; 1-3; 2-4; 3-5; 4-6. All the pairs in the first column from *CBS* 1766 appear in *CBS* 10996, though in a different order. In *CBS* 10996, the fourteen pairs of integers are preceded by the logogram SA, meaning 'string'. In her pioneering work on Babylonian music, Kilmer[20] interprets the even numbered lines between 11 and 24 - my column (1) – as a means of tuning by perfect fourth and perfect fifths each of the seven Babylonian heptachords. Smith[21] and Kilmer have interpreted the numbers in the second column as a method of further fine tuning, so as to sweeten the intervals of a third and bring the original Pythagorean tuning closer to Just tuning. This process is indicated by the pairs of green lines in figure 5.

When transcribing the seven heptachordal scales into modern letter notation, Kilmer presents them chromatically, within a single octave, in a manner described by musicologists as 'thetically'.

There is also another cuneiform tuning – or, more strictly speaking – modulating text: UET VII 74. When it was originally published by Gurney[22] in 1968, everyone assumed that the scales defined by the text were rising. However, later, when the musicologist Vitale[23] argued that the scales should be falling, the Assyriologist, Krispijn[24], proposed an amendment to line twelve, where a damaged sign, previously read as NU SU ('end of sequence') ought to have been read as *nusuḫ*(*u-um*), meaning 'to tighten'. The text now fell into two sections, the first involving the 'tightening' of certain strings, while the second involved their 'loosening'. The tablet therefore was describing a method[25] for modulating through all the seven heptachords,

20 Kilmer, A.D., (2001) 'Mesopotamia' *The New Grove Dictionary of Music and Musicians*, 2nd edition, 16, pp. 480-7.

21 Smith, J.C., and Kilmer, A.D., (2000) 'Continuity and Change in the Ancient Mesopotamian Terminology for Music and Musical Instruments', *Orient Archaeologie* 7, pp. 113-9.

22 Gurney, O.R., (1968) 'An Old Babylonian Treatise on the Tuning of the Harp', *Iraq* 30, pp. 229-33.

23 Vitale, R., (1982) 'La musique suméro-akkadienne, Gamme et notation musicale', *Ugarit-Forschungen* 14, pp. 241-65.

24 Krispijn, Th.J.H., (1990) 'Beitrage zur altorien-alischen Musikforschung I', *Akkadica* 70, pp. 1-27.

25 For details of the method see Crickmore, L., *op.cit.*

by means of sharpening or flattening one of the components of the tritone in the heptachord to which a nine-stringed instrument was presently tuned. Gurney published a revised transliteration in 1994[26]. But all this created a problem for Kilmer, since if the scales were falling, it would appear that she had been mistaken over her identification of the names of the heptachords. The present author[27] has suggested a resolution of this problem: that the Babylonian heptachords – unlike the ancient Greek octave species or our modern diatonic scales – must have been modal patterns of tones and semitones which remain unchanged, regardless of the direction of the scale. Greek modes and modern diatonic scales, on the other hand, are ladders of pitches, the modal patterns of which differ according to whether the scales are rising or falling. In figure 5, the names of the falling heptachords are shown in black, and their scales are to be read to the right; the rising heptachords are named in red, and the pitches of their respective scales have to be read to the left – thus reversing the directions shown in figure 3[28]. Modern letter-name notation has been used to define the relative pitches of the notes of each heptachord. This procedure was chosen for the sake of simplicity, since this makes it possible to define all seven heptachords using the white keys of a piano only. Musicologists call such a form of presentation 'dynamic', in contrast to Kilmer's 'thetic' or chromatic notation.

In adding the names of the seven Chaldean planets to the diagram, they have been ordered according to their diminishing orbital cycles to match (though not proportionately) the shortening of the musical string in the falling scales. Finally, the tone-numbers and the ratios of string-length defining each of the pitches in Just tuning have been included. It should be noted that whereas in Pythagorean tuning all tones are 9:8, in Just tuning there are two forms of tone: 9:8 and 10:9, and the semitone is 15:16.

A further question now arises: how can the heptagonal geometry of *CBS* 1766 be related to stone circles and henges, and in particular to the geometry of Castlerigg and the 'twelve-spoked wheel of the chariot of the sun'? Two books by John Michell can help: (1) *The Dimensions of Paradise*[29] with the subtitle '*the proportions and symbolic numbers of ancient cosmology*' and (2) *How the World is Made: the Story of Creation according to Sacred Geometry*. The opening sentence of the former reads:

> 'Ancient Science was based like that of today on number, but whereas number is now used in the quantitative sense for secular purposes, the ancients regarded numbers as symbols of the universe, finding parallels between the inherent structure of number and all types of form and motion'.

When reflecting on the differences between modern and ancient mathematics, one must additionally bear in mind that while the fulcrum of modern mathematics is zero, with positive numbers extending in one direction and negative numbers in the other, ancient mathematics knew no zero. Its fulcrum was the monad. From unity, numbers were conceived as an arithmetic progression upwards: 2, 3, 4, 5, 6.... and a harmonic progression downwards: 1/2, 1/3, 1/4, 1/5, 1/6 ... Michell's book is full of examples of how twelve combined with seven has been used to symbolize unions of differences such as heaven and earth, solar and lunar years, and mind and body.

Three, the first odd number – unity was not considered as a number in itself, but rather as the monad from which all other numbers are generated – plus four – the first square number – add up to seven, while three multiplied by four is twelve. The latter book, Michell's last and probably most beautiful, contains a section headed: *'Twelve and Seven: The Supreme Numerical Marriage'* (pp. 219-231).

Musicians are familiar with this mystery

in footnote 13, pp. 14-5, figures 5 and 6, and *op.cit.* in footnote 15, pp. 336-7, Appendix.

26 Gurney, O.R., (1994) 'Babylonian Music Again', *Iraq* 56, pp. 101-6.

27 Crickmore, L., (2010) *op.cit.* footnote 10.

28 In the modern world, musical pitch is defined by frequency and usually expressed in Herz (Hz.), signifying vibrations per second. Scales are conceived as rising – a convention adopted by both McClain and Kappraff. But since the ancients defined their scales in terms of ratios and reciprocals of string-length, it is sometimes preferable to reverse this procedure.

29 Michell, J., (1988) *The Dimensions of Paradise* Thames & Hudson.

through its expression in the seven white and five black keys of a modern keyboard.

Amongst Michell's illustrations is figure 196, in which a twelve-pointed figure is created by arcs, while the eighty-four divisions of a 7-pointed star are drawn by means of straight lines. At the end of this section, the caption to figure 207 reads: 'Seven and Twelve united and the Heavenly City (i.e. the New Jerusalem) revealed'. The earliest features of the Stonehenge Circle are the Aubrey Holes (c.3000 BC). Figure 6 indicates how this set of fifty-six holes has been interpreted by D.P. Gregg as the basis for both a heptagon and its seven-pointed star.

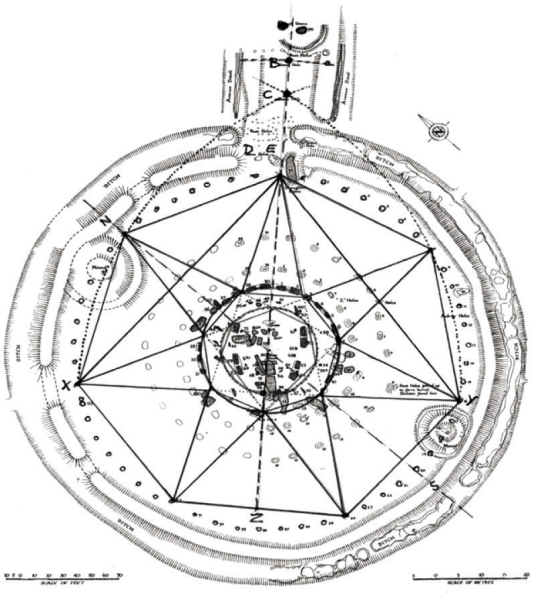


Figure 6. Gregg's Aubrey Circle heptagon to match CBS 1766.

The heptagon shown in figure 6 is only one of a number of geometrical interpretations of the Aubrey Holes at Stonehenge by D P Gregg[30].

The last of my examples of a geometrical marriage of seven and twelve comes from the Byrom Collection. The *Byrom Collection* comprises 516 geometrical drawings of which

John Byrom (1691-1763), a Jacobite sympathiser, was the custodian. These drawings were discovered by Joy Hancox, while she was writing a biography of Byrom. She has since also published a general book about them[31]. Figure 7 shows drawing number 291. Hancox interprets it as the geometry underlying an engraving by Theodore de Bry: *The Theatre of Human Life* (*Theatrum Vitae Humanae*, 1596). Frances Yates has reproduced this engraving in her book *Theatre of the World*, where she likens it to the speech of Jaques in Shakespeare's *As You Like It*: 'All the world's a stage....'. The geometrical drawing consists of two interlinked circles, the upper of which contains a seven-pointed star. In the lower circle, there is a set of diameters measuring: **30**, **32** **36**, **40**, (41, 42,) **45**, **48**, (51, **54** = **27** x 2, 56, and **60** = **30** x **2** units[32]. The numbers in heavy type appear as tone-numbers in my interpretation of *CBS* 1766 (Fig. 4), where, as ratios of string-length, they define the Babylonian heptachord *išartum* (falling).

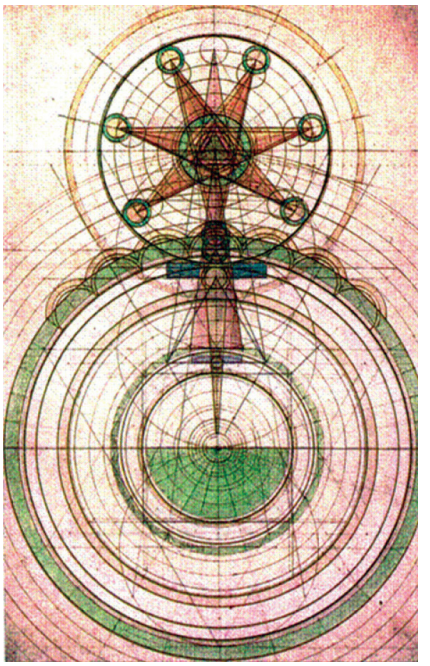


Figure 7. Byrom Collection 291: Reproduced with permission for purposes of scholarship ©Joy Hancox, The Byrom Collection, Cape, 1992.

The diameter numbers 42, 51 and 56 are Diophantine approximations for √2, (the musical tritone) used geometrically[33] to estimate the

30 Gregg, D.P., (2014) The Stonehenge Codes: A New Light on Ancient Science , Green Man Books, pp. 22.

31 Hancox, J., (1992), *The Byrom Collection*, Jonathan Cape.

32 Further details can be found in Crickmore, L., (2009). Appendix: *The Byrom Collection*', Harmonic Mythology: Nine Interdisciplinary Research Notes, *Arane*, Volume 1, *ICONEA* Publications.

33 Theon of Smyrna, *The Mathematics Useful for*

approximate length of the diameters of squares of sides 30, 36 and 40 respectively.

Finally, readers who are in sympathy with the mode of thinking underlying Ernest McClain's tone mandalas and tuning systems might care to try the following experiment. Take any pair of figures 2-7. Superimpose them in such a way that the centres of their circles coincide. Place a light behind them, and then rotate some point of symbolic significance in the lower diagram into the 12 o'clock/pitch D/ North position in the upper (or, in the case of Castlerigg to the first stone to the right of North), in the manner that seems most appropriate. Using these diagrams playfully in this way - as a kind of Jungian mandala for contemplation - this exercise can sometimes highlight fresh correspondences and connections for their viewer. Some of these new insights may even be contradictory, for as Joscelyn Godwin[34] admits, when writing about the Harmony of the Spheres:

> 'the way of understanding lies through finding the appropriate standpoint from which to consider every theory. Without that readiness to shift attitudes and to keep one's mind open to several different and sometimes contradictory levels of being, any study of planet-tone or zodiac-tone theories is limited to an academic approach which regards its subject as historically instructive but devoid of intrinsic truth'.

For readers of a more reductionist frame of mind, however, insights of the kind generated by the exercise described are likely to be deemed wishful thinking, at best, and at worst, sheer madness.

In another book[35], Godwin has traced the development of the concept of the Harmony of the Spheres through texts from the time of Plato up to the present day. He summarizes this process as 'a many-faceted commentary on the passage in Plato's *Timaeus*[36] that describes how the Demiurge fashioned the World Soul'. Plato's World Soul was, in essence, the arithmetic ratios of string-length that define the ancient Greek Dorian octave species in Pythagorean tuning. A thousand years earlier, the first seven tones of this mode, in Just tuning, already defined the Babylonian heptachord *nid qablim*. Ernest McClain's life's work and contribution to the debate over the Harmony of the Spheres has been summed up as follows[37]: 'He hypothesized prime number harmonics as the key driver and shaper of historical mythology'.

Conclusions

1. As Alexander Thom has suggested, the Castlerigg Stone Circle was in all probability an ancient astronomical observatory.

2. As Plato recognised (*Republic*, 530 c-e), astronomy and music have long been viewed as sister disciplines.

3. Since the seventeenth century, however, modern science has gradually 'untuned the heavens' and demythologized our understanding of the universe. Knowledge has thereby tended to be reduced to evidence within the limits of statistical mathematics.

4. Nevertheless, as the title of Kappraff's book *Beyond Measure* suggests, some of the most important dimensions of human life – including love, beauty and subjectively significant coincidences – are often ambiguous and rarely susceptible to the level of precision required by scientific method.

5. It seems unlikely that Ernest McClain's ideas will ever be considered mainstream. But for those of us who believe that at the core of his work there is some intrinsic truth, and who are minded to continue further to explore his hypothesized prime-number mythologies, a new name for such a developing body of interdisciplinary research will be required. Graham Fearnhead[38] has suggested: 'Harmonic Mythology'.

Understanding Plato, (translated from Greek/French edition of J. Dupuis by R & D Lawlor), San Diego (Cal.), Wizard's Bookshelf, (1979).

34 Godwin, J., (1987), *Harmonies of Heaven and Earth*, Thames & Hudson: 124-5.

35 Godwin, J., (1993), *The harmony of the Spheres: a Sourcebook of the Pythagorean Tradition in Music*, Inner Traditions International, Rochester, Vermont.

36 *Timaeus* 34-7. See also Crickmore, L., (2009), 'A possible Mesopotamian origin for Plato's World Soul', *Hermathena* 186, pp. 5-23.

37 Ernest Glenn McClain, Obituary, *New York Times*, May 6, 2014. See Crickmore, L., (2009 Research Note IX,)'Harmonic Mythology: Nine Interdisciplinary Research Notes', *ARANE*, Volume 1, pp. 63-4.

38 See Crickmore, L., (2009 Research Note IX,) 'Harmonic Mythology: Nine Interdisciplinary Research Notes', *ARANE*, Volume 1, pp. 63-4.

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Leon Crickmore was born in 1932 and educated at King's College, Cambridge, and the University of Birmingham. After working in further education, he became Dean of the Faculty of Arts at the North East London Polytechnic, and later HM Staff Inspector of Music. In the latter role, he was involved in the establishment of music as a subject in the National Curriculum, and of degree courses in the Conservatoires. In 1997 he was awarded an honorary fellowship of the Royal Scottish Academy of Music and Drama. Since 2000, he has been engaged in interdisciplinary research into the ancient science of harmonics, with particular reference to Babylonian and Greek tonal systema.

19-32

ANCIENT HARMONIC LAW

Jay KAPPRAFF

Abstract

The matrix arithmetic for ancient harmonic theory is presented here for two tuning systems with opposite defects: 'Spiral fifths' as presented by Nicomachus, a Syrian Neo-Pythagorean of the second century A.D., and Plato's 'Just tuning' as reconstructed by the ethnomusicologist, Ernest McClain, from clues preserved by Nicomachus and Boethius (6th century AD). These tables lie behind the system of architectural proportions used during the Renaissance, and their basic ratios now pervade modern science as the foundation of a 'string theory' formally presented first in Euclid. Calculation employs an early form of a log table governed by vectors of 2-3-4 in the first, and by 3-4-5 in the second.

The square root of 2 plays a central role in integrating these systems governing 12-tone theory from the perspective of four primes--2, 3, and 5 generate all ratios under the overview of 7— as disciplined 'self-limitation' within a 'balance of perfect opposites'.

Introduction

Nicomachus, writing about 150 AD, forms one of the best links to what survived from his day about Greek theory of numbers and music [1,2]. I shall describe how the sequence of integers shown in Table 1, and attributed to Nicomachus, defines perfect octaves, fifths, and fourths, the only consonances recognized by the Greeks, and lies at the basis of ancient tuning systems preserved by Ptolemy. A second table inferred by Plato but brought to light by the musicologist, Ernest McClain [3,4,5], will be shown to be the basis of the Just scale, another ancient musical scale. I shall refer to this table as the *Harmonic Matrix*, or simply as HM. In his books and papers, McClain has made a strong case for music serving as the *lingua franca* of classical and sacred texts, providing plausible explanations of otherwise too difficult to understand passages and providing metaphors to convey ideas and meaning. In this paper I will focus primarily on the mathematics and music at the basis of ancient musical scales, and this paper will serve as an introduction to the work of McClain. An original contribution of the author is a collaboration with McClain to create an algorithm for the construction of an approximation to √2 good to six decimal places using the musical scale. This approximation was used in Vedic India with a geometric derivation. The author will also demonstrate that the musical scale functions as a vector system simplifying musical computations.

1	2	4	8	16	32	64	...
	3	6	12	24	48	96	...
		9	18	36	72	144	...
			27	54	108	216	...
				81	162	324	...
					243	486	...
						729	...

Table 1. Nicomachus' table for expansions of the ratio 3:2 (as relative frequency).

The construction of ancient musical scales will be shown to be based strongly on primes: 2, 3, 5 and 7. There were no easy notations for fractions in ancient civilizations so the problem was to represent ratio in terms of whole numbers. To this Plato states:

> 'For surely you know the ways of men who are clever in these things. If in the argument someone attempts to cut the one itself [i.e., use a fraction], they laugh and won't permit it. If you try to break it up into small coin, they multiply…'
>
> *Republic* 525

Ancient musical scales must be assumed to be elegant solutions in smallest integers to this problem.

Two of Plato's foremost concerns were

limitation, preferably self-limitation, and balance of opposites. McClain states [5] that

> 'In political theory as in musical theory, both creation and the limitation of creation pose a central problem. Threatening infinity must be contained.
>
> What musicians have shown possible in tones, the philosopher should learn to make possible in the life political.'

As for the balance of opposites, in the *Republic*, Plato states:

> 'Some things are apt to summon thought, while others are not…Apt to summon it are those that strike the sense at the [same] time as their opposites.'
>
> *Republic* 524

One of the themes of this paper will be to show how the musical scale expresses the concepts of lmitation and balance of opposites.

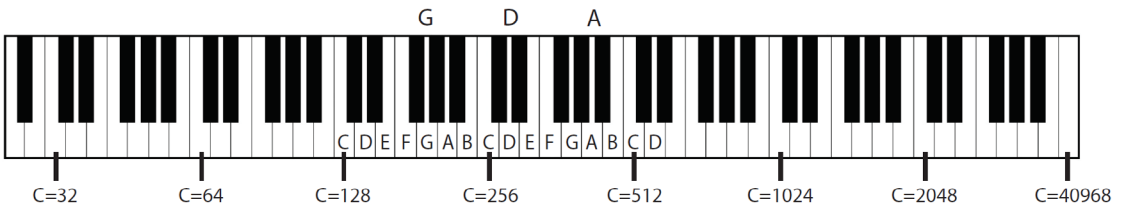


Figure 1. The piano keyboard

Our story begins with a look at the 88 keys of the piano shown in figure 1. Three tones, A, D, and G, have been highlighted. D is located at the center of symmetry of the black and white pattern of keys with G and A seven tones below and above D. The interval from D to G spans five tones DCBAG called a *falling fifth* while from D to A is a *rising fifth* (DEFGA). In the opposite direction these intervals are perfect fourths (DEFG and DCBA). The pattern of black and white keys repeats every 12 tones, called the *chromatic scale*, with the white keys repeating every seven keys and assigned the tone names DEFGABCD', the interval from D to D' being called an octave, and with the black keys augmented or diminished by sharps (♯) and flats (♭) to insert semitones between wholetones. The white keys produce heptatonic scales in seven different modes depending on the reference pitch. The black keys naturally provide five pentatonic modes, repeating on the sixth tone. Because of the periodicity of the musical scale, any tone can be chosen as the first or fundamental, and since any tone is perceived by the ear to be identical when played in different octaves, each tone of the scale refers to a pitch class of tones differing by some number of octaves.

The D mode, or ancient *Phrygian* mode, was the preferred self-symmetric scale of ancient times (notice the symmetric pattern of the keys on both sides of D in figure 1), and I will represent all scales in the D mode. The heptatonic scale beginning on C: CDEFGABC is the famous do-re-mi... scale, and common reference now in Western civilization. The intervals of the piano are *equal-tempered* so that beginning on any tone, after twelve perfect fifths one arrives again at the same tone seven octaves above or below in pitch.

Because of the periodicity of the chromatic scale, it can be comfortably represented on a tone circle as shown in figure 2 with semitone intervals between adjacent tones, and with pairs of semitones equaling a wholetone. Each semitone of the equal-tempered scale is worth 100 cents now assigned on a logarithmic scale with 1200 cents to the entire octave. The relationship between cents and frequency is discussed in the appendix. The equal-tempered tritone located at 6 o'clock has a relative frequency of √2. Proceeding clockwise around the circle leads to a rising scale while a counter-clockwise rotation leads to a falling scale. The numbers 0-12 on the clock positions number the tones of the chromatic scale, and they can be thought of as numbers in a mod 12 number system where any multiple of 12 can be added to one of the integers to get another tone in the same pitch class. I will refer to the numbers on the face of the clock as the *digital roots* of the pitch class more commonly called *principal values* in mathematics.

Notice that the digital roots of two tones symmetrically placed around the tone circle sum to 12. Such tonal pairs are called *complementary*.

Arithmeticization of the ancient musical scale begins with the numbers 1, 2, 3, and 4. The reference length of a vibrating string is arbitrarily assigned the value of 1 unit which is taken to be the *fundamental* tone when bowed or plucked. If the length of the fundamental is multiplied or divided by any power of 2, the resulting tone sounds to the ear identical in implied harmonic function to the fundamental by some number of octaves. In other words, shortening the string by half gives a tone one octave higher in pitch while doubling the length of the string results in a falling octave. Therefore each tone is a representative of a pitch class of tones all differing by a multiple of 2 in string length. That all tones in a pitch class sound alike to the ear in harmonic function is considered to be the miracle of music. In figure 3, if you place your finger at the 2/3 point on the string and sound the string it gives rise to a perfect fifth; lengthening the string by one half results in the interval of a falling fifth. If you shorten the string by one fourth a perfect fourth arises. The modern concept of relative frequency is reckoned as the inverse of the relative string length. A new pitch class at the interval of a fourth or fifth requires a new factor of 3. It is therefore easy to comprehend why the musical scale served well as a metaphor for the act of creation: 'Deity' commensurate with '1', doubling to 2 (the female number) as the 'Great Mother,' and both of these adding to 3 in 'procreation' of a 'male' son. Each new tone in this sequence requires another factor of 3, constituting the preferred 'Pythagorean tuning' of ancient Greece.

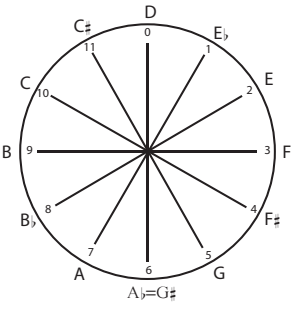


Figure 2. The Tone Circle.

2. The Tetrapolarity of the Ancient Musical Scale.

How does a new pitch class emerge from the number 3? It can do this in two ways: by relative string length or relative frequency. Let's first consider relative string length. In the ancient world, lacking the concept of a fraction, the integer 3 had four musical interpretations. First of all, 3 can be

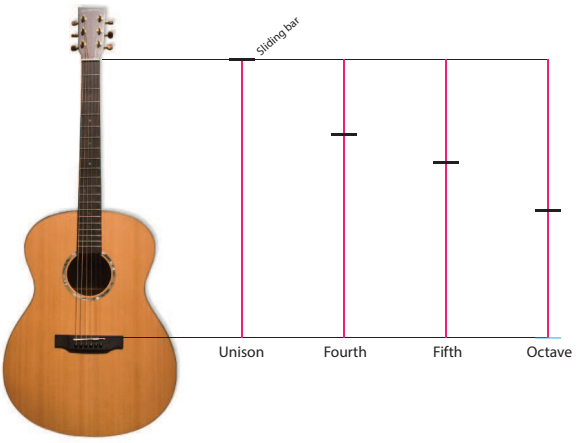


Figure 3. A sliding bridge on a monochord divides the string length representing the fundamental tone into segments corresponding to the perfect fifth (2:3) fourth (3:4) and octave (1:2).

associated with the ratio 3:1. This ratio is equivalent, after suppressing octave powers of 2, to relative string lengths 3/2 and 3/4 of a falling perfect fifth or a rising fourth. In this case, taking D to be the fundamental, 3:1 is represented by the pitch class G (a fifth below D). Alternatively, 3 can be associated with the ratio 1:3 which in turn corresponds to relative string lengths 2/3 and 4/3, a rising perfect fifth or a falling fourth, i.e., inverse ratios. Now with D as the fundamental, 3 is represented by pitch class A. So the integer 3 represents two pitch classes and takes on the four inverse interpretations : 3/2, 3/4, and 2/3, 4/3. Since the modern concept of relative frequency is the inverse of relative string length, the second interpretation relates to relative frequency. Here 3/2 and 3/4 are rising fifths and falling fourths respectively, while 2/3 and 4/3 are falling fifths and rising fourths respectively. The second interpretation is more familiar to musicians, and I will follow it for most of this paper. These four interpretations: rising scale, falling scale, relative string length, and relative frequency can be confusing to someone trying to orient themselves with regards to interpreting the musical scale in terms of integers.

The situation is analogous to the paradox of the relative motion of a pair of railway trains. The question one asks is: am I moving forward and the other train stationary, or am I stationary and the other train moving backwards? This paradox is then made more confusing by witnessing it in

a mirror in which forward and backwards reverse leading to the tetrapolarity: moving-stationary, and forward-backward.

3. The Greek scale.

Placing into the 2:1 octave, the fundamental D (1), octave D' (2), fifth A (3/2) and fourth G (4/3) in scale order and using relative frequency gives the rising scale:

D	G	A	D'
1	4/3	3/2	2

Multiplying by the common denominator, 6, results in the integer scale:

D	G	A	D'
6	8	9	12

A and G reverse when relative string length is used instead of relative frequency. Notice that these ratios are represented in Table 1 with the integers along each row expressing the ratio 1:2, the columns 2:3, and the right leaning diagonals 4:3. These rising fifths and fourths differ inaudibly in pitch from the piano fifths and fourths described above. In fact, equal-tempered relative frequencies of A and G are 700 cents and 500 cents, respectively, while corresponding 3:2 and 4:3 ratios, are 702 cents and 498 cents, respectively, slightly overshooting and undershooting. This extremely small discrepancy between tonal values is referred to in musical theory as a *comma*. Notice in figure 2 that the twin tones G and A occupy symmetric positions on the tone circle as do any pair of tones with reciprocal ratios. In this case, the symmetry is the result of the geometric sequence : 1/3, 1, 3 in which D as 1 is the geometric mean of AG as 3 and 1/3 respectively following harmonic law which states that *as relative frequencies multiply, intervals add*, i.e., 3 x 1/3 = 1 whereas 7 + 5 = 12.

In an alternative representation, let's derive the twin tones A and G from the middle at the fundamental D by moving along the tone circle 7 semitones in a clockwise direction from D to A and 7 counterclockwise semitones from D to G. A and G lies in the pitch classes: 3:1(A) and 1:3 (G).

G	D	A
1/3	1	3

1	3	9
4	6	9
8	6/12	9

In row 2 the three tones are represented in their respective pitch classes. In row 3, multiplication by 3 eliminates fractions. When the terms in column 3 are multiplied by powers of 2, the three integers of row 4 are obtained which duplicates column 3 of Table 1. When the fundamental is doubled to 12 in order to enclose or 'seal' the largest integer, 9, in the 6/12 octave, the twin tones multiply by powers of 2 to 8 and 9, with ratio 9:8, without changing pitch class. Rearranging the integers of row 5 in scale order we have again:

D	G	A	D'
6	8	9	12

This is read as a rising scale from left to right or a falling scale from right to left. Again, rising and falling, and A and G reverse when considering the numbers to be relative string length. Greek musical theory was founded on this formula. Observe from Sequence 2 that for the twin tones, A functions as the arithmetic mean of the octave DD' while G is the harmonic mean, reversing when the integers represent relative string length. The integer 9 is the arithmetic mean of 6 and 12 because, 9-6 = 12-9 whereas, 8 is the harmonic mean because: 8-6/6 = 12-8/12. Plato gave great significance to the Greek formula when he stated:

> '…In the potency of the mean between these terms (6,12), with its double sense, we have a gift from the blessed choir of the Muses to which mankind owes the boon of the play of consonance and measure, with all they contribute to rhythm and melody.'
>
> [*Epinomis* 991]

Also it seemed significant that the fundamental is 6, a perfect number, i.e. the sum of its factors 1, 2, 3 regarding which Plato states:

> '...for a divine birth there is a period comprehended by a perfect number.'
>
> [*Republic* 546]

4. The Nicomachus Table

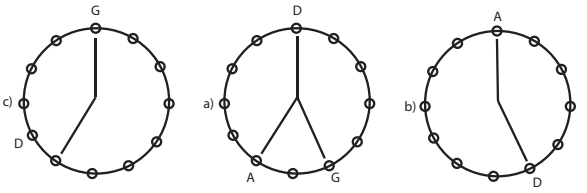


Figure 5. D as a) geometric means; b) arithmetic mean; c) harmonic mean.

The centrality of the Greek scale to musical theory can be appreciated by looking at figure 5. In figure 5a, D is the geometric mean between A and G as arithmetic and harmonic means, respectively. In figure 5b, A is elevated to the fundamental position on the tone circle, and now D is in the position of the harmonic mean of the octave AA'. In figure 5c, G is elevated to the fundamental, and D is now the arithmetic mean of the octave GG'. So we see that D functions in the tri-partite role of geometric, arithmetic, and harmonic mean. Once again, arithmetic and harmonic means change meanings when the scale numbers are interpreted as relative string length. This becomes evident when a segment of the Nicomachus Table (Table 1) is rewritten as follows:

1	2	4	8	16	...
3	6	12	24	...	
9	18	...			
27	...				

Table 2.

We see that any integer is the arithmetic mean of the two integers that brace it from above and the harmonic mean of the two integers that brace it from below. Notice how this is true for the Greek formula: 6 8 9 12, within Table 2. Also see how 12 plays the role of arithmetic mean of [8,16], the harmonic mean of [9,18] and the geometric mean of [6,24]. Notice the pattern of integers at the edge of Table 2.

I				1			
			2		3		
	4				9		
8						27	

or

II				1			
			2		3		
	4		6		9		
8		12		18		27	

Pattern I appears in Plato's *Timaeus* where it is referred to as the 'World Soul.' Pattern II above is the tetractys that figures greatly in Pythagorean thought symbolizing the importance of the first four integers in scale building. The architect Leon Battista Alberti reproduced the 'World Soul' pattern, which now appears in the Alberti Museum in Florence, using it as a template for the proportions of his buildings. He clearly states that he will look to the great architecture of antiquity to create his buildings, and says that 'what pleases the ear should please the eye.' Notice the hexagon of integers surrounding 12 in Table 2. Alberti took adjacent integers from this pattern to serve as relative length, width and heights of key dimensions within his buildings. From this table the musical proportions of Renaissance architecture had its origin [2,6]. I have also noticed that the integers in a sequence of matrices discovered by the Russian bio-physicist Sergey Petoukhov, and related to DNA coding are integers from column 4 of the Nicomachus matrix [7], a wonderful coincidence.

5. The Pentatonic scale.

The Greek scale was obtained by moving along the tone circle seven tones clockwise and counter-clockwise from D at its zenith. We can repeat

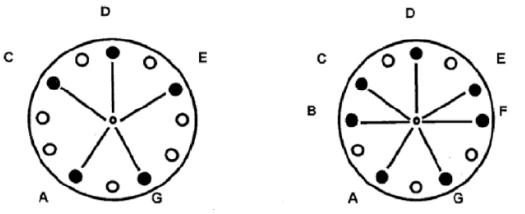


Figure 6, left above, a pentatonic scale. Fig. 7, right, a heptatatonic scale when inverted forms a pentatonic scale.

our analysis but now moving twice 7 clockwise and counter-clockwise from D. The two clockwise and counter-clockwise moves of seven tones produce the sequence of digital roots : 14, 7, 0, -7, -14. But on the mod 12 clock this sequence can be rewritten as the sequence of digital roots: 2, 7, 0, 5, 10 which

according to figure 2 produces the sequence: C, G, D, A, E as it appears on the tone circle of figure 6. These tones can also be derived from the five pitch classes: 1/9, 1/3, 1, 3, 9 as follows:

C	G	D	A	F
1/9	1/3	1	3	9
1	3	9	27	81
16	24	36	54	81
128	96	72/144	108	81

The fractions are eliminated in the second row by multiplying by 9. Multiplying the integers of row 3 by appropriate powers of 2 (the pitch classes do not change) we obtain the fourth row which replicates the fifth column of Table 1, an increasing sequence of fifths. Since the largest integer of this sequence is 81, the other integers in the fourth row can be 'sealed' in the 144/72 octave by multiplying them by appropriate powers of 2 without changing pitch class to reproduce Sequence 4. Arranged in scale order, a pentatonic scale in a rising octave emerges:

D	E	G	A	C	D
72	81	86	108	128	144

Recent research of Anne Bulckens has shown that principal dimensions of the Parthenon are reckoned by the integers of the pentatonic scale represented in column 5 of the Nicomachus Table [8,9].

6. The heptatonic scale

A heptatonic scale can be mapped from three clockwise and counterclockwise movements of 7 around the tone circle represented by the sequence: 21, 14, 7, 0, -7, -14, -21 or in terms of the digital roots as: 9, 2, 7, 0, 5, 10, 3 which, according to figure 2, is the sequence of falling fifths: B, E, A, D, G, C, F. These seven perfect fifths are represented in column 7 of Table 1 by their relative frequencies, reading from top to bottom. They can also be represented by the placement of the seven pitch classes: 1/27, 1/9, 1/3, 1, 3, 9, 27 into the octave limit 864/432 by the same process that we used to create the tones of the Greek and pentatonic scales. The symmetric heptatonic scale in the D mode is shown on figure 7 by the seven darkened circles. If the tone circle is inverted, notice that the five unmarked circles form the pattern of the pentatonic scale (see figure 6). Since these circles lie at the positions of the black keys of the piano, we see that the black keys represent all of the pentatonic modes. Semitone intervals are dissonant to the ear.

Because pentatonic scales have no semitone intervals, they give rise to pleasant sounding melodies however played, and they have been used to create much of the world's folk music. On the other hand the two semitone intervals BC and EF of the heptatonic scale must be cleverly manipulated by composers in order to create interesting musical patterns.

Toussaint [10] has shown that if one goes around the tone circle of heptatonic scales clapping on the black tones and pausing at the unmarked tones one gets one of the clapping patterns found in African rhythms. This was also recognized earlier by Rahn [11]. Different African clapping patterns are gotten by rotating the heptatonic scale pattern so that different tones are in the position of the fundamental.

7. The Chromatic scale and the Pythagorean comma.

The twelve tones of the chromatic scale are derived, beginning with D, from six clockwise and counter-clockwise perfect fifths or movements of seven spaces around the tone circle to produce the sequence of falling perfect fifths and their digital roots:

A♭	E♭	B♭	F	C	G	D	A	E	B	F♯	C♯	G♯
6	1	8	3	10	5	0	7	2	9	4	11	6

Note that the digital roots are multiples of 7 in the mode 12 system. The sixth clock-wise and sixth counterclockwise fifth at A♭ and G♯ overshoot the tritone at 6 o'clock on the tone circle in both directions. Although in modern equal temperament these tones would be identical, when the tones are reckoned by natural intervals of the harmonic series they differ by a barely audible pitch amounting to 24 cents (2 cents per fifth) or approximately a quarter of a semitone known as the Pythagorean comma. Six multiples of 3 and their inverses are defined

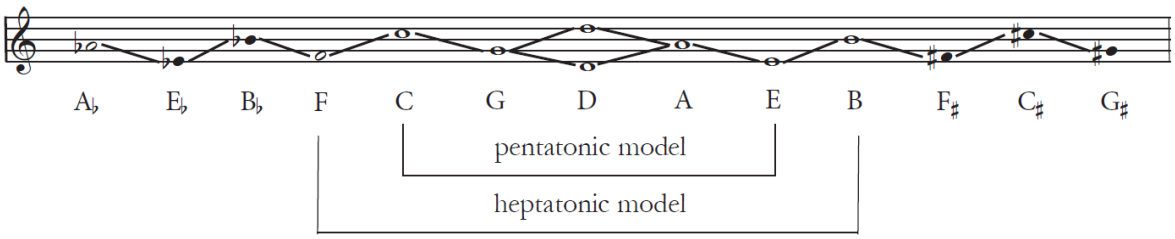


Figure 8. The serpent tuning of thirteen perfect fifths arranged according to increasing frequency.

with unity at the center: 1/729,....,1, 3, 1, 1, ...729. The sequence of relative string lengths for the pitch classes of these twelve tones: (1; 3; 9; 27; 81; 243; 729; ...., 177,147; 531,441), appear along the bottom edge of the Nicomachus Table 1. When placed in scale order in a single octave I shall refer to them as systems of spiral fifths. The thirteen tones (including a repeat of the tritone) of the chromatic scale, in terms of relative frequency, are pictured in figure 8, in a single octave, as the meandering form of a serpent eating its tail, since the first and thirteenth tones nearly coincide, missing by the Pythagorean comma.

8. The McClain Matrix

The ancients were aware that the integers of the spiral of fifths proliferated in their magnitudes. The string would have to be sealed in an octave great enough to contain the twelfth tone (177,147) of the chromatic scale.

McClain suggests that, in the Hebrew Scriptures [12], these huge integers were associated with *Anakim* or giants that walked the Earth and must be slain. So the serpent was cut in two places, C and E. As we will show in this section, the tones between E and G# and between Ab and C were multiplied by the ratio 81:80 (known as the syntonic comma). The tones between A♭ and C were multiplied by 80:81 the three strands of the severed rope were juxtaposed to form three rows of the Harmonic Matrix (HM) shown in figure 9a, with the integers representing the relative requencies of the twelve tones referred to as the Just scale, and with tones of the scale of *spiral fifths* along the center row of the three. The research of McClain suggests that this scale may have been known as far back as the 3rd millennium B.C. in ancient Sumer.

The tonal names of these pitch classes are presented in figure 9b when the fundamental at length 45 is chosen as D with uppercase letters for spiral fifths and lowercase for Just scale equivalents.

Since a♭ and g♯, both represent the tritone at 6 o'clock on the tone circle, g♯ at 2025 is eliminated and the largest integer is then 675. Therefore, the fundamental at D = 45 must be multiplied by 8 into the 720/360 octave in order to seal 675. The integers of figure 9a are now multiplied by the appropriate power of 2 to project them into the 720/360 octave of figure 9c. All twelve tones of the chromatic scale are now represented within a relative frequency limit between 360 and 720, a great reduction in size. The octave limits prevent the proliferation of tones. Note also that 360 represents the length of the ancient canonical year in various early cultures, a compromise between the solar year (365.25 days) and lunar year (354 days).

The fragment of figure 9a,

$$\begin{matrix} 5 & & & & 5 \\ 1 & 3 & \text{or} & 4 & 3 \end{matrix}$$

reveals the 3,4,5-relationship underlying the musical scale. Plato was aware of this when he stated in the *Republic*: '*four-three mated with five … produces two harmonies*.' All tones of the Just scale can be represented by merely adding or subtracting one part and doubling when necessary, e.g., 9 and 15 are derived from 3 as follows: 9 = (2 x 3) + 3, 15 = (2x3) + (2x3) + 3.

The integers of figure 9c can also be organized according to a system of three vectors and their opposites governed by the pairs of inverse ratios: 5:4, 8:5; 3:2, 4:3; 5:3, 6:5 as shown in figure 10. Beginning at any tone one can steer by this vector system to any other tone in the McClain Matrix

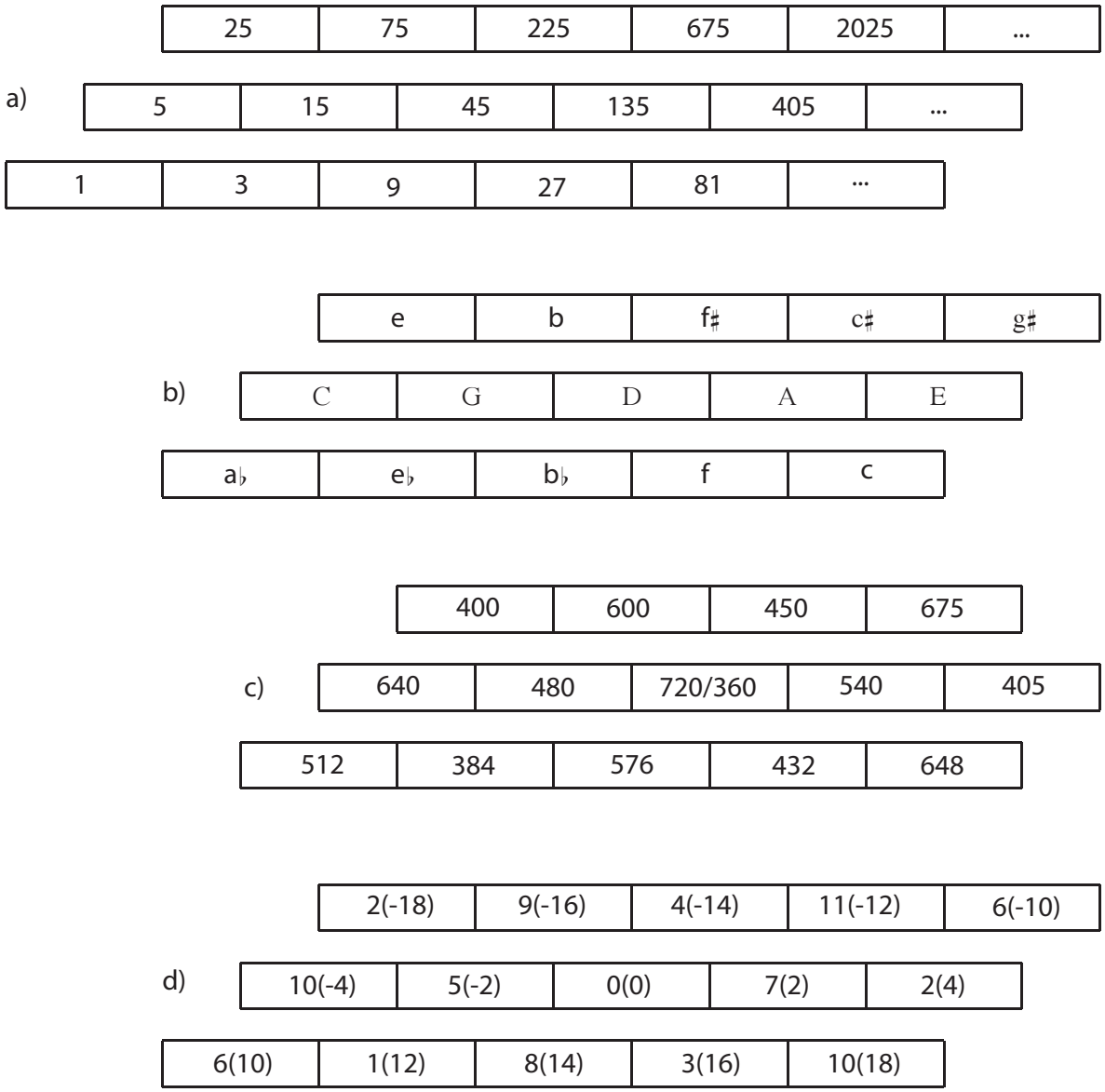


Figure 9. The McClain Matrix. a) Multiplication table of powers of 3 and 5; b) tones of the Just scale; c) integers are sealed in the 720/360 octave; d) digital roots of tones from the Just scale.

and derive the ratio corresponding to the interval between the origin and destination tones by multiplying the ratios along the path between them. One can determine its digital root of the interval by adding the digital roots in the square brackets [ ] in Figure 10 corresponding to the vector directions. The comma difference from equal temperament of the interval
can be determined by adding the numbers in the curved brackets ( ). Since multiplication and addition are equivalent operations, HM may have served as a kind of *proto-log* table enabling rapid computations to be carried out as far back as the 3rd millennium B.C.

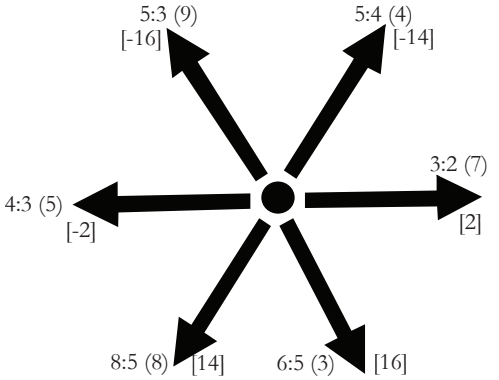


Figure 10. The system of musical vectors.

Example 1:

In figure 9b a move from D to e♭ by the route: D→G→e♭ results, using the vector system, in the ratio: 1 x 4/3 x 8/5 = 32/15 or, when placed into the octave, 16:15, the ratio of the Just semitone. The sum of the digital roots along this path are: 0+5+8 = 13 which has digital root 1 mod 12, and according to Fig. 2, corresponds to e♭. Any other path between the same two tones yields the same result.

Example 2:

A route from C to c in figure 9b is: C→G→D→A→E→c with ratios 3/2 x 3/2 x 3/2 x 3/2 x 8/5 = $8/5 = \frac{81 \times 8}{80}$. Dividing by the octave, 8, yields 81:80. Adding the commas along this path gives: 2 + 2 + 2 + 2 +14 = 22, corresponding to the syntonic comma of 22 cents. Assigning C the value 0 and summing the digital roots along this path: 0 + 7 + 7 + 7 + 7+ 8 = 36 with digital root 0. Of course, C and c represent the same tone with C from the scale of spiral fifths and c from the Just scale as shown in figure 9b and c where the ratio of c: C : 648:640 = 81:80. Similarly, the ratio of e: E= 400:405 = 80:81.

Example 3:

A♭ and g♯ would be the same tone on a piano so they differ in relative frequency by a comma. A route from A♭ to g♯ is: A♭→E♭→B♭→F→C→G→D→A→E→g♯ (see figure 8) with commas added along the path as 8x2 -14 = 2 cents, a comma known as a schisma.

The ratios of the twelve tones of the Just scale are listed in Table 3 along with the common names of the intervals and their digital roots.

The tones in Table 3 below are juxtaposed so that complementary or symmetric tones on the tone circle are adjacent. The major and minor thirds, major and minor sixths, fourth and fifth are the consonant intervals.

Deviations in cents of the Just scale from the equal-tempered scale appear in columns 5 and 12 (also the numbers in curved brackets in Fig. 9d) while deviations in cents of the Just scale from the scale of spiral fifths are listed in columns 7 and 14. Frequency values of tones of the Just scale in cents are found in columns 6 and 13.

9. McClain's Yantra Diagrams

The symmetry of the Just scale is best seen in the yantra diagrams of McClain. The simplest yantra diagrams are derived from a segment of the

0	D	0	1	0	0	0	12	D'	8^{va}	2/1	0	1200	0
1	e♭	t	16/15	12	112	22	11	c♯	$+7^{th}$	15/8	-12	1088	-22
2	E	2^{nd}	9/8	4	204	0	10	C	-7^{th}	16/9	-4	996	0
3	f	-3^{rd}	6/5	16	316	22	9	b	$+6^{th}$	5/3	-16	884	-22
4	f♯	$+3^{rd}$	5/4	-14	386	-22	8	b♭	-6^{th}	8/5	14	814	22
5	G	4^{th}	4/3	-2	498	0	7	A	5^{th}	3/2	2	702	0
6	g♯	3T	45/22	-19	590	-22							

Table 3. Tones of the just scale.

Harmonic Matrix (HM) and its inverse: where the integers with superlines denote inverses, e.g., $\overline{15} = 1/15$.

The integers in figure11a represent eight different pitch classes while the terms in figure 11b

	25			$\overline{27}$	$\overline{9}$	$\overline{3}$	$\overline{1}$
5		15	45		$\overline{45}$	$\overline{15}$	$\overline{5}$
1	3		9	27			$\overline{25}$

Figure11. A segment of the Harmonic Matrix; b) the segment of a) is inverted along with its integers.

are their inverses or tones symmetrically placed on the tone circle. Notice that in figure 11a, 9/15 = 3/5 whereas the symmetrically placed tone in Fig. 11b is $\overline{9}/\overline{15} = 15/9 = 5/3$.

Figure 11a is redrawn in figure 12a into the

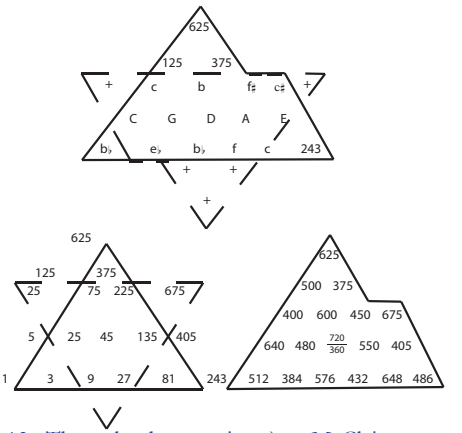
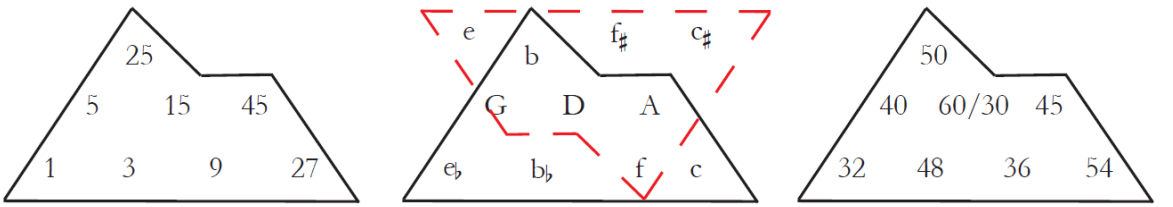


Fig. 13. The calendar matrix. a) a McClain yantra from a segment of HM; b) integers of HM are doubled into the 720/360 octave; c) intersection of HM and its inverse.



(a) Irreducible integers (b) Tonal reciprocals (c) Diatonic scale order

Figure 12. McClains' yantra diagrams. a) a segment of HM; b) integers are doubled into the 60/30 octave; c) a superposition of HM and its inverse.

form of a ziggurat or stepped pyramid. Since the largest integer is 45, the integers of figure 12c are sealed in a 60/30 octave. In figure 12b the two yantras in figure 11 are superimposed. Now the terms in common to both yantras correspond to those pitch classes whose inverses can be expressed as integers within the 60/30 octave limit, i.e., G and A, and b and f are the only symmetric pairs in the 60/30 octave, e.g., 50/30 = 5/3 while 36/60 = 3/5 and 45/30 = 3/2 while 40/60 = 2/3. So the yantra diagrams express, with the greatest economy, the two essential elements of Platonic philosophy, the importance of limits and the balance of opposites.

Figure 13 illustrates these yantras for the calendar matrix. You will notice that there are five pair of tones and their inverses in the intersection of this yantra diagram. They are the fundamental tone and five pairs of opposites at symmetric positions on the tone circle. Only the tritone, corresponding to $\sqrt{2}$, has no opposite.

These yantra disgrams have provided McClain with a kind of rosetta stone leading him to associate the musical scale with ancient sacred texts such as the Rg Veda, Legend of Gilgamesh, the Bible, and the works of Plato. A different interpretation arises depending on what number of HM is chosen to be the fundamental and what values are chosen as the octave limits of the fundamental. The yantra in figure 12 was associated by McClain with ancient Sumer since Anu at the head of the Sumerian Pantheon was given the number 60 while Enki, the mountain god was assigned 50, and Ea the god of fresh water was 40. McClain suggests that the Hebrews took this matrix and changed it for their own purposes. Reuben is Jacob's oldest son so he is assigned the first number, 1, of HM. Joseph is the twelfth son and he is assigned the twelfth largest number of the Matrix, 125. Benjamin the thirteenth son of Jacob gets the number 135. Since Jacob dies at age 147 there are no more sons for which to assign numbers. McClain feels that 147 came from the tail of greatest integer in the circle of fifths in Sequence 5 or 177,147. In the yantra of figure 12c,

the twin tones G and A are assigned to Isaac's twin sons, Jacob and Esau. Esau, who was born first, should have been assigned the larger number 45 leaving 40 for Jacob. However, Jacob was destined to be the father of the Hebrew nation and it was important that he should get the number 45. An elaborate deceit was then carried out by Jacob to steal Esau's birthright, and he was awarded integer 45. McClain suggests that the Hebrews took the Sumerian yantra and moved the fundamental one space to the right at 45 to build a new system. In this new system 45 was assigned to Jacob. The sequence 45→90→180→360→720 transformed 45 to the new octave limit of 720/360 of figures 13. As some justification for this numerology, McClain points out that Sarah gave birth to Isaac at age 90 and Isaac lived until 180. Notice in figure 13a how the five pair of inverse integers fit comfortably into a Star of David.

10. The Tetrapolarity of the Musical Scale Revisited.

Ancient obsession with inverse symmetry and numerical exactitude included a mastery of Spiral Fifths and Just tuning systems through all thirteen pitch classes in each, with only three tones (G-D-A) in common. A tetrapolarity results—with both vertical and horizontal planes of reflection —indifferent to right/left and up/down. McClain suggests that the three tones belonging to both systems were encoded in the name of David from the Bible using Hebrew gematria, where letters are given integer values, as 4.6.4 which can be read 2:3 vs 3:2, falling and rising fifths.

In figure 14 the tones of the thirteen pitch classes are plotted for the Just scale and the scale of spiral fifths against their deviations from the equal-tempered scale (see the numbers in parenthesis in figure 9d). A 'straight line' geometric progression of rising perfect fifths 3:2 and/or falling perfect fourths 3:4 (increasing or decreasing 2 cents per fifth) allows their 'Spiral' results to be imagined as a near approximation to equal temperament, graphed as a reference on the central horizontal axis. Five other pitch classes in each direction beyond the three, G,D,A, of the Greek scale, held in common, display Just alternate thirds of 5:4 at contrary angles to their spiral fifth relatives sharing the same names, but parallel to opposites above and below. In the four outer corners, the two systems converge to a schisma of 2 cents at the tritone--G-sharp with a-flat, and A-flat with g-sharp--more closely than pure and tempered perfect fifths at G and A (see Section 8 Example 3). Vertical dimensions are grossly exaggerated to make them visible in modern 'cents'. Notice that the values of the Just scale and scale of spiral fifths have opposite deficiencies and differ from each other by 22 cents, the syntonic comma. This 'dragonfly' projection offers a mainly 'subliminal' window on processing acoustical events in the soul.

11. The square root of 2.

Computation of an approximation to the square root of 2 was important for the astronomy, architecture, music, and mathematics of ancient civilizations. Although there was no concept of an irrational number, it was understood that the tritone of the musical scale provided an approximation to $\sqrt{2}$ where relative frequency of the equal-tempered tritone is $\sqrt{2}$ in an ideal sense. McClain has traced how this quest for an approximation to $\sqrt{2}$ provided a rich source of metaphors for the great sacred and philosophical works of antiquity including the Rg Veda, Hebrew Scriptures, and dialogues of Plato.

All tones of the ancient musical scales with digital roots of 6 provide approximation of $\sqrt{2}$. For example, A♭ and G♯ from the scale of spiral fifths provides two values: 729/512 = 1.423… and 1024/729 = 1.404…, where 512 and 1024 are powers of 2 representing unity. McClain suggests that since the tritone, also known as the devil's tone, $729 = 3^6$, and fundamental associated with Deity, $512 = 8^3$, in the spirit of number play from ancient times this may have given rise to 666 symbolizing the devil and 888 symbolizing Savior or Christ. The cornerstone of the McClain Table at a♭ = 512 provides the ratio to the fundamental at 720 as 720/512 = 45/32 = 1.406… while its inverse at g♯ is 64/45 = 1.422.

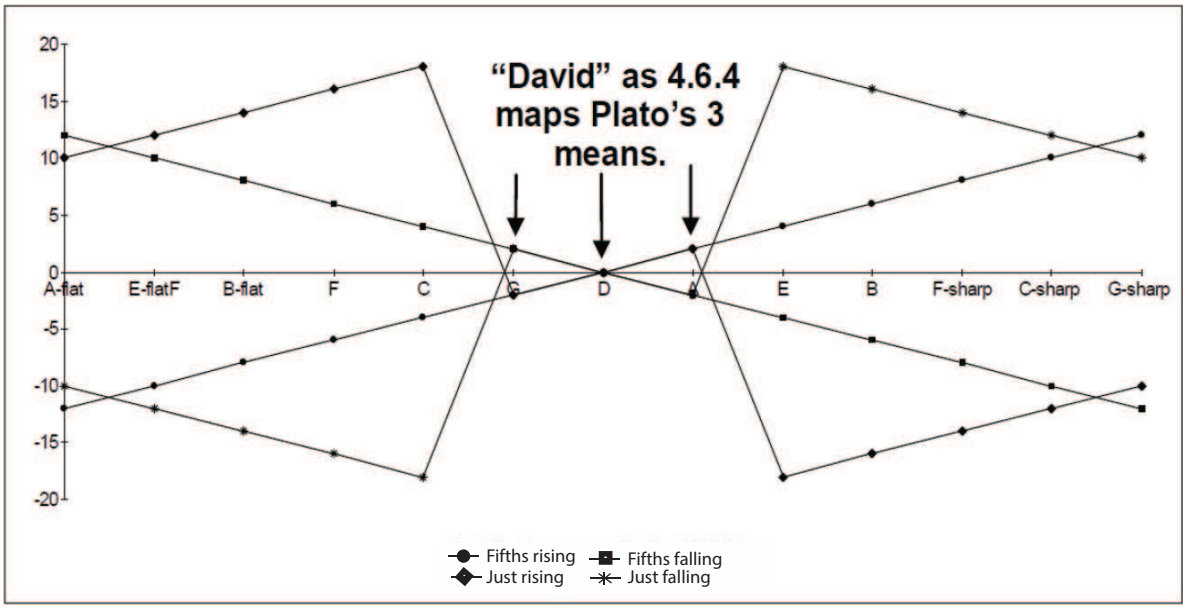


Figure 14. The tetrapolarity of the musical scale. Deviations in cents between the Just scale and spiral fifths and the equal-tempered scale.

The Sulba Sutra gives a Vedic construction of √2 stated as follows: Increase the measure by a third and this (third) again by its own fourth less its thirty-fourth part; this is the (length of) the diagonal of a square (whose side is the measure) [13]. Scholars have shown that this somewhat convoluted construction yields a result of √2 ≈ 577/408 = 1.414215... whereas the actual value of √2 = 1.414213... a result good to six significant places. McClain and I suggest an algorithm derived directly from the musical scale. The idea is that the tritone is surrounded on the tone circle by the fourth (4/3) on its right and the fifth (3/2) on its left, two inverse tones. Therefore, the average of these tones lies nearer to the tritone. We can then repeat this by finding the average of this new value and its symmetrically placed inverse, and continue this procedure. The following algorithm shows how this succeeds in deriving the Vedic approximation to √2:

a) Begin with 1 2.
Double: 2 4.
Insert 3.
First Approximation = 3/2.
b) Place 3/2 and its inverse in the octave: 1, 2/3, 3/2, 2.
Place in scale order in the octave: 1, 4/3, 3/2, 2.
Multiply by common denominator (2x3): 6 8 9 12.
Double: 12, 16, 18, 24. Insert 17.
Second Approximation = 17/12.
c) Place 12/17 and 17/12 in the octave.
Place in scale order in the octave: 1, 24/17, 17/12, 2.
Multiply by the common denominator (12x17): 204, 288, 289, 408.
Double: 408, 576, 578, 816. Insert 577.
Third Approximation = 577/408 which is the ancient Vedic approx. to √2 to 6 decimal places.

This algorithm may be continued indefinitely. Table 4 shows that these approximations to 2 are also the result of the double Pell sequence. This double sequence, studied by Theon of Smyrna, was the basis of the Roman system of proportions [3]:

	1	3	7	17	41	99	239	577
	1	2	5	12	29	70	169	408
No.	1				4			8

Table 4.

Any integer of this sequence is twice the previous integer added to the one before. Ratio of successive terms from either sequence approaches $\theta = 1 + \sqrt{2}$, referred to as the silver mean. The ratios of corresponding terms from each sequences approach √2, so we see that the ratio 577:408 is the eighth approximation to √2 in an accerlerated convergence.

12. The McClain Matrix and the modern theory of music.

Western music is built around a chord structure of the fundamental, major third, and fifth known as a *major triad* and the fundamental, minor third and fifth known as the *parallel minor triad*. Figure 9b when extended, exhibits major and minor triads in every key with the major triad as the upward triangle of tones consisting of a pair of tones from row 2 bracing the tone from above in row 3 while the minor triad is the pair of tones from row 2 bracing a tone from row 1. For example, in the key of C the major triad is: C, e, G while the parallel minor is: C, e♭, G.

13. Discussion and Conclusions.

We have shown that embodied within the ancient musical scale are primitive notions of modular arithmetic base 12, vector analysis, logarithms, symmetry, and the interplay between the geometric, arithmetic and harmonic means all accomplished within the realm of integers. McClain suggests that his table goes back to ancient Sumer in the 3rd millennium BC, and he cites the Gilgamesh legend claim that 'a three strand rope cannot be broken' as possible reference to the McClain Matrix. Although there is no direct evidence of the Harmonic Matrix or McClain's yantras appearing in ancient cultures, McClain has found many suggestive archaeological clues pointing in this direction. Thus we see that ancient musical scales through the ages have taken on numerous meanings in both scientific and cultural contexts, the result of its strong underlying mathematical structure.

Appendix

The equal-tempered relative frequencies, r, on the tone circle follow the geometric sequence:

1, $2^{1/12}$, $2^{2/12}$, $2^{3/12}$, $2^{4/12} = \sqrt[3]{2}$, $2^{5/12}$, $2^{6/12}$, $= \sqrt{2}$,...,$2^{11/12}$, 2 where the frequencies grow approximately like 6% interest, doubling after twelve tones. Note that the tritone and the major third correspond to relative frequencies of $\sqrt{2}$ and $\sqrt[3]{2}$ respectively. The digital roots are obtained as $12 \log_2 r = 1, 2, 3, ..., 11, 12$. The logarithmic measure of the length of an interval in terms of cents allots 1200 cents to the octave with each semitone of the equal-tempered scale given the value 100 cents. To express the length of an interval corresponding to relative frequency r in units of cents:

$$r_{cents} = 1200 \log_2 r = 1200 \times 3.322 \log_{10} r$$

Example: For a major third, $r_{cents} = 1200 \times 3.322 \log_{10} 5/4 = 386.3$ cents which deviates from the equal-tempered value of 400 by approximately -14 cents.

Notes

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33-50

THREE HEPTAGONAL SACRED SPACES

Sarah REICHART and Vivian RAMALINGAM

> 'Sacred buildings have always been based upon number.'
>
> Richard Heath
> *Sacred Number and the Origins of Civilization*[1]

Prelude in Eights

The number eight is recognized as a symbol of rebirth, regeneration and renewal. This comes from an understanding that the eighth, or octave, is understood as a return of the first, as in the musical octave, or the week's anniversary of a liturgical date. Thus we have eight-sided buildings for baptisteries, and even with mortuary shrines. These have a focus at the center of the plan and are often associated with domed structures, an import from the East. A full introduction to octagonal baptisteries, mausolea, and their relationship to the Anastasis in Jerusalem, as well as a discussion about European copies of the Holy Sepulchre church, can be found in Richard Krautheimer's Introduction to Mediaeval Iconography. A photo-documentary of French Rotunda Churches may be found at Dennis Aubrey's Via Lucis website[2]. Jouy de Veye discusses Crusader churches and early octagonal structures, along with his discussion of the church at Rieux Minervois[3], and of course there are plenty of centralized octagonal sacred spaces to be found in continental Europe and England, as buildings with domes became popular. None of these enter the discussion here, except for a hitherto unnoticed thread of ideas and imagery in some Reformed Churches, from 1600 to the present.

In the octagonal design taken up by the Dutch Reformed Church around 1600, the focus was not on the center, but on one of the eight sides, which held the pulpit. The congregation sat circularly along the other sides. Sitting in church was a new idea[4]. Before this, people often just stood about, with those needing a seat bringing their own.

This no longer worked with the lengthy homilies popular among dissenting ministers. In this way, the use of the sacred space was changed to emphasize a new relationship: from that of a priest leading his flock to that of a minister surrounded by his congregation. This idea was presented first in a church within an octagonal city-fort built for Mauritz of Orange/Nassau at Willemstad in Brabant (1597-1607)[5]. The Dutch India Companies spread the concept to their colonies throughout the world[6]. Peter Stuyvesant was responsible for a number of small octagonal wood buildings in Long Island and New Jersey in the 1650s to 1670s[7]. The Dutch plan also spread to England, where, in the mid-1750s, John Wesley saw an octagonal chapel in Norfolk and adopted it for the 14 Methodist churches he started to build in England[8]. Methodist octagons were built as late as 1909 in West Virginia (US)[9]. (But the Methodists seem to have forgotten the symbolic origin of eight: they describe their buildings popularly as having 'no place for the devil to hide.')

After this brief discussion of octagons we turn to the rare examples of heptagonal sacred spaces, and the symbolism of the number seven.

1 Vermont: Inner Traditions, (2007), p. 30.
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3 Jouy de Veye, A., 'Mémoire de M. Jouy de Veye sur l'église de Rieux Minervois.' *Congrès Archéologique de France.* Session 37, (1871), pp.117-53.
4 For a complete account of the introduction of Pews, see Alfred Heales, *The History and Law of Church Seats, or Pews*, Book I History, (1872).(Web reference).
5 Archimon, 'History and Styles: The first protestant churches (1597-1795),' *Archimon, the virtual museum of religious architecture in The Netherlands*, undated.
6 Photos collected by Sarah Reichart.
7 Drawings and descriptions collected by Sarah Reichart.
8 Wesley, J., *The Journal of John Wesley*, entries for November 5th 1757 (Norfolk), April 24th 1764, (Yarm). (Web reference).
9 Albert's Chapel, Sand Ridge West VA., (1909), photographed by Ralph Bennett, June (2012). See also 'Albert's Chapel'. (Web reference).

The Symbolic Meaning of Seven

The symbolism of the number seven has many dimensions and a rich history stretching back to Biblical times and earlier. Here is an attempt to sort out some of the different meanings.

Firstly, most commonly we find seven as representing a complete set of things: the Biblical seven days in the week[10], seven notes of the scale[11], seven planets[12], and their seven heavens (beyond which was the realm of the immovable stars and of God). Seven is such a strong limiting force that we use it in a basically arbitrary way: the Seven Seas, Seven Wonders of the Ancient World, or seven colors of the rainbow[13].

This limiting value of the number seven is defined by its relationship to the number eight, for eight is always the number of a new start: the musical octave, the liturgical octave (the eighth day after some specified holy day, *i.e.* a week later; or 50 as a Jubilee year, after 7 x 7 years). A building based on an octagon has the meaning of rebirth, so that the shape of an octagon becomes declarative for a baptismal font, a baptistry itself, or even a shrine for the dead awaiting rebirth in heaven.

In addition, the *Lexikon der mittelalterlichen Zahlenbedeutungen* (*LMZ* henceforth), (col. 495) identifies the use of numbers smaller than 10 as a figurative expression of very large numbers, as in 'many-fold', or a totality. Chief among these is the number seven, called in exegetical literature the *numerus universitatis*. St. Augustine observes that 'in this manner, '*Septies in die laudabo te*' (Ps.118: 164: 'I will praise thee seven times a day') is equivalent to '*Semper laus eius in ore meo*' (Ps.33:2: 'His praise shall always be on my lips'). Saint Jerome chooses to render the number seven in the Septuagint as 'many' in his Latin translation of scripture. An interesting example is seen in the Canticle of Anna, where the phrase '(*sterilis peperit*) *plurimos*' ('the barren shall bear many') is put for '(*h*)*eptos*' (seven)[14].

Secondly, another subset of meanings for the number seven, at least in Western religious expression, relates to Biblical ideas or qualities: the Seven Gifts of the Spirit: *sapientia, intellectus, fortitudo, scientia, pietas, and timor Domini*. (Isaiah 11:2) or the seven Pillars of Wisdom (Proverbs 8:12-14)[15]. If one goes back to the original Hebrew terms, one discovers that six of the terms are identical[16].Lost in translation is any sense of their

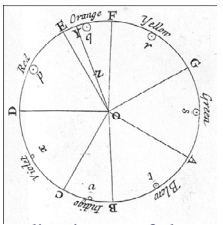
10 For how the Greeks associated the seven days of the week with the planets, see Richard Heath, *Sacred Number and the Origins of Civilization*, (2007), p. 22. For a possible biblical and Greek precedent in a Saturnian calendar, see *ibidem*, pp. 23-7.

11 For a study of how the biblical seven days are related to successive divisions of a vibrating string, see Howard Schatz, *The Lost Word of God*, Tone Circle Publishing, NY, especially 'An Overview of Creation,' (2007), pp. 113-9.

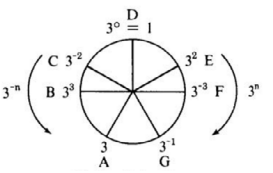
12 The Sun, Moon, and the planets Mercury, Venus, Mars, Jupiter and Saturn. See 'The Royal Game of Ur' - a board game with dice and seven tokens to be moved through 12 squares of the Zodiac. For an interesting discovery of the game scratched on the Assyrian bulls at the British Museum, see Irving Finkel, *The Ark Before Noah*, Doubleday, (2014), pp. 72-4.

13 The history of the seven colors is more interesting than the common understanding that Newton added indigo to make a total of seven. He actually first worked with five: red, yellow, green, blue, violet, which he arranged in a circle instead of linearly. 'But the number did not fit his conception that colors, like notes in music, expressed harmonies. A spectrum of colors, like a musical scale, he imagined, must have seven steps to make a full octave. To arrive at the seven requisite 'notes,' then, Newton inserted orange and indigo into his initial scheme, each addition representing a narrow 'half-step' appropriately spaced in the spectral 'scale'. The 'roygbiv' designation so familiar today thus not only reflects an arbitrary division of the spectrum, but also one shaped by a musical notion of octaves and the diatonic scale.' Douglas Allchin, Newton's Colors, section 'Newton's colors and Artists' Colors,' next to last page, SHiPS Resource Center for Sociology, History and Philosophy in Science Teaching,

Here is the diagram of Newton's colour wheel from the Wikipedia article 'Indigo.' If this wheel is rotated to put 'D' at the top, it is the same as Ernest McClain's basic tone circle diagram. Compare with his 'Circle of the Same,' *The Pythagorean Plato*, p. 66, fig. 25.



Indigo is one of the colours on Newton's colour wheel.



Circle of the Same

14 Meyer, H., and Suntrup, R., *Lexikon der mittelalterlichen Zahlenbedeutungen* (*LMZ*)., Wilhelm Fink, (1987), col. 495; col. 517-18.

15 *Vulgata*, Gifts of the Spirit (Isaiah 11:2); the Seven Pillars of Wisdom (Proverbs 9:1; list at Proverbs 8: 12-14).

16 Jewish Publication Society. Translations: Isaiah 11:2:

The spirit of *YHVH* shall alight upon him: a spirit of wisdom *chokhmah* and insight *binah*, a spirit of counsel *eitzah* and valor *gevurah*, a spirit of devotion *da'at*, also awareness or knowledge) and reverence *yir'ah YHVH*.

Prov. 8:12-14:

I, Wisdom *chokhmah*, live with Prudence *ormah*; I attain knowledge *da'at* and foresight.

equivalence. The seven-pointed 'Mystic Star' of the Methodists uses instead the New Testament Gifts in Rev 5:12: *virtus*, *divinitas*, *sapientia*, *fortitudo*, *honor*, *gloria*, and *benedictio*.

Also in Revelation one finds the presence of the number seven throughout. And through equating the Woman of Revelation with Mary Queen of Heaven, the number seven becomes associated with her veneration.

Thirdly, the makeup of seven as three plus four leads in a different direction. This paradigm is familiar in the *Trivium* and the *Quadrivium* as the seven Liberal Arts. Just as common was the idea of three representing heaven and four the earth. These ideas are much older than the Bible, but can be found there also[17]. They can apply to any group of seven which can be distinguished as spiritual and material, or heavenly and earthly. So we have the seven electors of the Holy Roman Emperor: three archbishops, four secular leaders[18]. And we might add the search for the Philosophic Stone: '...that infallible knowledge and wisdom which is afforded by divine illumination, [the] search for which is sometimes spoken of as the search for the quadrature of the circle, that is, for the extent or area of all sciences human and divine[19].' And does then the centuries-old conundrum of squaring the circle have an underlying meaning of reproducing heaven on earth?

Linear Sevens

Some examples of sacred sevens are a matter of straightforward arithmetic and require only counting: seven arches in the church portal at Ripoll[20]; seven steps (literal and figurative) to reach the Rosicrucian 'Cave' of Enlightenment[21].

Here is an explicit example from the late 8th century: 'By seven altars, by the seven candelabra, and by the seven lamps, the sevenfold grace of the Holy Spirit is understood[22].' This describes a new church built at Aniane in Languedoc. Aniane is a village northeast of Narbonne and west of Montpellier, and home of the Second Saint Benedict[23]. Here in 782 he began a grand new church at his monastery complex, which already contained a small church dedicated to Saint Mary.

In a *Vita* written by the monk Ardo shortly after Benedict died we get a description of the church and its fundamental basis of seven symbolism. Ardo recounts seven altars in the two churches, which demonstrate a sevenfold liturgical hierarchy: 1.The Trinity and The Lord Christ (Saint Salvator), 2. Mary the Bearer of God (*Dei Genetrix* or *Theotokos*), 3. Michael the chief angel, 4. Chief apostles Peter and Paul, 5. Stephen the first martyr, 6. Martin jewel of prelates, and 7. Benedict father of all the monks[24]. He states further: 'All the vessels in the building are consecrated to the number seven.' He notes that the seven candelabra are intricately made to be like those made by

To fear *yir'ah YHVH* is to hate evil; I hate pride, arrogance, the evil way.

Mine are counsel *eitzah* and resourcefulness *tushi'ah*, also a form of wisdom/knowledge); I am understanding *binah*; courage is mine *gevurah*.

Thanks to a communication from Rabbi Jonathan Slater. Lost in translation is the double occurrence and thus the equivalence of six terms:

Chokhmah: wisdom;
Binah: insight, understanding;
Eitzah: counsel;
Gevurah: valor, courage;
Da'at: devotion, awareness or knowledge;
Yir'ah: reverence or fear of God.

Four of these: *chokhmah*, *binah*, *gevurah* and *da'at* also appear among the ten *sephirot*.

17 *E.g.*, the earth is represented by four (as in the Four Corners of the Earth) in Rev. 7:1 and Isaiah 11:12. Three represents heaven or a circle of perfection (with circumference equaling three times the diameter) in 1 Kings 7:23 and 2 Chronicles 4:1.

18 From at least the 13th century the electors were: the Archbishops of Mainz, Trier, and Cologne; the King of Bohemia, the Elector Palatine, the Elector of Saxony and the Elector of Brandenburg. (Web reference).

19 Waite, A.E., *The Real History of the Rosicrucians*,(1887). Republished by Forgotten Books, (2008), p. 20.

20 These may represent the journey from outdoors through the seven heavens to the church interior, which represents heaven on earth - the realm of God. Canaan (own work) (2008). CC by SA 3.0. (Web reference).

21 Godwin, J., '*The Hermetic Tradition*,' with the drawing '*Cave of the Ancients*' by Robert Flood, Rosicrucian Digest, Hermetism 89:1 (2011). (Web reference).

22 *Benedict of Aniane, the Emperor's Monk: Ardo's Life.* Translated by Allen Cabaniss, Cistercian Publications, (2008), 17:6, pp. 78-9. For the Latin, see (Web reference).

23 Benedict went on to become Charlemagne's chief monk in charge of all Benedictine monasteries, and moved north near Aachen. The Aniane church, a basilica, was built at roughly the same time as the octagonal chapel at Aachen.

24 Benedict *Vita*, p. 78. The emphasis on the Trinity and Mary as bearer of God reflects a serious theological dispute with the 'Adoptionists' over the relationship of Mary and Christ. Benedict strongly favored Charlemagne's traditional position.

Bezaleel for the tabernacle[25], and in front of the altar hung seven lamps tended and filled with oil in the manner of Solomon's temple. These latter, of course, are equivalent to the Temple Menorah, now represented by a seven-fold candlestick, familiar to both Jews and Christians.

Of course with all these, more than one meaning could be easily applied to a single example. The critical study by Christian scholars of scriptural texts began, ironically enough, hand-in-hand with the commencement of the Crusades and the periodic expulsion of the Jews, whose sacred books and clergy were continually consulted, one way or another, by scholars in the employ of the ruling class. It reached its height in the 16th century, in the full sway of Protestant and Counter Reformation textual criticism. Thus, the designers of the buildings under examination here would have been fully aware of the variety of meanings that had been readily assigned to the number 7.

Constructing a Heptagon

Constructing a regular heptagon is a difficult geometric problem. It can not be done with compass and ruler. It is not represented among Platonic polygons. It is unrelated to constructions with elements of 1, 2, 3, 4, 5, 6, 8, 9,10, or 12[26].

Archimedes thought he had a solution, but it was based on an assumption he could not prove. This matter was discussed at length by Arab scholars in the ninth and tenth centuries. Jan P. Hogendijk has done a thorough study of unpublished Arabic manuscripts[27]. He has discovered a dozen constructions worked out by Islamic scholars, with five dating from the late 10th century. These use conic sections and other advanced solutions. These are basically two-dimensional constructions with little practical application. So although this information antedates the building of heptagonal churches, we can assume that it was not something the actual builders would have found useful.

A study by Garcia-Salgado, on the second-century Pantheon dome (2009) shows a very practicable way to make a heptagon with the same circumference as a given octagon[28]. This was required as the Pantheon base is octagonal, but the dome is heptagonal.

An ingenious compass and ruler approximation has been published at Wikimedia Commons[29]. Dennis Aubrey posts it on his rotunda church study[30], but there is no indication that it had been known in earlier times.

Towards the end of the sixteenth century people were becoming aware of new ideas and possibilities. Simon Stevin (1548 - 1620), employed by Mauritz of Nassau as a military engineer, was in charge of waterworks and military defenses[31]. His forte, however, was the construction of certain polygonal defensive forts, previously not possible. He rebuilt the fortifications of Coevorden (after conquest by Mauritz in 1592) in the shape of a heptagon - this was to represent the seven United Provinces of the new Dutch Republic[32]. Sometime later - in the 1630s? - several gardens were built at Enhien, just southwest of Brussels. One was called 'Le Jardin des sept Étoiles.' At its center was a lake with a heptagonal pavillion, and each radiating pathway terminated with a sculpture of one of the seven planets.

There is no doubt that by 1600 designing a heptagonal structure was no longer a big problem, and presented no difficulties for the designers of Scherpenheuvel or Stadthagen.

25 Exodus 37:17-22.

26 It is interesting that the absent 7 and 11 are crucial in the relation to diameter and circumference of a circle, with the useful and common relationship of 22/7. This is an approximate value for π still used today.

27 Hogendijk. J.P., 'Greek and Arabic Constructions of the Regular Heptagon,' *Archive for History of Exact Sciences,* Vol. 30, No. 3/4, (1984), pp. 197-330. Springer. (Web reference).

28 García-Salgado, T., 'The Heptagonal Layout of the Pantheon's Vault,' (2009). (Web reference).

29 Aldoaldoz, selfmade, 2010, 'Approximated drawing of a regular heptagon inscribed in a circle showing animated straightedge and compass', (Web reference).

30 Dennis Aubrey, Roronda churches (Web reference).

31 Stevin was a true genius, unfortunately not widely known today. Among his other accomplishments were the popularizing of decimal fractions, the insistence that 'one' was a number, tables of interest, the division of the equal-tempered musical scale, and a wind-driven vehicle. See *Wikipedia* 'Simon Stevin'.

32 For the fort plan at Coevorden, see S.J.Fockema Andreae et al., *Duizend jaar bouwen in Nederland,* Amsterdam, Allert de Lange, 1957-58. (Web reference).

I. RIEUX MINERVOIS: *Heaven and Earth*

The village of Rieux Minervois sits beside a tributary of the river Aude, between Carcasonne and Narbonne, in the Languedoc region of Southern France. 'Rieux' probably meant 'at the riverside', from Latin through Occitan. 'Minervois' is added to the names of a half-dozen towns in this area called the Minervois, after the central town of Minerve. Minerva itself would have been a Roman town, perhaps with a temple where Roman Legionnaires who settled here could make dedications of arms and gifts. (It is probably stretching a point to remark that the number seven was sacred to Minerva[33].)

This region of France has been called variously *Gallia* or *Gallia Narbonensis*, *Septimania*[34], *Gothia*, *Occitania* or *Languedoc*[35]. The area was controlled sequentially by Romans, Visigoths, the caliphate of Córdoba, and the Franks. The twelfth century was a tumultuous time in Languedoc: a time of Crusades, Templars, Cathars, the Albigensian Crusade with Simon de Montfort, and Saint Dominic with his Rosary. In addition there were Moorish scholars in adjacent Catalonia, and a renowned scholarly Jewish settlement in Narbonne.

The small heptagonal Église Sainte Marie here at Rieux Minervois dates from sometime in the mid-12th century, probably replacing an earlier structure. It sits crowded in the midst of the old town with red-tiled roofs all around. The original structure had a wall of fourteen sides. These are still discernible in the church plan.(Fig. 1) There is an inner ring of seven columns (Fig. 3), creating an ambulatory around the church between the two rings. The seven columns support a heptagonal dome (Fig. 4). Over the dome rises a heptagonal lantern. (Fig. 2) Over the centuries the original form has expanded with the addition of extra chapels, added by breaking through the walls and building outwards. The main entrance has been moved and replaced by a chapel.

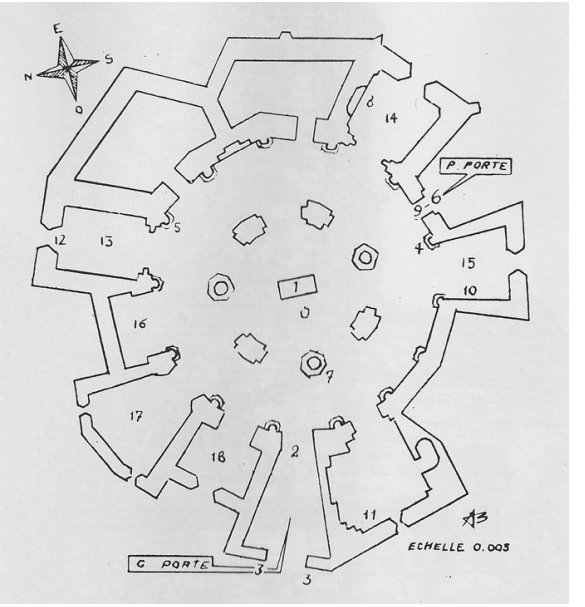


Figure 1. Rieux. Floor Plan.

The interior seven pillars are most remarkable: three round columns and four square pillars. Here in stone is a graphic representation of number three representing heaven and four the earth. The church also seems to be oriented to the east, if one takes a line from the center through the two adjacent square pillars.

The church first came to French national attention with the inspection by Prosper Mérimée in 1835. The first in-depth study was published in 1870 by Jouy de Veye[36]. He presents evidence that there was a church on this site dedicated to Sainte Marie by 1096 when the archbishop of Narbonne died while visiting Rieux. De Veye includes detailed descriptions, architectural drawings, and a comparison of various features with other early churches. He is persuasive in showing that this

33 Wescott, W.W., 'The Heptad. 7', *Numbers, Their Occult Power and Mystic Virtues*, London, *Theosophical Society*, (1911), pp. 72-84.

34 This is just an amusing coincidence - named so because veterans of Rome's VII Legion settled in the area, or perhaps for the seven major Roman cities here. *Wikipedia*, 'Septimania.'

35 Where the language uses 'oc' for 'yes'. Dante in De vulgari eloquentia: I:VIII, divides Romance languages by their word for 'yes' into the oc languages, the 'oïl' languages and the 'si' languages.

36 Jouy de Veye, A., 'Mémoire de M. Jouy de Veye sur l'église de Rieux Minervois', *Congrès archéologique de France*, session 37, (1871), pp. 117-53.

is not a Crusader church, but has elements that reflect Eastern architectural types.

A new line of approach arose after 1935 with the discovery of an unknown sculptor of distinctive style whose work is recognizable from Catalonia through Languedoc to Italy; it would seem along the Pilgrimage routes. Many studies are in progress on the work of this so-called 'Master of Cabestany'. One of his best-known examples is a capital at Rieux depicting the 'Assumption of the Virgin' (Fig. 5). This capital is to the right as you enter by the small southern door.

A close look at this capital, which exhibits the well-known Cabestany trademarks of long arms and fingers, triangular faces, and almond eyes, includes another special feature not much commented on: Mary sits in a *mandorla*. The mandorla is a symbol made when two circles overlap, making an almond shape. (This is also called the *Vesica Piscis*.) Here it represents an overlapping of heaven and earth. The mandorla usually surrounds a Christ figure[37], but in this case we have Mary. Either one can represent a connection between earth and heaven, or the material and the spiritual. This means that the symbolism of the seven pillars and of the Assumption capital are equivalent. The sculpture and the architecture are one: the joining of earth and heaven. It is possible that the Master of Cabestany was responsible for both.

More recent studies include that of Anna Thirion[38] who believes there was a change in plan from an earlier octagonal form to heptagonal. Michael Selvin had found a website (now discontinued) which reported that the diameter of the church is 28 Carolingian feet (7x4), with the inner ring of columns half-way between perimeter and centre[39].

Figure 2. Rieux, Église Sainte Marie

This anonymous site also considers the symbolism of pillars and Cabestany capital. The photographer Dennis Aubrey has added wonderful new photos to his Via Lucis website[40] - two are included here. Michael Selvin is the author of *Heaven Walker*[41], a historical novel depicting the times in which the Master of Cabestany lived. He will protest that his characters are fictional, but the vividly rendered background setting proclaims a full understanding of history and 12th century times.

A different interpretation of the symbolism is proposed by André Bonnery, the primary expert on the Cabestany sculptor. He suggests that the seven pillars represent the pillars of wisdom, and that Marie herself is Wisdom or 'La Sagesse', reflecting a common medieval interpretation, and that this has been true of the church at Rieux since its beginning. Perhaps both interpretations are

37 'Early depictions of Christ portray him as an infant within the vesica (in this context, it is usually referred to as a mandorla, meaning "almond shaped"), which represented the womb of Mary, and often, the coming together of heaven and earth in the body of Jesus (part man, part god[sic]). As such, it is also a doorway or portal between worlds, and symbolizes the intersection between the heavens and the material plane.' Symboldictionary. '*Vesica Pisces*' Symboldictionary.net, A Visual Glossary of Religious Symbols, p. 1127.

38 Anna Thirion, Changement de plan à Rieux-Minervois : du nouveau sur l'église romane Sainte-Marie. Carnets de la Recherche, 2009, pp.9-29. <halshs-00442038v2>

39 Personal communication from Michael Selvin.

40 Aubrey,D.,http://vialucispress.wordpress.com/2011/11/05/the-assumption-capital-of-rieux-minervois-dennis-aubrey

41 Selvin, M.B., *Heaven Walker: The Story of the Master of Cabestany*, (2013). (Selvin, M.B.)

valid: Mary as Wisdom and her Seven Pillars, and four plus three - the meeting of Earth and Heaven.

Figure 3. Rieux. Interior with Seven Inner Columns.

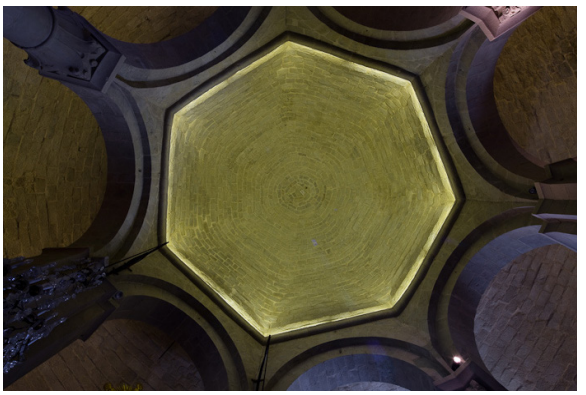


Figure 4. Rieux. The heptagonal Dome.

Figure 5. Rieux. The Assumption Capital.

II. SCHERPENHEUVEL: *The Queen of Heaven*

In 1609 construction was started on a heptagonal church, Onze Lieve Vrouw van Scherpenheuvel (Notre Dame de Montaigu) in the Barony of Diest in Brabant. Why this place? Why this time? Why this design?

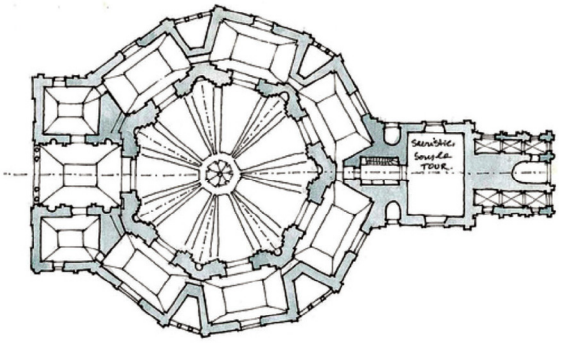


Figure 1. Scherpenheuvel. Floor Plan.

In the history of the Netherlands, Protestant and Catholic disputes over territory reached a peak as Mauritz of Orange gradually regained control over the Netherlands after 80 years of rule by the Spanish. Brabant was at the border and went back and forth many times.

Mauritz was by now Stadtholder of the Netherlands and had built an octagonal fort city at Willemstad in honor of his father, William the Silent. The church at Willemstad was an octagon and became the prototype of Dutch Reformed Church design. In 1609 the Barony of Dienst in Brabant was held by Philip Willem, the Spanish-raised half-brother of Mauritz. It was a stronghold of Catholicism - a most appropriate place for a major Counter Reformation statement.

At this time the Catholic Church was involved in Tridentine reforms and was increasingly promulgating the veneration of Mary. One might say that Mariolatry had started with the tradition of the vision of Saint Dominic in Prouille, Languedoc, in 1208. In this vision the Virgin appeared and gave him the Rosary. The idea of Marian intercession in all matters of health and welfare was advanced in 1571 when Pope Pius V instigated a crowd-based recitation of the Rosary throughout Europe

to pray for victory at Lepanto. Mary subsequently became revered as an intercessor in matters of health and welfare[42]. She was often depicted as the Queen of Heaven and equated with the Woman of the Apocalypse: 12 stars for a crown, standing on the moon, in the rays of the sun, and often a dragon underfoot[43]. The number seven permeates Revelation and was taken to be a number symbolic of Mary.

But there is another thread of ideas to be considered here - the construction of the so-called star forts, both for defense and as symbolic Ideal Cities. Starting this tradition was the octagonal Sfortzinda devised by Filarete but never built (1464?); Palma Nova, a nine sided fort-town, built by the Venetians in celebration of victory at Lepanto (1593); the heptagonal defenses of Coevorden built by Simon Stevin for Mauritz of Nassau, with each point representing one of the seven Dutch states of the new republic (1593); the octagonal town of Willemstad (with a grid plan) also for Mauritz (1597-1607); and finally the heptagonal town and basilica of Scherpenheuvel (1609-1627).

The erection of a heptagonal Basilica in the Spanish Netherlands 1609-1627 reflects the political situation in the Netherlands, the rapid increase in Marian veneration, and the related symbolic use of the number seven.

Here, at Zichem, a nearby venerable oak tree had already given rise to the worship of Mary as an intercessor and giver of miracles. The town of Zichem is named after the Biblical Shechem. Nearby, Scherpenheuvel (Dutch) or Montaigu (French) refers to the 'sharp hill' (Hebrew for 'shoulder') where Abram first stopped while entering Canaan. Near Shechem was the oak of Moreh, and near Zichem was a hill with a long venerated oak tree. These details fostered a reflection of a familiar biblical landscape.

Veneration of the oak tree at Scherpenheuvel predated Christianity. It was sanctified by an image of Mary hung from its branches, which could not be removed, and was considered responsible for miracles. In 1602 the image was taken to a small chapel and in 1604 the tree was cut down and a much larger church begun to honor 'Our Lady of the Oak.' Small carvings of the Virgin were made from the oak and given by priests at the dedication of new churches. One of these was given to Marguerite Bourgeoys for the dedication of Notre-Dame de Bon Secours in Montreal in 1672. The 'Fouencamps Médaille' survives from that time (Fig.5).

This new church was sponsored and developed by the Spanish Archduke Albert and his wife Isabella, sovereigns of the Hapsburg Netherlands between 1598 and 1621. The architect they retained in 1604 was Wenceslas Coebergher. Coebergher had just returned from a number of years in Rome where he became interested in the study of Roman antiquities. He also witnessed the building of several baroque churches in Rome in addition to the completion of the dome of Saint Peter's[44].

Coebergher is credited with the church plan, the heptagonal design evidently having been suggested by Albert. It started with a heptagonal garden or *hortus conclusus* to surround the chapel, which was then expanded to include a whole town (Fig. 2). The facade shows the influence of his Roman experience (Fig. 3). The church itself has an eastwest orientation. On the west is the entrance, taking one side. Three heptagon sides, both left and right, meet at an angle in the east where the high altar stands (Fig. 4).

42 One can see a nice example in the octagonal church Santa Maria della Salute in Venice, built to reflect Marian intercession in halting a devastating plague in 1630. The domed church is full of Marian symbolism. In particular, the high altar (1670) depicts the Queen of Heaven expelling the Plague. Not only is the crescent moon visible in the statuary group, but a chandelier above carefully places twelve (?) tassels as a crown, thus rendering a clear representation of Mary, Queen of Heaven, as the Woman of the Apocalypse defeating the dragon. (Web reference).

43 Art historians call depictions of Mary on the moon 'Madonnas of the Crescent Moon.' There are many such, but interest here in the full representation: Mary atop the moon with twelve stars circling her head, a serpent at her feet - a meme that lasted over four centuries. The Archdukes Albert and Isabella commissioned one such from Rubens, their court painter: 'The coronation of the Virgin' (Web reference).

44 It is tempting to think that Coebergher also observed carefully the dome of the Pantheon. Although this building has an octagonal foundation, the dome itself has graduated rings with 28 coffers each, rather than the expected 32 (4 per side times 8 sides). How this might have been done in a simple way is explored by Tomás García-Salgado in 'The Heptagonal Layout of the Pantheon's Vault' (Web reference).

The most complete study of Our Lady of Scherpenheuvel is *Scherpenheuvel, Het Jeruzalem van de Lage Landen* by Luc Duerloo and Marc Wingens. This beautifully illustrated volume has been reviewed by Susan Koslow[45], who explains much of the Marian symbolism in the church and its literary sources. The number seven is primary. The Church has seven sides, with seven altars. The high altar at the angle opposite the entrance (Fig. 4) has an 'Assumption of the Virgin', and the six side altars show other scenes in Mary's life. The Dome is decorated with hundreds of seven-pointed golden stars. There are also references to the Seven Joys and Seven Sorrows of Mary (the church had received a grant of papal indulgence in 1606, to celebrate the feast of Our Lady of Sorrows); the Rosary (there is a rosary walk outside); Litanies; and 50 (49 plus 1?) emblems relating Mary and this particular church to the Old Testament. These emblems are taken from a text by the Jesuit Jan David, titled *Pancarpium marianum*, which is dedicated to the Archduchess Isabella. They are enumerated at the Google Books website[46]. We can see, for example, the church entrance as 'Porta Coeli' and the ladder up the dome as 'Jacob's Ladder'. Others are more obscure, but all were explained at the time in a presentation pamphlet in Latin, Dutch, and French. We can see, for example, the church entrance as '*Porta Coeli*' and the ladder up the dome as 'Jacob's Ladder'. Others are more obscure, but all were explained at the time in a presentation pamphlet in Latin, Dutch, and French.

It is clear that this whole edifice - design, altars, emblems, is focused on the number seven as it relates to 'Our Lady' - Mary Queen of Heaven.

Figure 2. Sherpenheuvel. Town Map of Sherpenheuvel.

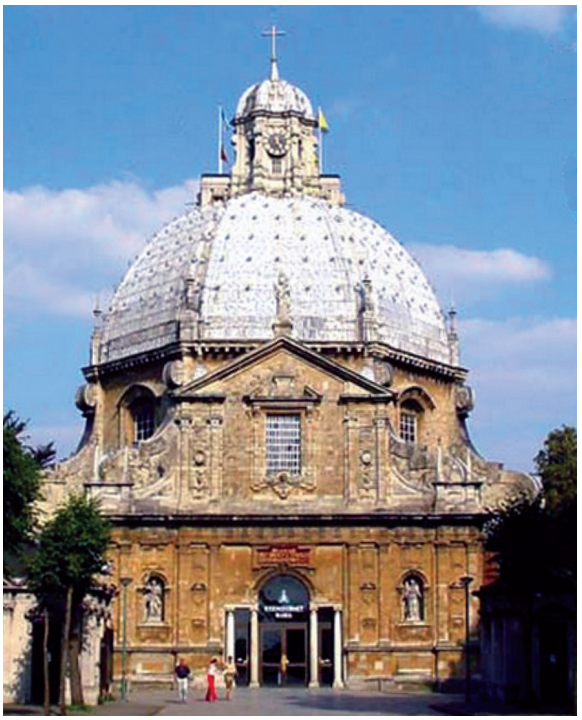


Figure 3. Scherpenheuvel. Facade and Dome covered with Seevn-pointed Stars.

45 Luc Duerloo and Marc Wingens. *Scherpenheuvel, Het Jeruzalem van de Lage Landen*. Leuven: Uitgeverij Davidsfonds, 2002. Reviewed by Susan Koslow. Historians of Netherlandish Art, HNA Review of Books, April (2004) (Web reference).

46 *Pancarpium Marianum* pp. 209-213 (Web reference).

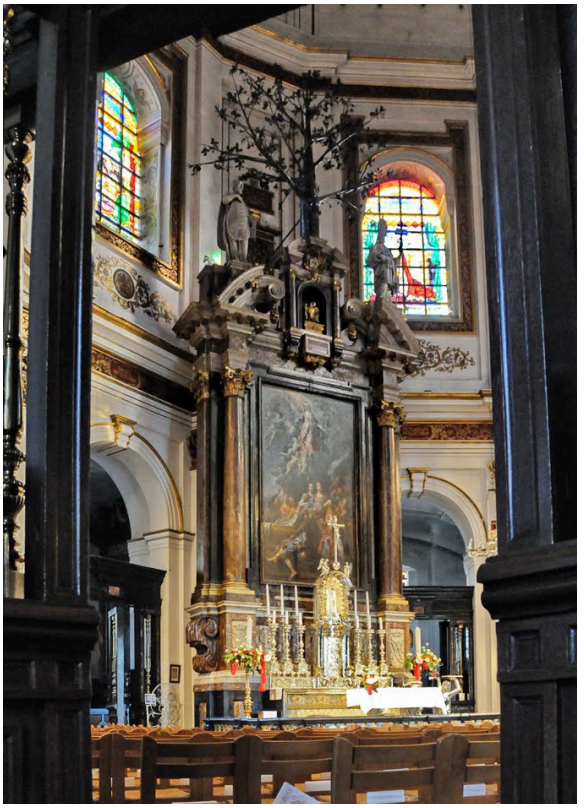


Figure 4. Scherpenheuvel. High Altar with Oak Tree.

Figure 5. Scherpenheuvel. Fouencamps Médaille: Virgin with Stars, Moon and Dragon.

III. STADTHAGEN
The Tomb of Christ

At the turn of the sixteenth century all Germany was embroiled in Reformation activity. With this came interest, even fascination, with matters occult, hermetic, esoteric, Kabbalistic, Masonic and Rosicrucian[47].

Caught up in this early fervor was Prince Ernst of Schaumberg who built a mausoleum at Stadthagen, Lower Saxony, which was intended to reproduce the Rosicrucian idea of Christ's tomb at the Church of the Holy Sepulchre in Jerusalem.

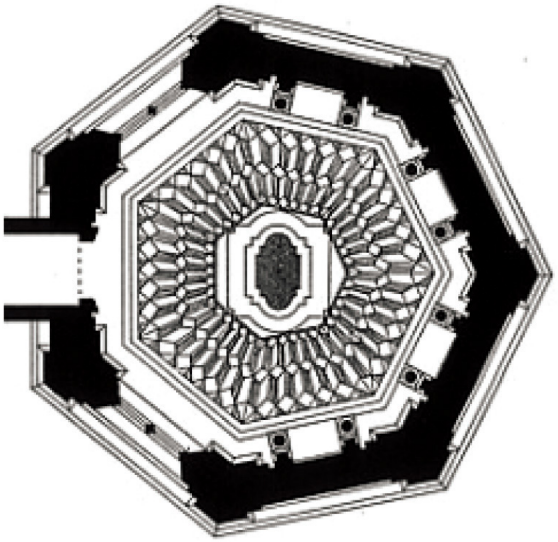


Figure 1. Stadthagen. Mausoleum Floor Plan

Rosicrucianism became important with the publication of its three manifestos, known in short as the *Fama* (1614), *Confessio* (1615), and the *Chemical Wedding* (1616)[48]. It was proposed therewith to institute a Universal Reformation, based on the collected insights of philosophers in both natural and spiritual realms. The number seven underlies Rosicrucian beliefs. This can be accepted without getting involved in the authorship of the manifestos or the reality of its protagonists.

The idea for the mausoleum's heptagonal design

47 For a complete overview, see Frances Yates, *The Rosicrucian Enlightenment*, Routledge and Kegan Paul, (1972). Republished by Routledge Classics, (2002).

48 For a new translation of the Chemical Wedding, see

may have been Prince Ernst's; the mausoleum itself was based on the reported discovery of the tomb of Christian Rosencrantz as related in the *Fama*:

> 'In the morning following [after finding a hidden entranceway] we opened the door, and there appeared to our sight a vault of seven sides and seven corners, every side five foot broad, and the height eight foot. Although the sun never shined in this vault, nevertheless, it was enlightened by another sun ... which was situated in the upper part in the center of the siding. In the midst, instead of a tombstone, was a round altar... This vault we parted in three parts, the upper part or siding, the wall or side, the ground or floor ... the upper part was divided according to the seven sides in the triangle which was in the bright center... Every side or wall is parted into ten squares, every one with their figures or sentences... the bottom again is parted in the triangle... Every side or wall had a door for a chest wherein there lay divers things, especially all our books...[49]'

In 1620 construction on the mausoleum began (Figs. 1 and 2). Prince Ernst had earlier engaged Giovanni Maria Nosseni (1545-1620), architect for the Elector of Saxony. For a central monumental statuary group of Christ Triumphant, he employed the well-known Adrian de Vries (1556-1626) then sculptor to HRE Rudolph II. The fourteen musical angels of the dome are by the Dutch painter Anton Boten (Fig. 3) who took over when Nosseni died. Nosseni had an architectural interest in 'invisible music[50]' that he might have considered here. The Mausoleum was connected via the chancel to the church of Saint Martini in Stadthagen, Lower Saxony. Prince Ernst's father and his two wives are buried in Saint Martini, so this was planned to continue a family tradition. The church faces east, as does the mausoleum, though with an axis slightly off. There are of course seven sides, and seven triangles leading up to a central lantern and motif. In the center is Ernst's tomb, topped with a statue of Christ Triumphant (Fig.4). The statue faces the entrance; four of the seven sides behind the statue contain epitaphs for Ernst, his wife Hedwig, and his two parents. Over the entrance as well as on the two sides left and right of the entrance are windows, making a set of three: thus seven sides altogether. Once again we have the arrangement of three plus four: heaven and earth make seven.

The seven triangular panels of the dome are beautifully decorated with a pair of musical angels each[51]. At the very top center is a triangular symbol clearly meant to represent God, or Wisdom, or some manifestation of the Highest (Fig. 5). It has three arrow-like projections, with rays in between. It is possible that the three projections may refer to the Trinity, as well as to the three top emanations of the Sephiroth: Keter, Binah, and Chokmah, taken over by the Kabbalists and Rosicrucians. Here they are described by Adam McLean in his article on the 'Kabbalistic Lehrentafel of Antonia of Württemburg':

> 'The Supernal Triad Kether, Chokmah, Binah, is indicated by the three crowned female figures at the centre of the dome of the temple. Although they cannot be seen on the detail of the illustration here, on the original painting the head of Kether is surrounded by a nimbus of seven eyes, Chokmah by seven stars, and Binah by a nimbus of seven flames, which provides an interesting interpretation of the usual meanings of these Sephiroth. Kether at the Crown of the Tree has the gift of seeing into the higher reaches of the spiritual world, Chokmah bears the Wisdom of the Stars, and Binah has the fire of illuminated Understanding[52].'

Today Freemasons and their associated Orders retain this meaning of the number seven - that it represents the Seven Liberal Arts[53]. Here at Stadthagen we see how this blends intimately with Christian symbolism.

Joscelyn Godwin, *The Chemical Wedding of Christian Rosenkreutz: Introduction and Commentary* by Adam McLean, Magnum Opus Hermetic Sourceworks #18, Phanes Press, (1991).

49 Waite, A.E., *The Real History of the Rosicrucians*, (1887), republished by Forgotten Books, (2008), pp. 52-3.

50 Spohr, A., 'This Charming Invention Created by the King: Christian IV and his invisible music,' *Danish Yearbook of Musicology*, (2012), pp. 21 (Web reference).

51 For an account of musical performances, see http://www.sn-online.de/Schaumburg/Stadthagen/Stadthagen-Stadt/Broschuere-erklaert-Renaissance-Musik Last viewed: 26/08/2014.

52 McLean, A., '*The Kabbalistic-alchemical altarpiece in Bad Teinach*', Hermetic Journal 12, Summer (1981), pp 21-6 (Web reference).

53 Communication from Worshipful Brother Charles

Figure 2. Stadthagen. Mausoleum of Prince Ernst.

Figure 3. Stadthagen. Heptagonal Dome with 14 Musical Angels.

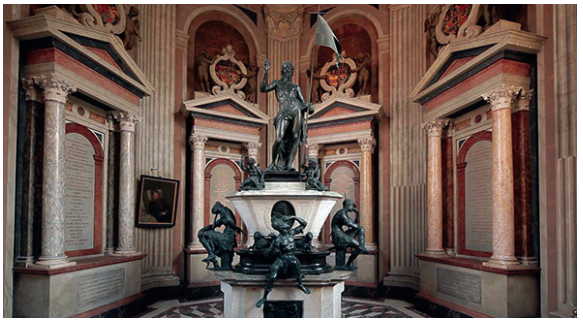


Figure 4. Stadthagen. De Vries, Statuary Group: Christ Triumphant with four Epitaph Panels.

Figure 5. Stadthagen. Mystical Center of Dome.

The Tomb of Christ in Jerusalem

Most churches are known to face the east, toward the Holy Land in general, or Jerusalem in particular[54]. Rotunda churches refer specifically to the Church of the Holy Sepulchre in Jerusalem, as

Vail, P.M.

54 For the antiquity of facing due east, see 'Before Stonehenge; the Ness of Brodgar,' National Geographic August (2014), p. 37 (note 4: Where earth and heaven meet). For a recent statement on facing east, see Cardinal Joseph Ratzinger on priests facing *Ad Orientem*: it is the 'cosmic sign of the rising sun which symbolizes the universality of God.' *The Spirit of the Liturgy*, Ad Solem, (2006), p. 64. For an early church facing Jerusalem, see the Hagia Sophia in Constantinople, the orientation of which towards Jerusalem was slightly altered later to face towards Mecca. For sunrise on a particular day, see the proposal by Fred Gettings that the Church of San Miniato al Monte in Florence has an axis pointing toward sunrise on 28 May 1207 when an alignment or constellium of five planets occurred in the sign of Taurus. Fred Gettings, *The Secret Zodiac*, Arkana, (1989), p. 5; diagram p. 111. This is part of a more general assumption that churches are oriented to sunrise on the day of the Saint to which they are dedicated, evidently disproved (in England) by Ian Hinton, in 'Churches face East, don't they?' *British Archaeology* 94 May/June (2007).

Krautheimer notes in his Iconography[55]. It is no surprise that our three heptagonal sacred spaces follow this trend; the Mausoleum itself is a special case as it so clearly refers to the very tomb of Christ in Jerusalem.

The Jerusalem site itself has an interesting history, as related by Wendy Pullan in her study of its development[56]. She discusses Hadrian's Venus temple in Aelia Capitolina, and the possible connection to the 'Venus' temple at Baalbek. By 326 Constantine had cleared the Roman site and replaced the temple with the Church of the Holy Sepulchre[57]. This was characterized uniquely by a rotunda or anastasis enclosing the tomb of Christ, with Golgotha nearby. The rotunda survived a number of burnings and almost total destruction in 1009, being rebuilt each time.

The Anastasis has eight piers (in four sets of two), and 12 columns, three between each set of piers. From this we can see how the 8, 12, or even the total of 20 pillars of the Anastasis of the Church could be copied in European rotunda churches, as Krautheimer has so amply discussed[58].

Inside was the Edicule, covering the actual tomb. The original Edicule was built over many times, each rebuilding encasing the previous one. To help understand its arrangement, recent scholars have turned by way of analogy to the 'Venus' temple at Baalbek. Especially important is the comparative study by J. Lauffray[59]. In examining the relationship between its circle and square elements, he has unexpectedly hit upon the venerable and ancient ratio for expressing π: 22/7[60], for in this case the diameter of the circle is equal to the length of the entrance or Pronaos. The circle diameter of 7 units is laid on the circumference at the entrance. This leaves 15 units which are divided into five groups of 3, making a harmonious join of circle and square. This is simple and elegant.

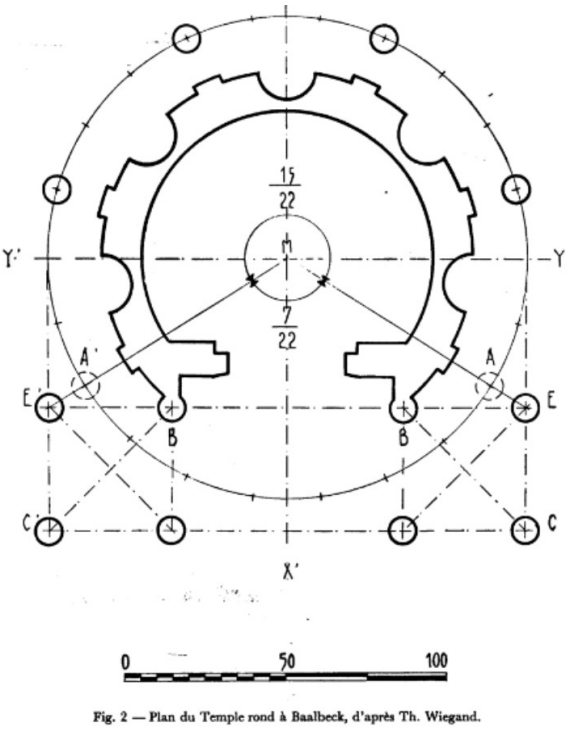


Fig. 2 — Plan du Temple rond à Baalbeck, d'après Th. Wiegand.

Figure 6. Baalbeck, Lebanon, Plan of the round temple by Th. Wiegand.

The Narbonne Model

There is a three-dimensional model of the Jerusalem Edicule at the Museum in Narbonne. It is about 3 feet high, made of local stone, and dates from the 5th century[61]. Here Lauffrey diagrams it as a structure similar to Baalbek: based on a circle and square with related diameter and side[62]. The measurements (5/17 and 12/17) and Lauffray's display are not quite right, though the idea is the same as Baalbek's 22/7. Perhaps these are just approximate. It is not clear whether the sculptor was using 22/7 or perhaps the simpler formula 3/1.

55 'Among the great number of edifices erected throughout the Middle Ages with the intention of imitating a highly venerated prototype, one group is particularly suitable for establishing the nature of a mediaeval copy: the imitations of the Holy Sepulchre at Jerusalem. They exist not only in great numbers but also depend on a model which is still relatively well preserved and can easily be reconstructed in its original aspect. These copies were built all over Europe from the 5th to as late as the 17th century.' Krautheimer, 'Introduction to an Iconography of Medieval Architecture' *Journal of the Warburg and Courtauld Institutes*, the Warburg Institute, Vol. 5 (1942), p. 3.

56 Pullan, W., 'Regeneration and the Legacy of Venus: Towards an Interpretation of Memory at Early Christian Golgotha,' *Memory and Oblivion*, Springer (1999), pp. 592-602.

57 For the fourth-century floor plan, see Pullan, p. 596, figure 2.

58 Krautheimer, R., 'Introduction,' pp. 1-33.

59 Lauffray, J., '*Le Mémorial Sancti Sepulcri de Narbonne. Mélanges de l'Université Saint-Joseph*, Vol.38, (1962), p. 199-217.

60 Lauffray, J., p. 206, footnote 2.

61 Rey, R., 'La Memoria Sancti Sepulchri du Musée de Narbonne,' *Comptes rendus des séances de l'Académie des Inscriptions et Belles-Lettres*, 93:1, (1941), pp. 21-25. See photos also at Lauffray, two unnumbered pages following p. 200.

62 Diagram, Lauffray, J., the unnumbered page before p. 202, figure 1a.

Here is another attempt to unite circle and square.

After a most through examination of all sources, Lauffray concludes:

> 'La Memoria Sancti Sepulcri di Narbonne parait bien la plus fidèle des représentations de l'édicule constantinien. Le plan de cet édicule ne devait pas être carré, mais polygonal... Ce plan est à rattacher à un type architectural syrien dont le temple rond de Baalbeck est un example [63].'

So, a polygonal Edicule. With a square front. at the back. Pilgrims reported square, round, or polygonal forms; nine or eleven columns. This seems to have depended on one's point of observation. Certainly a seven-sided visualization is possible: four parts of equal length on the back; a rectangular set of three for the Pronaos - the entrance and its two sides. It is even possible that the Rosicrucians got a whiff of this idea, and that the tomb of Prince Ernst at Stadthagen is a truer representation of the Tomb of Christ than one first believed.

Conclusion

Sacred spaces are set apart from the demands of the day-to-day, to evoke in the believer a sense of the Holy, a place where heaven seems to touch the earth. In a time when it was believed that God has disposed all things according to number, weight, and measure [Wisdom 11:21], the quantifiable was a constant reminder of this gift, and the capacity to understand these signs a constant proof of grace.The idea that numbers could explicate the buildings we have considered here is reciprocated by the idea that the buildings explicate the numbers, just as the elements of the heptagon both confront and balance each other, each corner facing a corresponding side seven times.

In the reflection of seven in the rotunda presentation, the π relationship of dome and entrance of Stadthagen, modeled after the Tomb of Christ in Jerusalem (comparing the Venus temples), can be rewritten as (7x3)+1/7, so that 7 is both the end and the point of a new beginning. Rieux Minervois mingles the elements of 3 and 4; Scherpenheuvel mingles the elements of 1 and 6; and Stadthagen combines 3 + 4. Domes are associated with 3 and heaven; the earth is represented by 4. Seven pillars are essential at Rieux Minervois (3 round and 4 square); they are visible at Scherpenheuvel and at Stadthagen at the corners, but the idea of 7 sides is to be emphasized. Further, the architecture amplifies the documentary evidence for the unique Marian interpretation assigned to each edifice: the Mary of Rieux Minervois is Wisdom, with her Seven Pillars; the Mary of Scherpenheuvel is Queen of Heaven; and the Mary of Stadthagen represents on earth the Supreme Wisdom of the seven gifts of the Holy Spirit. The buildings are both a manifestation and a gloss on the range of meaning expressed by the number seven.

63 Lauffray, J., p. 217.

Photo credits

Rieux Minervois:

Figure 1.
Floor Plan. Mrugala, Fabrice. Rotonde, Plan (dessin de André Baraille).jpg. http://medieval.mrugala.net/Architecture/France,_Aude,_Rieux-Minervois/Rieux-Minervois,%20Rotonde,%20Plan%20%28dessin%20de%20Andre%20Baraille%29.jpg Last viewed: 12/08/2014.

Figure 2.
Yeza (own work). 'Église Sainte Marie'. GFDL http://commons.wikimedia.org/wiki/File:Rieux_Minervois.JPG Last viewed: 12/08/2014.

Figures 3 and 4.
Interior with Seven Inner Columns; The Heptagonal Dome. Aubrey, Dennis. Photos 1 and 2. http://vialucispress.wordpress.com/2011/11/05/the-assumption-capital-of-rieux-minervois-dennis-aubrey/ With permission. Last viewed: 12/08/2014.

Figure 5.
Baldiri (own work). 'Assumption Capital.' GFDL. https://commons.wikimedia.org/wiki/File:Rieux-Minervois-Capitell-2.jpg Last viewed: 12/08/2014.

Scherpenheuvel:

Figure1.
Floor Plan. Michel, Gérard 2008. 'Montaigu. Basilique Notre Dame'. Flickr, https://www.flickr.com/photos/gerard_michel/3087359208 With permission. Last viewed: 17/08/2014.

Figure 2.

Town Map. Google Earth. 'Bedevaartsoord (pilgrimage-place) Scherpenheuvel'. Aerodata International Surveys.

Figure 3.

Facade, Dome with 7-pointed Stars. Hermans, Paul 2009. 'Scherpenheuvel gezien van straat.' GNU. Wikimedia Commons, http://commons.wikimedia.org/wiki/File:Belgie_scherpenheuvel_basiliek02.jpg Last viewed August, 16, 2014.

Figure 4.

High Altar. Van Aerschot, Alphons 2009. 'Basiliek Scherpenheuvel Hoofdaltaar.'

Figure 5.

Fouencamps Médaille: Virgin with Stars, Moon and Dragon. 'Attribué au graveur François de Poilly (1623-1693) avec la collaboration de Claude François dit le Frère Luc (1614-1685), "Médaille" du baron de Fouencamps, Notre-Dame de Bon Secours, Vierge-Mère à l'enfant Jésus emmaillotté terrassant le dragon, mai-juin 1672, plaque de gravure en cuivre, 11 x 9 cm, Montréal, Musée Marguerite-Bourgeoys, Chapelle Notre-Dame-de-Bon-Secours. © Photo professeur Robert DEROME.'.www.er.uqam.ca/nobel/r14310/NDdBS/Images/Inverse/tg.jpg With permission. Last viewed: 24/08/2014.

Stadthagen:

Figure1.

Floor Plan.

http://www.renaissance-stadthagen.de/Zusatzinfo mausoleum 7-Eck.htm Last viewed 6/5/2014.

Figure 2.

Mausoleum of Prinz Ernst. Beckstet 2010 (own work). 'Mausoleum von Süd, links der Chor der Martinkirche.' CC BY SA 3. http://www.weserrenaissance-stadthagen.de/index.html?page=http://www.weserrenaissance-stadthagen.de/Zusatzinfo%20mausoleum%207-Eck.htm

Figure 3.

The Heptagonal Dome, with 14 Musical Angels. Carr, Dale C. Groenigen, NL. With permission.

Figure 4.

DeVries: Statue with four Epitaph Panels. Coenan, Mike. Second item at http://www.filmschmiede-schaumburg.de/Abgeschlossen.shtml With permission. Last viewed August, 18, 2014.

Figure 5.

Mystical center of Dome. 'Winfried Berger an der Kern-Orgel in St. Martini') YouTube video 2013. (SBR, Screen Shot at 8:10). http://www.youtube.com/watch?v=hW-kyUj6Y40 Last viewed: 17/08/2014.

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fn 7) http://www.yarmmethodist.church.uk/history.aspx
fn 8) Wikipedia, 'Albert's Chapel'
fn 13) http://www1.umn.edu/ships/updates/newton1.htm
fn 18) http://en.wikipedia.org/wiki/Prince-elector
fn 20) http://en.wikipedia.org/wiki/Santa_Maria_de_Ripoll#mediaviewer/File:Ripoll05.jpg Last viewed: 19/08/2014.
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fn 27) http://www.jstor.org/stable/41133724 Last viewed: 25/08/2014.
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fn 38) http://halshs.archives-ouvertes.fr/docs/00/44/20/38/PDF/articleenligne.pdf Last viewed: 26/08/2014.
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fn 42) For the statuary itself, see: www.wga.hu/d/de_corti/altar_d.jpg
fn 43) http://commons.wikimedia.org/wiki/File:Peter_Paul_Rubens_079.jpg-Last viewed: 26/08/2014.
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Allchin: www1.umn.edu/ships/updates/newton1.htm
Archimon: www.archimon.nl/history/firstprotestantchurches.html
Aubrey: www.vialucispress.wordpress.com/2013/05/22/the-rotunda-churches-dennis-aubrey/
David: books.google.com/books? id=38PWzSh3W-UC&printsec=frontcover&source=gbs_ge_summary_r&ad=0#
Fockema: www.dbnl.org/tekst/fock001duiz01_01/fock001duiz01_01_0025.php
García-Salgado: www.perspectivegeometry.com/sitebuildercontent/sitebuilderfiles/ThePantheonsVault.pdf
Heales: www.archive.org/details/historylawofchur00heal/
Koslow: profkoslow.com/publications/Scherpenheuvel.html.
Spohr: www.dym.dk/dym_pdf_files/volume_39/volume_39_013_033.pdf
symboldictionary.net
Thirion: halshs-00442038v2
Wesley: www.ccel.org/ccel/wesley/journal.html
www.yarmmethodist.church.uk/history.aspx
Wescott: sacred-texts.com/eso/nop/nop14.htm

Dedication

In the 1970s I was a student at Brooklyn College, where I was introduced to Ernest McClain and his wonderful ideas in 'The Pythagorean Plato'. I renewed contact with him about five years ago, and was allowed to listen in on his conversations with a most amazing assemblage of creative minds, with some of whom I have become true Internet Friends. Previously, I had become interested in number symbolism in architecture. I was searching for the source of some octagonal Dutch Reformed churches around New York built in the 1650s. Ernest was a wise counselor, offering encouragement and enthusiasm as I tracked down the octagonal Reformation prototype at Willemstad (1597-1607), and then a surprise - the nearby Counter-Reformation heptagonal basilica at Scherpenheuvel (1609-1627).

This sent me on a hunt for other heptagonal sacred spaces. He called me his 'mugwump' as I shared interesting tidbits, and asked, 'I wonder where this journey is taking you?' This paper is part of my answer. One of the best outcomes of renewing my contact with Ernest was getting to know Vivian Ramalingam. We discovered that we had had the same professors at Brooklyn College, and that we had both given AMS papers in Philadelphia in 1984. It was Vivian who, aware of my 'Septomaniac' obsession, spotted the church at Rieux Minervois, which led to this study. Her interest, collaboration and editorial assistance made the work a joy. Her friendship, wit and humor have been a wonderful addition to my life. Ernest has given me a lasting gift.

Sarah B. Reichart

Sarah Reichart's friendship is the last gift given to me by Ernest McClain. When I arrived at Brooklyn College in 1955, 'Mac' was the Director of the College Band, and I was his first chair trumpet. His unfailing encouragement sustained me throughout my experiences in higher education. He was the very first audience for my public performance of the Hindemith Trumpet Sonata, and I was his first experience with this music. Touchingly, he did not fail to remind me often of this moment, almost until the day he died. I was the first person with whom he shared his groundbreaking discovery of the immutable nature of 'Yellow Bell'.

I too had pursued an interest in architecture and number, so that meeting Sally, both of us devotees of Ernest's exhilarating on-line discussions, was a natural fit. My own first endeavors in this area resulted in a study of the relationship between Brunelleschi's Dome atop the Cathedral of Florence, and *Nuper rosarum flores*, Guillaume Dufay's motet for the Dedication (1436); first presented in 1996, and then at the Renaissance Society of America's 'Y2K' Conference in Florence. Here, I proposed that the inversion of the pair of Tenors, and the acceleration of the motet expressed by means of mensuration and rhythm, represented the anatomy of Mary's swelling, pregnant belly.

Together, our work demonstrates and may help to perpetuate Ernest's conviction that number is an infallible guide to the science and thought of the past.

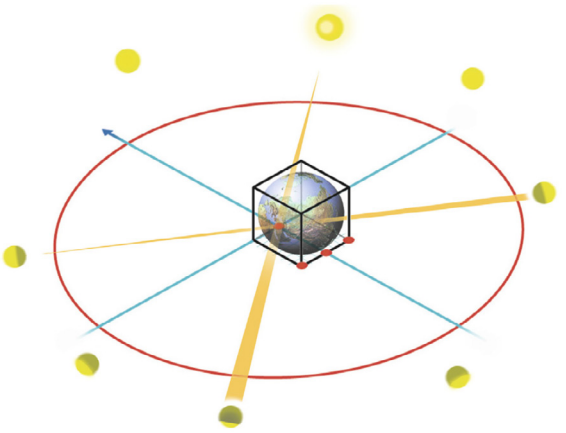
Vivian S. Ramalingam

51-60

PATTERN OF SETTLEMENTS PACED FROM 1 TO 9

The measure of a settlers' cosmology as deciphered from the allegory of Icelandic leather manuscripts

Pétur HALLDÓRSON



Introduction

In this paper, I write my fascination that mankind traces its ancestry to the uplands of the Western Rift valley, around lake Victoria, the headwaters which feed into the White Nile, in equatorial Africa - in other words, to the center of the earth. From there, rivers flow west to the Atlantic, and east to the Indian ocean and from there, early hominids followed their river valleys in all directions, to conquer the earth.

Around lakes Victoria and Edward, relics of man-made tools have been found. They were brought there from a long distance by the so-called 'Nutcracker Man'. According to C14 dating, these finds suggest that the hunters, who made them, possibly knew, and used the seasons in a specialized way, around 2 million years ago. Man had begun to make himself at home in a time-factored complex geometry which was far from simian - a complexity which would deepen and widen his brain[1].

Studying tools of early man from this area, archaeologist Alexander Marshack might have discovered a technique which would have helped crack the code of some of man's earliest intellectual activities: the counting tool stemming from a system of lunar metrology and its marking, ritually, on the Ishango bone[2]. Thus, apparently, as far back as 30,000 B.C., the Ice Age hunters of Europe were using a complex notation system which had already evolved, a tradition which would have already been thousands of years old at that time[3].

Methodology

Research by the Icelandic scholar Einar Palsson into the conceptual background of the Sagas told in 13th century Icelandic manuscripts has revealed a prehistoric image of creation marked in the land as early as Iceland's 9th century, with exquisite geometric and mathematical precision.

Einar argued that the Icelandic Saga literature was rooted in Mediterranean, Celtic and German mythologies, as they appeared in Nordic sagas, Njálsaga and Hrafnkelssaga, to name but two, and were imbued with Pythagorean/Platonic ideas about numbers and proportions as the organizing principles of the cosmos, as it was with other neighbouring mythologies. Pálsson's theories can be grouped into four categories: the mythology theory of Saga origins, the theory of allegorical Sagas, the landscape cosmogram theory, and the socio-mythological theory.

The Icelandic pagan society was conceived as a mirror image of the heavenly organization where 36 goðar represented the heavenly circle

1 Marshack, A., *The Roots of Civilisation*, Weidenfield & Nicolson, London, (1972), pp.72-5

2 Ibid., pp. 72-5.

3 Ibid., pp. 32-3.

and constituted a king in the mythological sense. The goðar were the vehicles of the mythological knowledge associated with the ritual landscape.

Orientation and time-counting was a vital part of the ritual landscape. Implementing it onto the geometry was to apply it to the face of a dial, a sun dial, on the ground with 12 as north. The wider the prairie, the delta, the more sophisticated would be the time measure read from the sun watch.

On deltas of major river systems, or on flat areas, there is evidence of some of mankind's oldest settlements. On these choice lands for cultivation, the geodetic system, Einar read from the allegory of Icelandic skin manuscripts fifteen places all over the world. It appears to have been a device for time counting and orientation. It had a universal size, they all are placed on major river system deltas. They were anchored to nine markers and their magnificent size was 216,000 feet in diameter.

Mankind began to produce their earliest works of art and engaged in spiritual behavior such as a burial ritual around 2.6 million to 12,000 years ago. It can be assumed that around the same time the Cosmic Image as orientation device was initially paced, it became firmly tied to key landmarks with all units of measurement modeled on the human body and proportions found in the path of heavenly bodies. Accurate measurements of time and space, death and rebirth became principal concerns. Subsequently, as the poet said, 'they fused with esthetics and thoughts which stir in the psyche and gradually assumed divine attributes'.

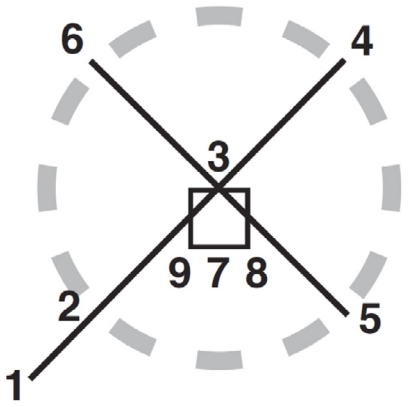


Figure 1. The pattern of Cosmic inage's nine markers.

A Cosmic Image was aligned to a 'foresight', a 'Triple Rock', which was located outside the south-west sector of the land which was to be cultivated (marker 1). The land was measured from a 'Primeval Hill' (marker 2), aligned to the Triple Rock, through the center (marker 3), to a landmark where sunrise on the longest day of the year, summer solstice, was marked (marker 4). Sunrise at the winter solstice appeared south-east from the center, and straight east from the 'Primeval Hill'. From that spot (marker 5) a second line was similarly measured to a point associated with the point of sunset at the summer solstice (marker 6). These two axes were 216,000 feet long and crossed at the center of the area which was to be settled. From the center area, for orientation north-south, 36,000 feet (2 x 18,000) were paced due south to marker 7. From there was placed 18,000 feet due east (marker 8), and 18,000 feet due west (marker 9), thereby an orientation was established north, south, east, west. A sacred area was placed at the center, and was measured in relation to the area of the horizon. An area lined by the horizon which was both circle and square, which seemed to have symbolized the Earth.

This ancient approach is akin to a yardstick which applies not only to the ancient sun watch in Iceland. Recent researches outlined in these pages established that similar reference frames were used in antiquity in all parts of the globe. It brought to light an orientation'blueprint' where cultural areas were built. Such an image of creation might have initiated the famous Stone Age constructions in Europe and underlined cultivated areas of the world. Hence Jelling in Denmark; Saint Denis in France; the Vatican in Rome; Marathon, in Greece; Marble Arch and The Tower of London; Glastonbury, Avebury and Stonehenge in Somerset, and last but not least, Bergþórshvoll in Iceland. All were rooted on a grid, suggesting their location followed a measured plan which predated them.

Since the dawn of recorded history, symbolic imagery of religion, in most countries, has mirrored the same set of basic ideas regarding proportions. An English manuscript from the time of Athelstan, king of England, whose rule overlapped with the age of Icelandic settlements dated from 874, says that laws pertaining to the king's holy verge dictated that it should be measured from a center

3 miles, 3 furlongs, 3 acres, 9 feet, 9 hands, 9 barley corns, the sum of which equating to 3 minutes of the circumference of the earth. An area with 3 minutes radius has 6 minutes (6´) diameter. This depicts an area of the same size as found at the center of Cosmic Images.

The proportions measured so precisely were inherited from the royal surveyors of Athelstan's grandfather, king Alfred, and much earlier, they agreed with ancient Egyptian measurements whereby the Earth was divided into belts of 6 minutes each[4]. This geodetic survey appears to have been an archaic heritage perhaps as old as man's first attempt to stake a claim to his surroundings.

Much later in the evolution of religious ideas, when temples were erected on those spots of geometric measure, they were usually constructed in symmetric fashion, north-south, east-west, orientated, with gardens in their immediate surroundings extending the notion in symbolic form far into the distance as may be seen in many manors in Europe. This symmetry is akin to numerological concepts found in the Kabbalah which aims to connect man's finite world to the infinite God.

In modern terms the proportion assigned to the area within the horizon equals the horizon as seen by a 6 food tall man standing on level ground. The center area is conmmensurate with a 6 x 6 minute square[5]. (1/60 of a degree of the circumference of the earth, that is 6´ or 36,000 feet).

The squared circle, as the center of the Cosmic Image, appears to have been a sacred ground. It was specifically stated in 9th century English laws saying that utmost care had to be taken when traveling inside it. Within it no one could lift a weapon in anger; that was deemed a crime against the king.

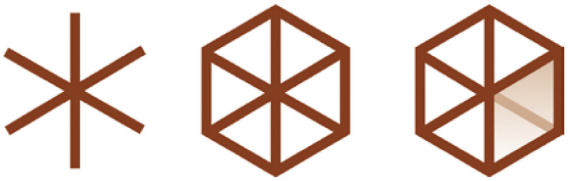


Figure 2. The Kabbala cube: perfect form of creation.

The Kabbalah offers a numerological explanation of the original concept of the cube as a symbol for the almighty, derived from a single point where 6 points on the circumference represent the 6 days of creation. The point at the center is the day of rest. Six internal lines link the points to the centre. Thus, the cube has therefore 8 angles of which only 7 are visible, because the 8th faces the eye, and unites the visible central point with the eighth point, lying behind it. The 6 outer points represent the visible form of the divine power, the seventh, the central point, is a symbol of the divine. The 8th, the invisible original point, is necessary to complete the cube which can only be understood spiritually[6]. When the eye beholds the cube's three-dimensional form, it perceives the moment of perfect creation.

The orientation device, the Cosmic Image, was paced on land from markers one to nine and gradually became a geodetic system which was used for time counting. In the following pages seven samples of the universal Cosmic Image are shown of which the Icelandic is explained in detail.

The Cosmic Image in south Iceland

1. Þrídrangur

Off the country's south coast rises the rock Þrídrangur, 'Triple Rock', from the sea as a tailor-made reference point for the 'Cosmic Image' of south Iceland, governing its geographical location.

The ley-line of the 'Cosmic Image' which extends from 'Triple Rock', measures 216,000 feet from Bergþórshvoll (2) to Stöng (4) as the sunrise marking spot at the summer solstice.

4 Livio Catullo Stechini's Appendix to Peter Tompkins, *Secret of the Great Pyramids*, Galahd Books N.Y., (1971), pp. 343-5.

5 U.S. Coast Guard Addendum to the United States National Search and Rescue Supplement, 27 March 2006, estimating maximum Reception Distances. (1) Determining the R.F.F. Audio Antenna Heights. (p. 5).

6 Bond, F.B., Lea, T.A., *Gematria*, Rilko, London (1977), p. 26.

2. Bergþórshvoll

The actual line from the 'Primeval hill' to Stöng (4) ('Rod' in the English language, 'Herm', in Greek) extended from the 'Primeval hill' marking the sun's lowest altitude in the mid-winter sky, from where it began its ascent on the great sun-watch.

When the sun set behind Bergþórshvoll on the darkest day of winter, a one-year circle was completed. It was the advent of a new year. The sun began to rise again on the horizon, and the fields in the great delta flat-lands in south-Iceland were set to bear fruit once again. The ley line to Stöng symbolized cosmic fertility as it connected these two poles of the Cosmic Image.

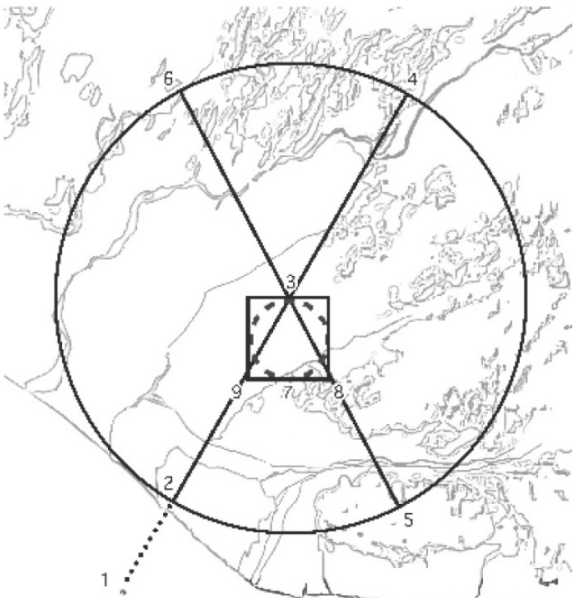


Figure 3. Steinkross.

3. Steinkross

Solstice lines crossed at Steinkross (3), the center of the land that was to be settled. There, a nation in its birth assumed the measure of 'Central Time' and orientation. The measurement ritual was performed in spring when night and day were of equal length. The surveyors came to Steinkross after having measured 108,000 feet from the 'Primeval hill' to unite the 'Spirit of Man' with the 'Sacred Powers' of the cardinal directions. Steinkross became the sacred Center of the settlers' 'Cosmic Image', tied to the symbol of the Polar Star. The 'Cosmic Image' revolved around this place. Steinkross was the starting point for the measurement of the center cube, cube 216. Once that had been done, it was allowed to bury the chieftains who walked into Helgafell to die, for there was the 'Mill of the World', the 'Eye of Mímir'.

4. Stöng

The settlement of Iceland was well documented, as is seen in Landnámsbók, 'The Book of Settlement'. The name of the farmer who lived at the omega point of Stöng at the north-east sector of the 'Cosmic Image', was called Gaukur Trandilsson. 'Gaukur', (cock) and 'stöng' (rod) were well known procreation symbols in ancient Europe.

The ceremony of the *hieros gamos*, the sacred marriage, refers to sexual fertility relationships between deities of agricultural societies, where a ritual re-enacted the holy sexual intercourse between the fields and the sun. The land appeared to its king as a goddess and the king married his kingdom.

The most important duty of priests or kings was to ensure a prosperous future for their subjects. Their tools for that purpose were the holy creation ritual. Such a ceremony may have been held at the brightest day of summer, related to the first harvest, and was performed with mythical symbols, some of which etched in the land. However, the ancients might have articulated the purpose of the ceremony; it appears to have been the creation of a microcosmos of correct proportions.

5. Goðasteinn

Law and order reigned in the invariant rhythm of the stellar spheres. For thousands of years men had looked at the heavens and identified certain stars with the most distant landmarks and fixed directions. Paths of the stellar spheres associated various attributes of nature with specific regions in the sky and transformed groups of stars into mythical images.

The measure from Goðasteinn in Eyjafjallajökull to Skálholt is 216,000 feet. This line together with the 'Stöng' line, marked the ancient 'Cosmic Image' in Rangárhverfi. Viewed from Steinkross, the sunrise at Goðasteinn at Eyjafjallajökull reveals the winter solstice.

6. Skálholt

Sunset at Skálholt opposide Goðasteinn marks the summer solstice; and Whitsun on the Icelandic Cosmic Image. Goðasteinn, stone of the Gods, commanded the beginning of both winter and summer.

Landmarks in this sector are evocative of the red and white Egyptian Ox which ruled that part of the heavens corresponding to that sector of the sun associated with spring and summer, during the Egyptian dynastic period, around 4000 B.C.

The Ox of heaven was linked to Osiris in ancient Egypt. Osiris, or Apis, was both red and white because the Ox was a symbol of the united Egypt, comprising the white state of the upper and the red state of the lower Nile. Names in this sector of the Icelandic sun watch sight agree with the myth reflected in landmarks linked to the white color of the approaching summer in this part of the country, as seden at Hvítá and Hvítárnes, White river, and White peninsula. The grandson of the original settler of this land, Gissur, was also called 'the white' when he preached Christianity to the Icelandic people and donated the land on which was build the country's first church, Patrimonium Petri, or the Church of Peter, later named Skálholt. This ancient Egyptian idea was probably also reflected in a similar idea with a similar name; Patrimonium Petri, the first Basilica of the pope, called Sutri(6), on the same spot within the Cosmic Image of Rome on the Tiber delta and which features later in our story.

7. Helgafell

Mount Helgafell (7) is located 36,000 feet, or 6 minutes of the circumference of Earth, where the sun is in the sky at noon, south of Steinkross, on the south side of the squared area and acts as the defining factor of the Cosmic Image' north-south.

The mountain is one of the most interesting concepts of the whole web which builds-up the Cosmic Image system. The Icelandic sagas tell of men walking into Helgafell to die[7]. On those occasions Helgafell is said to have opened up towards the North, serving as the path of life beyond the earthly one.

From Helgafell's highest point one can see the whole of Rangárhverfi's Cosmic Image. On a rock a chute appears to have been hewed, directing the eye to the north, towards the north's chief symbol in the Cosmic Image, the Pole Star. The same principle can be seen in Paris with the 'Cosmic Iamage' where stands the Observatoire (7). There, this line is also decisively moulded into the surrounding area. The ornamental gardens of the Luxembourg palace point the way north from the Observatoire to the symbol of the Pole Star, the Mill of the World, symbolised by Saint Denis (3). In front of the Observatoire is where Daviouds' statue, 'Four Corners of the World' stands. From the context of ideas englobing the Cosmic Image, it becomes clear why the sculpture was located at this place: there is no other fitting location for the 'Four Corners of the World' in Paris. That is where the gate into the existence beyond is found, at the intersection of this axis of Time, on the line which gives 'stability and persistence'. The north ley line starts from this place to end in a symbolic fashion at the Saint Denis church (3) 36,000 feet or 6 minutes further to the north. Just as Saint Denis is the same symbol as Steinkross (3) in the French Cosmic Image, so a corresponding symbol for the North will be found at this spot in every Cosmic Image. That is one of the hypotheses of this paper.

8. Þríhyrningur

Due east of Helgafell (7) is Mount Þríhyrningur: The place of Starkad in Njássaga and his unkind sons, Thorgeir and Thorkell, who coveted people's material posessions.

The name, Þríhyrningur, means 'triangle', and it symbolizes number 3 in the Cosmic Image web of ideas, which stands for Fire in the Dictionary of Symbols. It is the chief symbol of a New Sun, New Day. When Mount Triangle is viewed from Hof (9) it has three pinnacles, but when viewed from neighbouring areas it has four or more pinnacles. Mount Þríhyrningur was a mountain with three pinnacles in the Cosmic Image only when viewed from Marðarhaugur at Hof (Temple), in the west-east alignement of the Cosmic Image markers; 9 – 7 – 8.

7 Íslensk Fornrit, Hið íslenska fornrita félag, IV Reykjavík, p. 9-10.

Mount Þríhyrningur was appropriately named in the context of the geodetic measure of a Cosmic Image.

9. Marðarhaugur, Hof, Temple

Ketill hængur, Iceland's first law-speaker, lived at Hof, the 9th and last marker of the Cosmic Image. His son, Mörður, whose name's symbolic meaning stands for 'Earth', sums up the notion of a Cosmic Image measure. In it, Mörður Burrow stood as Earth's Burrow. There, we encounter the same idea as seen at Asbanke (9) located at the same points of the Danish Cosmic Image, Saint Cloud (9) for the French one, and at the Church of Saint Peter (9) in the Italian example, to name but a few.

In the south of Iceland, the Cosmic Image is found in strikingly beautiful fashion in communion with the layout of the land. Its emplacement was determined by Þrídrangur, Triple Rock (1), which denoted the origin of the alignment with Stöng (4), 216,000 feet long, measured from Bergþórshvoll (2). Goðasteinn (5) at Eyjafjallajökull lies to the east from Bergþórshvoll. From Goðasteinn there are 216,000 feet to Skálholt (6). The Center (3) is at Hekla's lava field. Six minutes south from there lies Helgafell (7), offering a panoramic view of the entire Cosmic Image. Viewed from Hof (9), three tops crown Mt. Þríhyrningur (8), aligned with the top of Helgafell. The length of this alignment is also six minutes. Þjórsá ties the system to the symbol of the vernal equinox to the west, where a young summer sun is linked to the river in the land of Kálfholt, the Calf hill.

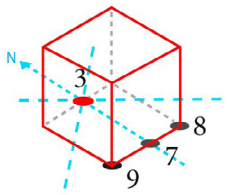


Figure 4.

Steinkross	Iceland, S.
Hít	Iceland, W.
St. Denis	France
Jelling	Denmark
The Devil's Bed & Bolster	Somerset, England
Copetts Wood/St. Pancras	London, England
Settabagni	Rome
Saint Peter's cript	St. Peters, Rome
Nekrotafeio	Athens
Memphis	Egypt
Har HaMenuchot	The Holy Land
Morgano	Italy. N

Steinkross is the center of the Cosmic Image in Rangárvellir, Iceland. Figure 4 shows its corresponding ideological places in other Cosmic Images.

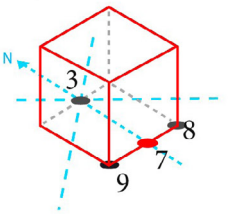


Figure 5.

Helgafell	Iceland, S.
Tindaborg	Iceland, W.
Observatoire	St. France
Lysholt	Denmark
Heaven's gate	Somerset, England
Marble Arch	London, England
San Lorenzo fuori le Mura	Rome
Stairway, Saint Peter's cript	St. Peters, Rome
Nekrotafeio	Athens
Heliopolis	Egypt
Church of Nativity	The Holy Land
Salzano	Italy

Helgafell is the decider of the north – south orientation within the Cosmic Image in Rangárvellir. Figure 5 shows its corresponding ideological places in other Cosmic Images.

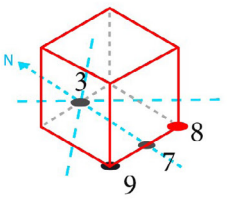


Figure 6.

Þríhyrningur	Iceland, S.
Unknown marker	Iceland, W.
Porta Dorée	St. France
Nyhöje	Denmark
Robin Hood Bower	Somerset, England
The Tower	London, England
Piazza de Cupis	Rome
Right side Saint Peter's cript	St. Peters, Rome
Moni Karea	Athens
Unknown marker	Egypt
Church of Nativity	The Holy Land
Trevisto	Italy. N

Mount Þríhyrningur is the 8th marker of the Icelandic Cosmic Image in Rangárvellir. Figure 6 shows its corresponding ideological places in other Cosmic Images.

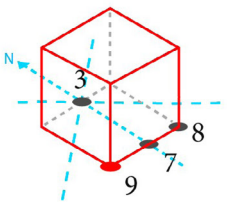


Figure 7.

Hof	Iceland, S.
Unknown marker	Iceland, W.
Saint Cloud	St. France
Asbanke	Denmark
The Bushes	Somerset, England
Acton	London, England
Saint Peters Basilika	Rome
Left side Saint Peter's cript	St. Peters, Rome
Pireas	Athens
Saqqara	Egypt
Nahalin	The Holy Land
Noale	Italy. N

Hof is the 9th and last marker of the Icelandic Cosmic Image in Rangárvellir. Figure 7 shows its corresponding ideological places in other Cosmic Images.

Figure 7. Comparison of the four markers (3, 7, 8, and 9) of the center area of 11 Cosmic Images universaly. In every Cosmic Image the sacred center, marker 3, was a burial ground.

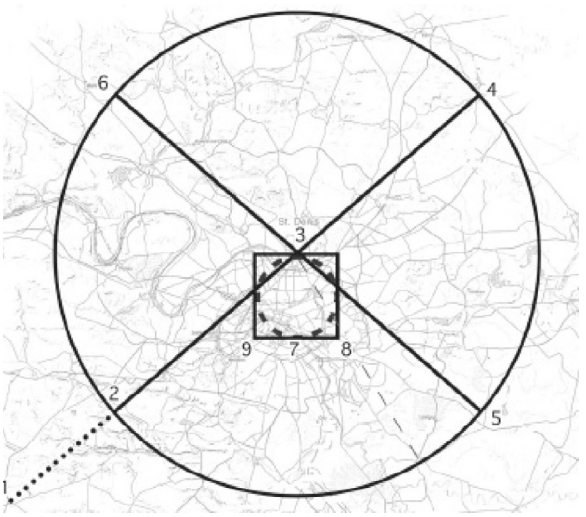


Figure 8. The Cosmic Image around Paris.

Main alignments of the Cosmic Image around Paris are: 1. Rambouillet is the fore-sight of the Cosmic Image. 2. Port Royal is the Primeval Hill. 3. Saint Denis is the Sacred Center. 4. Ermenonville. 5. Église Saint-Étienne church by the Brie-Comte-Robert Castle near Belle Croix. 6. Cormeilles en Vexin. 7. Observatoire. 8. Porte Dorée. 9. Saint Cloud.

The distance between Port Royal and Ermenonville, as the crow flies, is 216,000 feet (about 66.6 km). The distance between medieval Église Saint-Étienne church, with its original stained glass rose window symbolizing the solarcycle, to the medieval church in Bréancon is 216,000 feet (about 66.6 km).

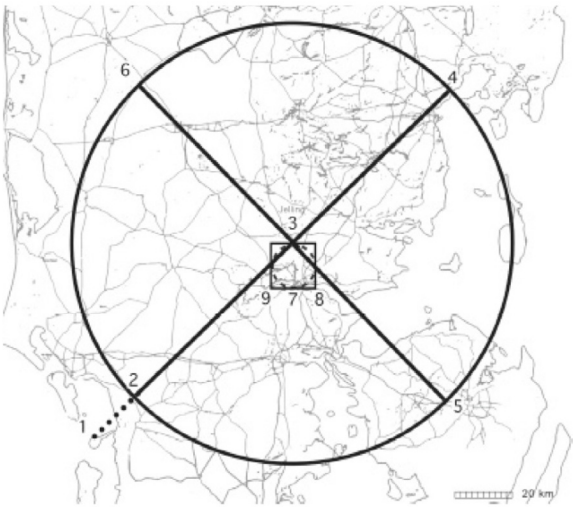


Figure 9. The Danish Cosmic Image.

The Cosmic Image of Denmark is tvice the size of other Cosmic Images, perhaps due to the vast open spaces of the Danish peninsula - as on the Nile delta.

Main alignments of the Cosmic Image in Denmark: 1. Mandö as the fore-sight of the Cosmic Image. 2. Ribe, the Primeval Hill. 3. Jelling is the sacred Center. 4. Arhus. 5. Odence. 6. Trehöje. 7. Lysholt. 8. Nihöjen. 9. Asbanke.

The alleged distance between Ribe and Arhus, as the crow flies, is 432,000 feet (2 x 216,000 feet, about 133.2 km). The alleged distance between medieval Odence and Trehöje is 432,000 feet (2 x 216,000 feet, about 133.2 km).

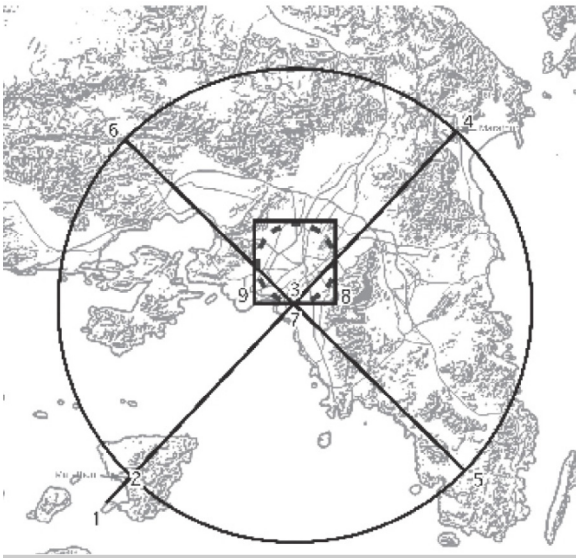


Figure 10. The Cosmic Image in Greece.

1. Nea Moni, the fore-sight of the Greek Cosmic Image. 2. Marathon on Égina island is its Primeval Hill. 3. Nakretofeio is the First Cemetery in Athens, the sacred center of the Greek Cosmic Image. 4. Marathon is 216,000 feet northeast from Marathon on Égina island, with a 45° angle on latitudes. 5. Olympus. 6. Levka is 216,000 feet north east of Olympus. 7. Obscure marker. 8. Moni Karea 9. Piraeus.

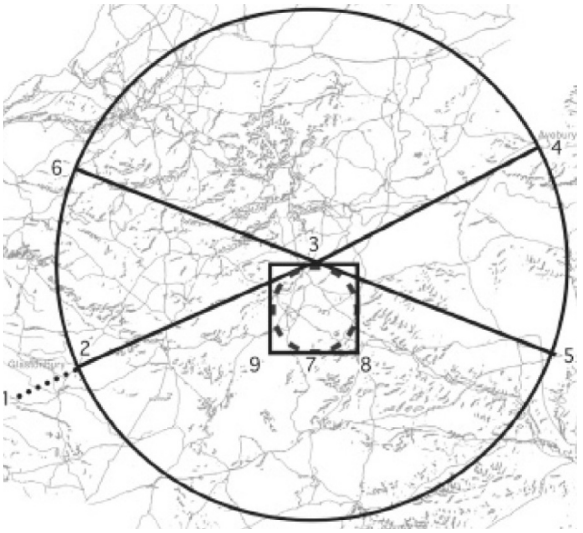


Figure 11. The Cosmic Image of Somerset.

Main alignments of the Cosmic Image in Somerset are: 1. Burrows Mump as its fore-sight. 2. Glastonbury, the Primeval Hill. 3. The Devil's Bed and Bolster is the sacred Center. 4. Avebury. 5. Stonehenge. 6. Bristol. 7. Heaven's Gate. 8. Robin Hood's Bower. 9. The Bushes.

The distance from Glastonbury to Avebury is 65,200 m, and from Stonehenge to the archaic burial chamber in Nailsea, Bristol is 64,600 m (64,130 m = 216,000 feet). These alignments point to sunset and sunrise at summer and winter solstices. They intersect at the stone circle, Devil's Bed and Bolster, from which there are 10,837 m (6 minutes of Earth's circumference) to Heaven's Gate in magnetic south. These are measured proportions between some of the oldest manmade stone structures in Europe, rooting them on a grid which suggests their location, is according to a measured plan which predates them.

The Cosmic Image of the Latium delta is 216,000 feet in diameter. Its nine markers are: 1. Lido di Faro is the fore-sight of the Etruscan-Roman Cosmic Image. 2. Isola Sacra is its Primeval Hill. 3. Settebagni is its sacred Center. 4. San Maria in the Sabine Mountains. 5. Rocca di Papa. 6. Sutri. 7. San Lorenzo fuori le Mura. 8. Piazza de la Cupis 9. Saint Peter's Basilica in the Vatican.

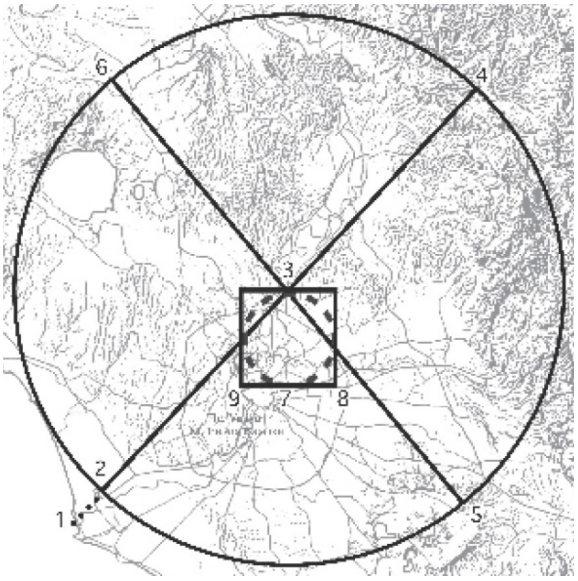


Figure 12. The Cosmic Image around Rome.

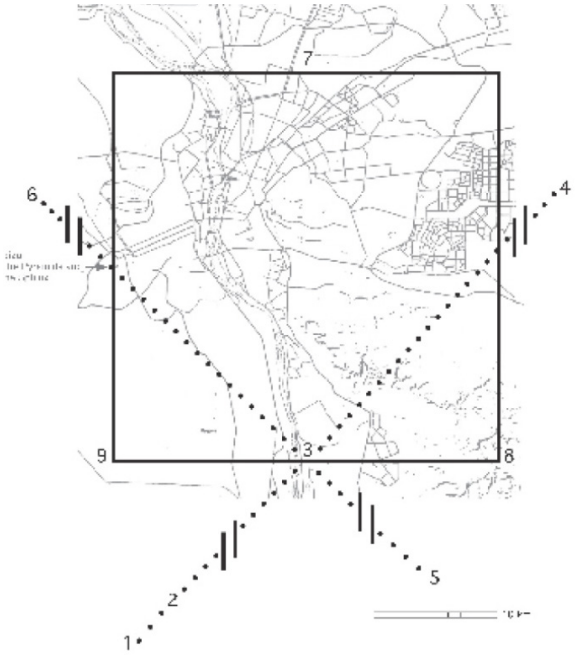


Figure 13. The Cosmic Image of Egypt.

1. The fore-sight of the Cosmic Image of the Nile delta is assumed to be somewhere in the south-west desert. 2. Medinet Maadi is the Primeval Hill, 216,000 geographical cubits, 97.2 km, south-west of Memphis with a 45° angle of latitude. 3. Memphis is assumed to be the sacred center of the Egyptian Cosmic Image. 4. Pelusium is 1000 Stadia, or 400,000 Cubits north-east of Memphis, with a 45° angle of latitude. 5. The Monastery

of Saint Anthony is 162 km with a 47° angle of latitude south-east from Memphis. 6. Alexandria is about 108 minutes of the earth's circumference, 203 km, north-west with a 46° angle of latitude. 7. Heliopolis is 6,000 Royal cubits (31,5 km) north of Memphis 8. (Unknown marker) 9. Saqqara.

Pétur Halldórsson is an artist. The programme introduced here is a research into the roots of Icelandic culture, based on the work of the Icelandic scholar Einar Pálsson. Ideas connected with specific areas within societies are examined with a view to ascertain the relationship between them. The question is whether it is possible to probe the meaning of myth and symbol and to connect those to specific geodetic measures of a sun watch (Cosmic Image) with a diameter of 216(000) U.M. This entails marked cardinal and solstice directions that define a specific system paced on land. Such material constitutes a large part of our sources and cannot be ascertained unless we understand which part of a narrative is based on myth related to a universal background, the stars. Traditional methods do not apply for this task, to find the shape of the sun watch we apply the working method of a landscape artist.

More information see: www.peturhalldorsson.com

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THE METONIC CYCLE AND THE PARTHENON: *The Tropical year of the Parthenon and Lunar year durations*

Anne Maria Louisa BULCKENS

Introduction

Jay Kappraff had found some basic similarities between a Vedic ceremonial altar construction (known as the Rathacakra Citi Chariot Wheel) and the Parthenon when using 120 Dactyls as one large unit. He asked me to explore if and how the Parthenon designers might have represented the duration of a Solar and Lunar year in the Parthenon, the main temple on the Acropolis of Athens. This article gives the answer to this question. The outcome is that the Parthenon design elements, including the careful placing of its railings, express accurately the duration of the Tropical year and of the Lunar year, similar to - but more accurate than - Meton's findings of 432 BC. According to Meton, 235 Lunar months are equal in duration to 19 Solar years. (A Solar year is the time it takes the earth to revolve around the sun, and this was very hard to measure. The Tropical year is, for example, the time it takes from Summer solstice to Summer solstice or from equinox to equinox. Meton measured the duration of the Tropical year. Solar years and Tropical years are about equal in duration.) It means that after 19 Tropical years, the moon and sun return to the same positions, relative to the earth, as 19 years earlier. The phases of the moon, such as new moon, full moon, etc., are on the same dates as 19 years earlier. In the Metonic cycle, a synodic Lunar year (consisting of 12 synodic Lunar months) is nowadays defined as 354.367 days long and hence Meton's Tropical year is 365.2467 days long, since:

A Tropical year of 365.2467 days x 19 = a Lunar year of 354.367 days x 235/12.

Meton's works have not survived, but some later sources wrote some snippets about Meton's cycle[1].

The Parthenon foot is 343 millimeter and consists of 16 Dactyls per foot (Dactyl is henceforth abbreviated by D). Measurements are based on the accurate measurements taken by Francis Cranmer Penrose. One Penrose foot is equal to 304.6845 mm, and hence one Penrose foot is equal to 14.212688 Ds.

The Parthenon designers aimed to show the length of a Lunar year and the length of a Tropical year in the Parthenon. For this purpose, I found that they decided that an area of 120 x 120 square D represents one day in time, and I call this one 'Parthenon Day Unit'. So, 14400 square Ds represent one day in time, and the Parthenon designers followed the system timewise by which there fit 24 hours of 60 minutes in one day, with 60 seconds per minute. Thus, one 'Parthenon Day Unit' is equal to 14400 square Ds and this represents one day in time which counts 1440 minutes. It follows then that one square D represents $1/10^{th}$ of a minute, which is equal to six seconds in time.

These are the area relations to time: D^2

1 Parthenon Day Unit = 120 D x 120 D = 1 day

14400 D^2 = 1 day

14400 D^2 = 1440 minutes (with 60 minutes per hour in 24 hours per day)

1 D^2 = 1440 minutes / 14400

1 D^2 = 6 seconds

1 D^2 = 1/14 400 x 1 day (with 24 hours in 1 day).

1 D = $\sqrt{6}$ seconds, for which (49/20) x 4800/4801 is an excellent approximation.

1 cube D= 6 seconds x (49/20) x 4800/4801 = 70560/4801 seconds or 4.9 / 28806 days.

To express the duration of the Tropical and Lunar years, the principle idea in the Parthenon is

1 http://www.encyclopedia.com/doc/1G2-2830902934.html

that its designers regarded a specific stylobate area as base. To this base, they added other areas found in the cella to arrive at the duration of the Tropical and Lunar years.

The stylobate, and even the cella, with to the east its naos and to the west its opisthodomos, had been designed and built to specific measurements. So, the area of the stylobate, the naos and the opisthodomos had already been decided. And when adding areas at the naos or opisthodomos, it was impossible to arrive at the accurate duration of a Tropical year and a Lunar year. The solution to this problem was brilliant. The Parthenon designers decided that for the Tropical year, there would be 2 specific areas, and the average of these 2 areas is the area that has to be added to the specific stylobate area in order to convey the length of a Tropical year. They follow a similar procedure to show the duration of a Lunar year.

How did they show these additional measurements for these additional areas? By carefully placing a railing around the statue of Athena, a railing with a specific width and a specific distance from the east stylobate edge. For the Lunar year measurement, they used almost the same width as this railing around the statue of Athena, and they had to place an additional railing much more in front of the statue of Athena.

Finally, the Parthenon even represents interesting multiple Tropical and Lunar years when volumes are considered.

When measuring and finally taking the average of these areas and volumes, the outcome is that the Parthenon builders represent the Metonic cycle carefully. The Parthenon was finished in 432 BC, and the Greek Meton wrote about the duration of the Tropical year and Lunar years also in 432 BC. So, at that moment, the duration of Tropical and Lunar years were a hot topic! This article will show how precisely the Metonic years are found in the measurements of the Parthenon.

I write everything in the following calculations in the decimal system, because that is what we are used to nowadays. Yet it is quite feasible to compute all this in the system of seconds, or minutes or days. I cut off the time in years after the sixth decimal. In fact, already the fifth decimal is a fraction of a second, so the sixth decimal is a very small fraction of a second. I used an online calculator because it allows for more decimals than a hand calculator, so the calculations are more accurate.

In my previous work, I found that the Parthenon measurements followed as 'ideal' the 4/6 = 6/9 proportion, when allowing a large margin of inaccuracy of up to 0.2%. I find that these 'ideal' measurements were then altered for specific reasons. When in this paper I employ the accurate measurements of the Parthenon, instead of only using round Ds, and I employ eighths of Ds and sixths of Ds, then the results are astonishing! The Parthenon builders were certainly able to build to such high level of accuracy using eights and sixths of Ds, since the Parthenon construction attests of this, as is explained by Dr. Manolis Korres[2]:

> 'Recent observations and measurements have added much to our knowledge of the celebrated precision with which the Parthenon was constructed: for instance, the resting and bearing surfaces of the middle drums of all the columns are so perfectly flat that at no point is there a divergence from the mathematical plane of more than 1/20 of a millimetre ... These surfaces are so perfectly parallel that along the perimeter of any of the drums its height fluctuates by no more than 15/100 of a millimetre. And - wherever they have survived - the joints in the crepidoma are so hermetically tight that they cannot be examined even when seen under a microscope with a power of x 60. It is clear, then - at least to those sufficiently familiar with the quality of engineering work - that even the most refined structures built in other eras (including our own) often lag far behind the constructional precision of the Parthenon.'

The specific stylobate area that is considered

For the length, the south side is considered, since the south refers to the sun. It is 3242.25 D. 228.154 Penrose feet is 69514.987 mm. When the 10 millimeters marble growth of the south stylobate perimeter due to creep are subtracted, then this measurement is accurate within one millimeter.

For the width, the average between the east stylobate and the west stylobate is considered, because the sun goes from east to west. It is

2 Korres M., 'The Architecture of the the Parthenon' in *The Parthenon and its Impact in Modern Times*, Panayotis Tournikiotis (ed.), Melissa Publishing House, (1994), p. 66.

1439.625 D. This average is 101.351 Penrose feet, which is 30880.078 mm. When the 18 millimeters (average) marble growth of the east and west perimeter due to creep are subtracted, then this measurement is accurate within 1 millimeter.

This stylobate area is 3242.25 D x 1439.625 D x (1/120 D x 1/120 D) Parthenon Day unit = 324.140566 Parthenon Day units = 324.140566 days.

Tropical year measurements in areas.

On top of this stylobate area there are two areas taken at the east side, and when the average area is added to the stylobate area, then the average duration of the Tropical year of the area is determined. The sun rises at the east, so the Tropical year measurements are to be found at the east side.

First measurement (See figure 1).

Width

The width involves the colonnade that surrounds the statue of Athena in the naos.

It is the width between the raised slabs upon which these columns stand.

32.233 Penrose feet = 458.117 D = 458.125 D (0.16 mm off).

Length

The length also involves this colonnade and it goes from the entrance of the naos to the slabs upon which this colonnade stands that goes also behind the statue of Athena.

Towards the left side in the naos: (6.87 + 56.335 + 27.11) Penrose feet = 90.315 Penrose feet = 1283.618 D = 1283.625 D (0.15 mm off).

Area of the first Tropical year measurements in Parthenon Day Units

458.125 D x 1283.625 D x (1/120 D x 1/120 D) Parthenon Day unit = 40.837548 Parthenon Day Units = 40.837548 days.

Second measurement (See figure 2).

Width

The width involves the railing that surrounds the statue of Athena; it goes from one side of the railing to the other side of the railing.

(32.233 – 2.898 – 2.9) Penrose feet = 26.435 Penrose feet = 375.7124 D = 375.666… D (0.98 mm too short).

Length

The length involves this railing that surrounds the statue of Athena.

It goes from the east stylobate edge to the back of the railing that surrounds the statue of Athena.

(17.138 + 17.78 + 6.87 + 56.335 + 27.11 - 13.688) Penrose feet = 111.545 Penrose feet = 1585.326 D = 1585.333… D. (0.44 mm off).

Area of the second Tropical year measurements in Parthenon Day Units

375.666… D x 1585.333… D x (1/120 D x 1/120 D) Parthenon Day unit = 41.358117 Parthenon Day Units = 41.358117 days.

The average of these two areas gives: (40.837548 + 41.358117) days / 2 = 41.097832 days.

The days of the stylobate need to be added to find the duration of the Tropical year. Thus, 41.097832 days + 324.140566 days = 365.238398 days.

This is the duration of the Tropical year as expressed in the average area of the Parthenon: 365.238398 days.

Lunar year measurements in areas

On top of the stylobate area there are two areas taken from the west side, and when the average area is added to the stylobate area, then the average duration of the Lunar year of the area is determined.

The sun sets in the west and then night starts. So, the Lunar year measurements are to be found taken from the west side. Since the Metonic cycle is also related to the Tropical year, the lengths reach from the west to well within the naos in the east side of the Parthenon.

1) The twelve-month Lunar year (See this area shown in drawing 3).

Width

The width involves the same railing that surrounds the statue of Athena; it goes from one side of the railing to the other side of the railing. It is 375.75 D.

At one side, the railing is 2.898 Penrose feet away from the slabs upon which the naos columns

stand, whereas at the other side, the railing is 2.9 Penrose feet away from the slabs upon which the naos columns stand. And the actual width of 375.7124 D is about right in the middle between 375.666… D and 375.75 D. I propose this width is designed this way on purpose.

For the Tropical year calculation the width has to be regarded as 375.666… D (0.98 mm too short), whereas for the Lunar calculations the width has to be regarded as 375.75 D (0.81mm too long).

Length

The length goes from the interior face of the west wall of the opisthodomos to the west-end of the 'spine' of the statue of Athena - the 'spine' keeping the statue of Athena upright was a huge pole in the statue put firmly into the floor of the Parthenon. This central 'hole' for the statue is a crucial place in regard to the Parthenon measurements and design.

Penrose did not provide measurements of the five stones in length underneath the statue of Athena, and he did not draw them equally in size. If he made his drawing perfectly to scale, then the west-end of the 'hole' for the statue of Athena measures from the east side, 16/25 of the total length of the five stones measuring 8.535 Penrose feet, which is 5.4624 Penrose feet. And the length from this west-end of the 'hole' for the statue of Athena to the west-end of these five stones underneath the statue measures (8.535 – 5.4624) Penrose feet = 3.0726 Penrose feet.

a) The length near the opisthodomos door

The entire length from the east stylobate edge to the face of the interior west wall of the opisthodomos is:

(17.138 + 17.78 + 6.82 + 98.145 + 3.025 + 43.767 near the door of the opisthodomos) Penrose feet = 186.675 Penrose feet.

There needs to be subtracted the length from the east stylobate edge to the west-end of the 'hole' for the statue of Athena:

(17.138 + 17.78 + 6.87 + 56.335 + 27.11 – 16.108 – 8.535 + 5.4624) Penrose feet = 106.0524 Penrose feet.

So, the length from the west-end of the 'hole' for the statue of Athena to the wall face at the opisthodomos door is:

(186.675 – 106.0524) Penrose feet = 80.6226 Penrose feet = 1145.8638 D = 1145.875 D (0.24 mm off if the west-end of the 'hole' for the statue of Athena is at exactly 16/25 of the length of the five stones).

Area of the twelve-month synodic Lunar year (Measurements in Parthenon Day Units):

375.75 D x 1145.875 D x (1/120 D x 1/120 D) Parthenon Day unit = 29.900175 Parthenon Day Units = 29.900175 days.

The days of the stylobate need to be added to find the duration of the synodic Lunar year.

29.900175 days + 324.140566 days = 354.040741 days.

b) Alternative length to calculate 254 sidereal Lunar months: the length near the cella side wall

The length from the west-end of the 'hole' for the statue of Athena to the west-end of the five stones underneath the statue of Athena measures 3.0726 Penrose feet.

The further length to the interior face of the west wall of the opisthodomos, near the cella side wall is:

(3.0726 + 16.108 + 14.265 + 71.334 – 17.33 – 6.81) Penrose feet = 80.6396 Penrose feet = 1146.1054 D = 1146.125 D (0.42 mm off if the west-end of the 'hole' for the statue of Athena is at exactly 16/25 of the length of the five stones).

Area of the twelve-month Lunar year (Measurements in Parthenon Day Units).

375.75 D x 1146.125 D x (1/120 D x 1/120 D) Parthenon Day unit = 29.906699 Parthenon Day Units = 29.906699 days.

The days of the stylobate need to be added to find the duration of the sidereal Lunar months.

29.906699 days + 324.140566 days = 354.047265 days.

2) Thirteen month Lunar year (See figure 4).

The length of the 12 month Lunar year could never be seen, since the socle and the statue of Athena covered it. They also placed a large railing way in front of Athena, and then the procedure is the same as was needed to find the duration of the Tropical year shown in the Parthenon.

Hereto, they used a '13 month Lunar year'. In the 'Metonic cycle' seven extra Lunar months are inserted in order to fit exactly in a span of 19 solar years since (19 x 12) Lunar months per Lunar year + seven Lunar months, are 235 Lunar months in total. Thus, a year where they add 1 Lunar month can be regarded as a Lunar year with 1 intercalated month, called an 'embolismic' month, inserted to it, so that it is a Lunar year of 13 months, and this is what the Parthenon measurement alludes to. It portrays the 'Lunisolar system' which Meton wrote about: 19 solar years are equal to 235 Lunar months. Meton's aim was to make it a base for a Greek calendar to be used by astronomers. There is no evidence that can support that he aimed to make it the official calendar for Athens.

A Lunisolar calendar is a calendar that combines the phases of the moon and the seasons into one calendar. Later sources mention that Meton proposed to round off this duration of 19 Tropical years and 235 Lunar months to 6940 days. (This would probably count 125 months of 30 days and 110 months of 29 days.) The phases of the moon would then happen on the same dates as 19 Tropical years earlier, when counting the days each year for example from solstice day onwards.)

How accurate is such calendar?

225 Synodic months count 6939.688415 days and 19 solar years count 6939.6016 days. So, the true difference between the Solar and Lunar year is about 2 hours and 5 minutes in a span of 19 years. This does not mean that 19 years later, the moon (for example the full moon) would rise and set with a time difference of several hours, but it does mean that after 19 years the full moon would rise and set later by several minutes.

The Metonic cycle is also the base for the Hebrew calendar. Later, in Christianity, the sequence of 'Golden Numbers', used for determining the date of Easter, begins at one when the new moon falls on the first of January. That year was one and these years were considered so important that it was said that these years (multiples of 19 years) were inscribed in golden letters on a temple in Athens, hence the term 'The Golden Number'. In the sixth century AD, the Parthenon temple became the Orthodox Cathedral of Hagia Sophia ('Holy Wisdom'), while many centuries later it became a Catholic church, befittingly dedicated to 'Saint Mary of Athens'. It would be so nice if these 'Golden Numbers' were actually inscribed in this one particular 'Athenian temple-turned-into-church': the Parthenon.

Width

The width involves the same railing as the railing that is used to show the duration of the Tropical year and the 12 month Lunar year. It goes from one side of the railing to the other side of the railing.

26.435 Penrose feet = 375.7124 D = 375.5 D (0.81 mm off).

Length

The length goes from the centers of the third columns of the colonnade all the way to the edge of the stylobate at the west side. There would not have been anything particular about these columns and therefore, to put the emphasis on these particular columns, a railing was installed connecting these columns and demarcating the most sacred area for the statue of Athena. Behind these columns, the colonnade counts seven columns at each side per story, with seven constituting the virginal number, befitting the statue of Athena 'Parthenos' (meaning 'Virgin') and 28 columns in total of the two rows of two stories, perhaps alluding to 28 days (four weeks of seven days) as an easy to count Lunar month. In front of the railing, in the more 'worldly area' of the naos, there are four naos columns on the ground floor, with the number 4 symbolizing 'Victory'. Moreover, as was said, there stand four columns in front of the railing on the ground floor of the naos, six columns in total stand behind the statue of Athena between the rows of columns, and four columns of the upper story stand in front of the railing, bringing out the symbolism of 4 - 6 - 4.

This length measures 2298.5 D

(8.55) (average interaxial of naos columns since Penrose does not provide the measurement of this one particular interaxial) + 25.662 + 8.535 + 16.108 + 14.265 + 71.334 + 17.27) Penrose feet = 161.724 Penrose feet = 2298.533 D = 2298.5 D.

Area of the second Lunar year measurements

375.75 D x 2298.5 D x (1/120 D x 1/120 D) x Parthenon Day Unit = 59.976484 Parthenon Day Units = 59.976484 days.

The days of the stylobate need to be added to find the duration of the Lunar year.

59.976484 days + 324.140566 days = 384.117050 days.

So, 384.117050 days is the duration of the Lunar year of 13 months as expressed in the Parthenon.

Thus, its Lunar year of 12 months is 384.117050 days x 12/13 = 354.569584 days long, according to this area in the Parthenon.

The average of the two Lunar areas gives the duration of the Lunar year in the area:

a) Area needed to determine the synodic Lunar months

(354.040741 days + 354.569584 days) / 2 = 354.305162 days.

This is the duration of the synodic Lunar year as expressed in the average area of the Parthenon: 354.305162 days.

b) Area needed to determine the sidereal Lunar months

(354.047265 days + 354.569584 days) / 2 = 354.308424 days.

This is the duration of the longer Lunar year as expressed in the average area of the Parthenon: 354.308424 days. It helps to determine the sidereal Lunar months.

The volumes in the Parthenon representing multiple Lunar and Tropical years

Since the stylobate area was too small to represent a Tropical or Lunar year, other areas from the cella had to be added to the stylobate area. Since volumes are large, multiple Lunar and Tropical years are directly represented in the Parthenon.

With a square D representing 6 seconds, it follows that a D is the √6 Ds x √seconds. Seconds are regarded as the base unit 1. So, a root second is 1. When considering volumes, then 1 cube D is 6 seconds x √6.

What is the √6 approximation of the Parthenon?

Since there is in the stylobate an obvious√2, which indicates that the Parthenon designers know the Pell series of √2 (such as 17/12, 41/29, 99/70, 239/169, 577/408, …), they use in this case 140/99 (which would accompany 99/70 to form 2) as an approximation of √2, and since in the Parthenon the accompanying √3 value of 693/400 x 4800/4801 is also present, they use for √6 this multiplication of √2 x √3, which is (49 / 20) x (4800/4801).

((49/20) x (4800/4801) x (49/20) x (4800/4801) = 5.99999973969)

Thus, (49 / 20) x (4800/4801) is an excellent approximation of √6.

It follows that, using this value for √6, that 1 cube D represents 6 seconds x (235200/96020) seconds, which is (70560/4801) seconds. This is equal to (1176 / 4801) minute, and it also represents (19.6 / 4801) hour. It is also equal to 245/1440300 day, which is 49/288060 day, which is 4.9 day per cube D/28806 (for ease of calculations, written in the final multiplications as the third factor in the row of multiplications).

The measurements considered in the volumes are shown in drawings 5, 6 and 7.

The volumes representing Lunar years

Volume representing 660 Lunar years

This volume relates to the volume of the entire cella.

Athena, in Greek gematria, is 66. In the Parthenon one often finds tenfolds of important numbers. Thus, Athena herself, residing in the cella, is in the Parthenon related to the moon.

Height

The height is the height of the cella walls, including its entablature. (The height of the entablature is the same above the walls as above the porch columns.)

(33.076 + 3.424 +3.324 + 1.128 + 1.78) Penrose feet = 42.732 Penrose feet = 607.33658 D = 607.333… D (0.07 mm off).

Width

The width is the width of the cella, including

its walls.

(63.01 + 3.83 + 3.83) Penrose feet = 70.67 Penrose feet = 1004.4106 D = 1004.333… D (1.66 mm off, but towards the east, the naos becomes narrower. For example, the upper cella platform measures at the opisthodomos 71.33 Penrose feet, while at the naos this platform only measures (32.233 + 19.525 + 19.55) Penrose feet = 71.308 Penrose feet)

Length

The length is the length of the cella, including its walls and 'plinths'. The considerable thicknesses of the 'plinths' in the Parthenon are never arbitrarily. These 'plinths' are in fact the lowest course of fatter stones upon which the narrower walls stand.

(6.82 + 98.145 + 3.025 + 43.803 + 6.81) Penrose feet = 158.603 Penrose feet = 2254.1749 D = 2254.1666… D (0.18 mm off).

Volume representing 660 Lunar years:

2254.1666… D x 607.333… D x (4.9 day per cube D / 28806) x 1004.333… D =

660 Lunar years x 354.372658 days per Lunar year.

Volume representing 464 Lunar years

In ancient Greece and in Greek gematria, there was, according to Michael Schneider, Athena celebrated as 'The Mother' ('H MHTHP'), which is 464.

Height

The height to be considered is the height of the exterior temple from the center of the stylobate to the top of the cymatium of the pediment.

58.902 Penrose feet = 837.1557 D = 837.125 D (0.66 mm off).

Width

The width is the width of the colonnade of the naos, including the columns.

(36.243 + 3.656) Penrose feet = 39.899 Penrose feet = 567.07203 D = 567.125 D (1.14 mm off).

Length

The length is the length from the east stylobate to the end of the naos, including the partition wall and its 'plinths'.

(17.138 + 17.78 + 6.87 + 56.335 + 27.11 + 14.265 + 3.785) Penrose feet = 143.283 Penrose feet = 2036.4365 D = 2036.375 D (1.32 mm off).

(1440 D x 140/99 (accompanying 99/70 as approximation of √2) = 2036.3636 D.)

Volume representing 464 Lunar years:

837.125 D x 567.125 D x (4.9 day per cube D / 28806) x 2036.375 D =

464 Lunar years x 354.423087 days per Lunar year.

a) The synodic Lunar year (the average of its three Lunar years) in the Parthenon is 354.366969 days.

(354.305162 days + 354.372658 days + 354.423087 days) / 3 = 354.366969 days in one synodic Lunar year.

Hence, since there is no written evidence of that era stating the exact length of a synodic Lunar year, the Parthenon might very well be, as structure, the first instance that attests of a synodic Lunar year of almost 354.367 days (less than three seconds shorter than the synodic Lunar year of 354.367 days).

The Parthenon synodic Lunar year of 354.366969 days x 1/12 is 29.530580 days.

The true mean length of the synodic month is 29.530589 days.

This Parthenon Lunar year average represents synodic months, and the Parthenon mean length of the synodic Lunar month is less than one second shorter than the true average synodic Lunar month. Hence, it is about 99.99996 % of the true average synodic Lunar month.

b) The longer alternative Lunar year (the average of its three Lunar years) in the Parthenon is 354.368056 days, and it helps to determine the sidereal months.

(354.308424 days + 354.372658 days + 354.423087 days) / 3 = 354.368056 days in one Lunar year.

The volumes representing Tropical years

For the Lunar years, there is a representation of 660 Lunar years and 464 Lunar years in the Parthenon. For the Tropical years, there is a representation of 3 x 660 Tropical years and 3 x 464 Tropical years in the Parthenon.

Athena (66 in gematria) is celebrated in the Parthenon in three ways; as Athena 'Parthenos' ('Virgin') (Parthenos is 515 in Greek gematria and the naos colonnade is 515.125 D wide between the column centers); as 'Pallas' Athena ('Maiden') - 'Pallas' is 342 and also 343 in Greek gematria, and this alludes to the 343-mm Parthenon foot; and as Athena 'Nike' ('Victory'), since in the Parthenon frieze there is a Nike depicted among the Gods, and since also the statue of Athena probably held a Nike in her hand, and since there were probably also Nike statues on the corners of the temple roof.

Volume representing 1980 Tropical years = 3 x 660 Tropical years.

Height

The height is the total central height from the lowest step of the stereobate to the top of the cymatium, at the east front.

The west side gives:

(1.693 + 1.693 + 1.814 + 58.902) Penrose feet = 64.102 Penrose feet = 911.06172 D = either 911 D or 911.125 D (1.36 mm off).

If the east side (where the pediment does not survive) would have had the same pediment height as at the west side, then:

(1.693 + 1.693 + 1.814 + 34.218 + 10.793 + 11.257 + 1.208 + 1.43) Penrose feet = 64.106 Penrose feet = 911.11857 D = 911.125 D (0.14 mm off).

Width

The width is the width of the east stylobate.

101.341 Penrose feet – 17 mm growth due to creep at the east side = 1439.5 D (less than one mm off).

Length

The length is the stylobate length at the north side, the side along which the public on the Acropolis walks.

228.141 Penrose feet - 22 mm growth due to creep = 3241.5 D (less than 1 mm off).

Volume representing 1980 Solar years

911.125 D x 1439.5 D x (4.9 days per cube D / 28806) x 3241.5 D =

1980 Tropical years x 365.244421 days per Tropical year.

Volume representing 1392 Tropical years = 3 x 464 Tropical years

Height

The height is the height, at the corners of the temple, of the corner columns and the entablature.

(34.253 + 10.793) Penrose feet = 45.046 Penrose feet = 640.22474 D = 640.25 D (0.5 mm off).

Width

The width is the width of the stylobate at the statue of Athena.

It is 1440 D. This is one arc second of their desired meridian earth circumference of 40 007 520 meters.

Length

The length is the average stylobate length (the average between the north length and the south length).

The average length is (228.154 + 228.141) Penrose feet / 2 = 228.1475 Penrose feet = 69513.006 mm. Their average 16 millimeters due to creep need to be subtracted. So, the average stylobate length is 3241.875 D (less than one mm off).

(This length may or may not be the central stylobate length.)

Volume representing 1392 Tropical years:

640.25 D x 1440 D x (4.9 days per cube D / 28806) x 3241.875 D = 1392 Tropical years x 365.243278 days per Tropical year.

Volume representing 343 solar years:

The other volumes to determine the Tropical year were 3 times 660 years and 3 times 464 years. This volume stands apart as it is only once 343 Tropical years, 343 is a number that also refers to virginity since it is 7 x 7 x 7, and 7 is the virginal number since 360 degrees of a circle circumference cannot be divided by 7. It is connected to 'Parthenos', 515 in Greek gematria. The vertex angle at the corner of a regular heptagon is 51.4128…degrees (360 degrees divided by 7), which is close to 51.5. According to Michael Schneider, the similarity is no coincidence, since there is a connection between Greek gematria and the ratios of geometry, from which the term 'gematria' is derived.

Pericles gave the inauguration speech of the Parthenon in 438 BC during the celebration of the Panathenaic games. The construction of the Parthenon had begun in 447 BC, and Pericles inaugurated the Parthenon even when it was not yet completely finished: its statue of Athena was perhaps finished and put in place (which happened in 438 BC or 437 BC), but the sculptural decoration on the temple was only completed by 432 BC.

It is logical to presume that Pericles gave his speech to inaugurate the temple in front of the main Parthenon front, which is the east side. 343 years earlier, around 680 BC or 681 BC, for the first time coins were minted in Greece[3]. Also in this period the long Lelantine war was going on between two city-states, and this long war wore these cities out economically. (Thucydides wrote about this war: "The war between Chalcis and Eretria was the one in which most cities belonging to the rest of Greece were divided up into alliances with one side or the other.") Thus, knowing that the opisthodomos was going to house all the moneys from the Delian League, Pericles implicitly referred to the first minting around 680 BC and how good it is to be strong and united to fend off the enemy in order to secure future prosperity (while Pericles had used the 'left-over money' of the contributions of the free city-states to the Delian League to pay for the construction of the Parthenon). That may be the reason why Pericles inaugurated the Parthenon in 438 BC, although the construction of the Parthenon was not yet entirely finished: this volume of 343 solar years employs the width of the east stylobate, the length of the opisthodomos, and the overall temple height. If this was intentional, it means that the Parthenon designers, and Pericles who was informed, already knew a good length for the Tropical year before Meton did. The average of the three Tropical volumes gives a Tropical year of 365.243213 days.

Height

The height to be considered is the height of the exterior temple from the center of the stylobate to the top of the cymatium of the pediment.

58.902 Penrose feet = 837.15574 D = 837.125D (0.66 mm off).

Width

The width is the width of the stylobate at the east side.

It is 1439.5 D (within one mm correct).

Length

The length goes from the 'plinth' of the partition wall of the opisthodomos to the 'plinth' of the other wall of the opisthodomos. So it is the length of the interior ground floor of the opisthodomos. It measures 611.1666… D.

43.003 Penrose feet = 611.1882 D = 611.1666… D (0.46 mm off).

Volume representing 343 tropical years:

837.125 D x 1439.5 D x (4.9 days per cube D / 28806) x 611.1666… D = 343 tropical years x 365.241942 days per Tropical year.

The Tropical year (the average of its 4 Tropical years) is 365.242009 days.

(365.238398 days + 365.244421 days + 365.243278 days + 365.241942 days) / 4 = 365.242009 days in 1 Tropical year.

Whereas in the formula by Meton, 235 Lunar months are equal to 19 Tropical years and he put these equal to 6940 days, these are not exactly equal in duration in the Parthenon.

The Lunar year of 354.366969 days x 235/12 = 6939.686476 days, and the Parthenon Tropical year of 365.242009 days x 19 years = 6939.598171 days. The difference is 0.088305 days, which is 2 hours and 7.1592 minutes difference in 19 years.

Nowadays it is known that the time it takes for the earth to revolve around the sun is 365.24219 days and that after 19 years, there is a difference of 2 hours and more than four minutes with the duration of 235 Lunar months.

In the era of the Parthenon it was impossible to measure the time it takes for the earth to revolve around the sun, but I found a method to measure the duration of a Tropical year, for example, the time it takes from spring equinox to spring equinox. And the Tropical year is about as long as the time it takes for the earth to revolve around the sun[4].

Days can vary over time. For example, around 350 million years ago, an earth year was around

3 http://www.collectorsweekly.com/world-coins/ancient-greek)

4 http://www.scientificamerican.com/article/earth-

385 days, "meaning not that it took longer for the planet to revolve around the sun, but that a day-night cycle was less than 23 hours long." Since then, the day-night cycles became longer as we now have days of about 24 hours long, but in a more recent era, as ice began melting about 13,000 years ago, the ground rose underneath, making earth rotate faster again. This makes the earth rotate faster around its axis at an average rate of 0.6 seconds per century. Since 1972, 24 leap seconds have already been added to the atomic clock that has exactly 24 hours in a day (with the latest leap second added in 2008), in order to maintain a solar year of 365.24219 days. The Parthenon was built about 24.19 centuries before 1972, so the days were about 14.514 seconds longer than the days counting from 1972 when 24 leap seconds had to be added. In other words, when the Parthenon was built, the average Tropical year was about 9.486 seconds shorter than the solar year of 365.24219 days, which was thus about 365.242080 days long.

How could the Parthenon researchers measure a Tropical year with accuracy?

The construction of the forerunner of the Parthenon had been halted when the invading Persians had sacked Athens and destroyed its temples. Later, the Athenians swore an oath before the battle of Plataia in 479 BC not to reconstruct those temples that had been burnt and ruined by the Persians for thirty years. According to Susan Woodford, this would give them plenty of time to focus on defensive works and economic recovery, while the sight of these ruins might also induce national consciousness. In the book that I will publish, I explain how this gave the 'Parthenon researchers' for the new Parthenon that was going to be built ample time to do many experiments. I explain how the Parthenon researchers found the circumference of the earth by an ingenious method of trial and error, slowly zooming in on the true circumference of the earth, year after year. They wanted the width of the stylobate of the new temple that was going to be built to measure one arc second of the meridian circumference of the earth. In this long period they also studied the distances to the moon, and also these findings are accurately portrayed in the Parthenon. In this article, I explain here below how they might have measured the duration of the Tropical year during these many years of research.

I construed the following method, showing an early attempt of how the Tropical year could have been measured at first. The Parthenon researchers might perhaps have developed a far more elaborate mechanism later.

Several cultures in antiquity celebrated the equinox days: the days when night and day are equal in length.

The Parthenon researchers decided that the spring equinox and autumn equinox would be both 24 hours long. How can they measure this? On the day of the spring equinox (or several days later), they measure the time from sunrise to the sunrise of the following day. Of course the sun will rise at spring some minutes earlier the next day, because in the spring season the days get longer, so this time span is somewhat shorter than 24 hours. To even this out, they also measure the length at autumn equinox (or several days later), when the sun rises later each day since the days shorten during autumn. In order to measure this, they build two identical hourglasses, made out of the same materials, the same diameter and filled with the same material. At the diameter of the hourglass, they build a mechanism to open and close it, so they can close it when they stop their 'hourglass clock'. They start their hourglass clock at spring equinox sunrise (or some days later) and stop their hourglass at the next sunrise. They do the same with the second hourglass at the fall equinox (or some days later). They remove the material that had not gone through the diameter. And then they collect the material that had gone through both hourglasses and they divide it evenly in two (by weighing it or by filling up two containers in an identical manner). Now they have two volumes of material that each represent 24 hours. They fill their two hourglasses with it. The following year, at the sunrise of springtime, they let their first hourglass run, and from the moment it empties, they let the second hourglass run. They then have ample time to turn

rotation-summer-solstice/

their first hourglass, and they hence measure time, with their alternating hourglasses each representing 24 hours, for 365 days. During the night after the 365th day, they bring one hourglass outside and from the moment the other hourglass is empty, they will let their hourglass run till it is exactly sunrise of the 366th day and then they close the diameter. During this year they construct a prism. They took the volume of one hourglass that represents 24 hours, put it in a container, measured it, and then put it back in the hourglass, and then they filled this container in exactly the same way as the original with a third volume of the same material, so they now had a third volume to experiment with. With this volume, representing 24 hours, they have to construct a prism (with one transparent side) that is two units wide, two units long and 360 units high. And this volume that represents the 24 hours has to fit in it exactly. So they have to try and try, till they have found the unit that matches their 24 hour volume. They certainly can construct such prism in a year's time. So, when they then empty their last hourglass that has a part of 24 hours in it, they put it in this prism. They can read the height by which they fill their prism. Every 15 units height filled in their prism of a base of two units x two units = 60 minutes, equal to one hour. Every unit height is equal to four minutes in time.

The Parthenon Solar year is 365.242009 days.

Say, if they fill 0.242009 part of a day in such tall but narrow prism, they will notice that it is altogether 87.125 units high, constituting 87.125 units x 4 minutes per unit = 348.5 minutes = 5.808333… hours. So, 0.242009 day x 24 hours/day = 5.808216. So it is five hours and 48.5 minutes, which is five hours and 0.808333… of one hour, which is about equal to 0.808216 (0.42 seconds difference). So, with the volume of their hourglass that they had collected during the last partial day, they had filled their prism up to a height of 87.125 units. And working only with eights and sixths of Ds, the Parthenon Tropical year of 365.242009 days is the closest approximation possible of five hours and 48.5 minutes.)

What is the accuracy of such method if it would have been executed in a perfect manner[5]?

Filling in the city, the months March and the months September, show that during the days of the spring equinox and quite a lot of days after spring equinox, the days shorten by two minutes and 27 seconds per day in Athens since sunrise is two minutes and 27 seconds earlier during those days than the previous day. During the day of autumn equinox and quite a lot of days after autumn equinox, the days lengthen by two minutes and 25 seconds per day in Athens. So, their two 24 hourglass clocks last in total 48 hours minus two seconds, which is 24 hours minus one second per hourglass. One second x 365 days is 6 minutes and five seconds.

So in all, in the era of the Parthenon, the year was about 365.24208 days long. If the Parthenon researchers would have executed the procedure I explained to perfection, their year would be six minutes and five seconds shorter, so it would be 365.237784 days long. Perhaps each day they waited just a little too long until the first hourglass would completely empty before opening the second hourglass, and perhaps it took too long to completely open the mechanism at the diameter of the hourglass each day? Compared to a Tropical year of 365.24208 years in that era, their outcome of a Solar year of 365.242009 days (or 365.2420138 days) is about 99.999980 % of the duration of the actual Tropical year of that era. They might have repeated this experiment several times, and then they could take their average Tropical year, which is in the Parthenon (working with sixths and eights of Ds) 365.242009 days.

I used Athens as an example, but it was not at all in Athens that the 'Parthenon researchers' made their secret observations. Besides, Meton had also made an attempt to calculate when the solstices and equinoxes were. He was quite erroneous in that regard. Yet, it was not necessary to know the exact days of spring and autumn equinox, since a good many days afterwards, sunrises still differ by the same amounts in time per day.

The Parthenon was built with tremendous accuracy, and fortunately it was also measured by

5 http://www.timeanddate.com/sun/greece/athens?month=9&year=1991

Penrose with tremendous accuracy. I adhered to these measurements with great accuracy, and the outcome of it all is that the Tropical year and Lunar year in the Parthenon are expressed in a consistent way: the volumes express numbers of years that bring out the number symbolism (by using Greek gematria) that befits this temple: Athena, celebrated as maiden-virgin and as 'The Mother' of all Athenians. For the areas of the Tropical year and the Lunar year, it sufficed to build one railing around the statue, albeit a railing with a somewhat ambiguous width, and one railing more in front of the statue.

In other words, the Parthenon shows that it was not impossible to measure a Tropical year and a Lunar year with an excellent accuracy. The Parthenon Tropical year is 0.00002 % too short, and the Parthenon synodic Lunar year is 0.00004 % too short. Hence it is in line with using 1 'clock mechanism' (since the synodic months can vary rather substantially from month to month).

Did the Parthenon designers know the duration of the Tropical year before the temple was constructed? The volumes of their Tropical years have a small range from 365.241942 days to 365.244421 days, with as average 365.243213 days. So, when they placed the railing around the statue of Athena, their average solar year merely became less than two minutes shorter.

In regard to the duration of the average synodic Lunar year, the measurements of the building elements were already decided upon, and these measurements were also determined by other specifications. Thus the Parthenon designers had to place a big railing at the third naos columns to compensate for the large volumes of the synodic Lunar years in order to represent an accurate duration of the Lunar year.

The moon in the background of the stars.

Diodorus Siculus of the first century BC wrote in Book II:

> "... They also say that the moon, as viewed from this island, appears to be but a little distance from the earth and to have upon it prominences, like those of the earth, which are visible to the eye. The account is also given that the god visits the island every nineteen years, the period in which the return of the stars to the same place in the heavens is accomplished; and for this reason the nineteen-year period is called by the Greeks the year of Meton. ..."

Thus, in this quote, the Metonic cycle was also related to the moon reappearing in the same star background after 19 years.

This refers to the shorter sidereal month: the sidereal month marks the return of the moon to the same point in the sky. To find out the duration of the sidereal month, it is needed to measure the angle between certain stars and the moon at a certain date and time, and notice that the star background after 19 Tropical years is about the same. It is a pity that the writings by Meton do not survive. Would he also have specified that 254 sidereal months fit into 19 solar years, or did he only write that the star background of the moon is also the same after 19 years? There are scholars who claimed that it is impossible that in that era, there could have been accurate knowledge about the sidereal month. The true duration of the average sidereal month is about 27.321662 days (27 days, 7 hours, 43 minutes, 11.6 seconds).

Interestingly, the face of the interior west wall of the opisthodomos is, at the door entrance, closer to the east stylobate. Hence, from the west-end of the 'hole' for the statue of Athena to the face of the interior west wall of the opisthodomos measures, at the door, 1145.875 D. This helps to indicate the synodic average Lunar year, which is in the Parthenon 354.366969 days, while this length taken further away from the door helps to determine the average sidereal Lunar month, which is (in the Parthenon) 354.368056 days x 235/12 = 254 sidereal Lunar months. Thus, 235 synodic Lunar months are about equal to 254 sidereal Lunar months.

This Parthenon Lunar year of 354.368056 days x 235/12 = 6939.707769 days. And 254 sidereal months (235 + 19) also fit in this time span, since 6939.707769 days / 254 = 27.321684 days. This Parthenon duration of the sidereal month is less than two seconds longer than the true mean duration of the sidereal month of 27.321662 days, indicating an accuracy of 100.00008 %.

The sidereal period is = (Elapsed Time) x

360°/ (Change in angle between moon and star).

If all this is no coincidence, is it possible that the Parthenon researchers could have measured the duration of the sidereal months with an accuracy of 100.00008 % with a 'clock mechanism' that may have had an accuracy of 99.99998 % when measuring the Tropical year? The difference in accuracy between the Parthenon Tropical year and the Parthenon 254 sidereal Lunar months is merely 0.0001 %[6].

There has been discovered a Greek complex mechanism, called the Antikythera mechanism.

The artefact was recovered from a shipwreck found near the Greek island Antikythera. It is believed to have been designed and constructed by Greek scientists. The instrument has been dated, according to a more recent view, to 205 BC. "After the knowledge of this technology was lost at some point in antiquity, technological artefacts approaching its complexity and workmanship did not appear again until the development of mechanical astronomical clocks in Europe in the fourteenth century."

Scientists are still studying the device. I wonder if the device can be older, or if the Parthenon researchers had already made a similar device about 245 years earlier, since the duration of the Tropical year and the Lunar months are so tremendously accurate in the Parthenon.

As I will explain in my book, I found that the Parthenon also represents in a very consistent and simple manner, completely in line with the design concept of the Parthenon, the distances to the moon: the average distance to the moon, the distance to the moon at normal perigee, the distance to the moon at normal apogee, the distance to the moon at the closest perigee that might ever have been recorded by ancient cultures (Babylonians or Egyptians or Vedic Indians) and the distance to the moon at the farthest apogee that might ever have been recorded by ancient cultures (Babylonians or Egyptians or Vedic Indians). To top it off, the Parthenon reserved an ideal distance of 60 of their actual earth radii for the statue of Athena, and the Parthenon even shows which solar eclipse they witnessed (from various places) in order to find the distance to the moon. In many ways, this was a most fortunate eclipse.

The Parthenon average distance to the moon

(The semi-major axis of the elliptical orbit of the moon around the earth) is, from the center of the earth, exactly 384 427.8144 km, exactly 60.375 of their earth radii. The Parthenon researchers employed a unique π that is commensurable with their earth circumference, a π equal to (630 / 200) x (375 / 376), with altogether 'Athena Promachos' equal to 630 in Greek gematria). The true average distance to the moon is about 384,403 km.

Thus, the Parthenon average distance to the moon is about 100.0065 % accurate. The Parthenon researchers could have found all these moon data by observing the full moons for many years aided by the tool of a magnifying glass. The use of an additional mechanism might have helped.

Interestingly, the Parthenon average distance (from the surface of the earth) between the farthest possible moon distance (ever recorded) and the closest possible moon distance (ever recorded) is in the Parthenon equal to the semi-major axis distance of the moon as seen from the surface of the earth x 144/145. This distance is in the Parthenon, when expressed in their moon diameters instead of their earth radii, 108 moon diameters. As Jay Kappraff once explained to me, this happens to be exactly the distance to the moon that the Vedic Indians maintained: according to the Vedic Indians the moon is 108 moon diameters away from the surface of the earth. (This figure is somewhat less accurate.)

I discovered how the Parthenon designers also found an accurate meridian circumference of the earth, more than 150 years before Eratosthenes (c. 276 BC – c. 195/194 BC) would make an 'inaccurate guess' compared to the accuracy of the Parthenon. I found, after 16 years of tackling the problem, how there is a simple but time consuming solution to find the circumference of the earth by which the Parthenon designers could find the size of their accurate meter. The Parthenon designers wanted

6 https://en.wikipedia.org/wiki/Antikythera_mechanism

their earth to measure 40,007,520,000 millimeters, and their aim was to construct their meter by which the earth would measure this circumference. They wanted the circumference of the earth to be made out of multiples of the integers in the decad and nothing else, with the circumference being equal to 1 x 2 x 3 x 4 x (5 x 5 x 5), x 6 x (7 x 7) x 8 x (9 x 9) x 10. Here 5 recurs thrice in order to get us to millimeters; 7 stresses the virginity of Athena; and 9 symbolizes Poseidon. Since Poseidon only appears twice in this circumference while Athena appears thrice in this circumference, Athena won the contest over the matron- or patronship of Athens. (This contest is depicted in the west pediment of the Parthenon). This circumference in millimeters is also equal to (7 x 7 x 7) x 9 x (60^4), this last number in this equation being what Ernest McClain calls the "sovereign number" in music.

To protect their Virgin, nothing of this knowledge got out to the world: their Parthenon foot remained sacred and secret, since Athena could now descend to the Parthenon to protect all Athenians because the stylobate – at the statue of Athena – measures 1 arc second of this earth circumference now divisible by sevens, but she was not to be raped by the world!

The west-pediment face, where this contest between Athena and Poseidon is depicted measures 10 Parthenon feet (0.17 mm too short), the columns plus entablature measure between the sixth and seventh columns of the fronts exactly 40 Parthenon feet, the length of the opisthodomos (from its plinth) plus its porch measures 60 Parthenon feet (1.13 mm too long), and the stylobate width at the statue of Athena has to measure 90 Parthenon feet. The actual meridian circumference of the earth measures, according to the IUGG standard, to the nearest millimeter 40,007,862.917 mm.

In a way, with their earth circumference consisting of thousands of millimeters, the Parthenon designers adhered to the metric system. The west-pediment face measures 10 Parthenon feet, and in the central frieze scene where the 12 Olympian Gods are seated, Athena as Mother sits on the sixth stool when counting from the right and when counting from the left, Athena

Representation of the west-pediment of the Figure. Nashville reconstruction.

The west pediment depicts the contest between Athena, to the left, and Poseidon to the right.

The legend goes that by planting an olive tree, Athena won the contest over Poseidon and hence the matronship over Athens. However, nowhere in the Parthenon is reference made to an olive tree. In this Parthenon depiction, Athena surely looks like 'Athena Promachos' constituting a value of 630 in Greek gematria. 'Athena Promachos' is the warrior maiden who leads battles in a disciplined manner (in contrast to her brother Ares, the patron of 'violence, bloodlust and slaughter').

As can be inferred from this depiction, Poseidon probably came by horse to this contest, whereas Athena came by a horse cart on wheels: the measuring wheels that were needed to win the contest by appearing trice in the circumference of the earth, a circumference found by years of steadfast rationality and perseverance, using a π of (630 / 200) x (375 / 376).

All this is just to show that the knowledge of the Parthenon designers was far ahead of common knowledge by scientists. For example, to know the north-south distance between places, the Parthenon designers used an easy formula that Eratosthenes never seemed to have used, yet the Parthenon designers could check whether this formula was correct.

Meton turned 'his' cycle into a generelized – and hence inaccurate - formula making 19 Tropical years equal to 6940 days and equal to 235 Lunar

months, whereas the Parthenon shows far more accurate durations. Likewise, about three centuries after the Parthenon was built, Hipparchus (c.190 BC – c. 120 BC) generalized the formula that the Parthenon designers used far more accurately: in the formula of Hipparchus, to find the distance to the moon by analyzing solar eclipses, Hipparchus stated that the angle of the moon diameter as seen in the sky is equal to the angle of the sun diameter in the sky, and equal to 0.5 degree. After years of observing the moon in the sky, the Parthenon researchers knew that they could easily see with the naked eye that at times the moon appears large in the sky (hence nearby) and at times the moon appears small in the sky (hence far away). So they knew that the moon does not always look like having the size of 0.5 degree in the sky. When observing the moon, they used the formula of proportional triangles. It did not even matter if during the first years, they would not yet have had their exact size of a centimeter: as long as they noted down accurately how many moon diameters fitted in the distance between the observer and the circular plate that they held in front of them in order to just cover the circle of the moon. It was this ratio that mattered in the formula of proportional triangles.

Of course, these observations make clear how many moon diameters the moon is away from the surface of the earth, but it does not provide a clue of what the actual size of the moon is and what the actual distance to the earth is. To find this out, it was needed to study a solar eclipse.

When observing a most fortunate eclipse, probably at four places, the Parthenon designers just had to look at the sun the following day: they then could estimate that on that day there fitted 342 large sun diameters in the 180 degrees in the sky (or 171 sun diameters in 90 degrees in the sky). And during the eclipse, they had lined up a one centimeter circular plate with the partial contour of the moon. They noticed that when they held that circular plate 1044 millimeter away from their eye, the contours lined up quite well and the moon was hence 57 of their earth radii away from the surface of the earth. When they set this distance equal to exactly 1044 mm, they knew they were quite accurate ánd that everything became hence measurable in easy earth radii. When they set 1044 mm as the distance of their one cm circular plate, they realized that the average (semi-major axis) distance seen at 108.75 cm from the observer, is simply 25/24 of 104.4 cm, hence their average distance of the moon to the surface of the earth is 59.375 earth radii. They see the average perigee distance at (82 / 87) x the average distance, equal to 102.5 cm, and they see the average apogee distance at (92 / 87) x the average distance, being equal to 115 cm. And their average of the furthest and nearest distance to the moon is simply (30/29) x 104.4 cm, equal to 108 cm. In fact, they might have observed all these findings with a half centimeter circular plate, so that all distances to the observer are halved and so that these are adequate distances to put a magnifying glass in between the eye of the observer and the 5 mm circular plate. All these findings are portrayed in the Parthenon in a simple manner.

I propose that such astronomy and geography might have been regarded as 'sacred' by the temple researchers and designers who were also astronomers and geographers, and they imbued the temple with this sacred knowledge in a way that was befitting to Goddess Athena. Hence their knowledge was secret and remained secret. Thus it is no surprise to me that their knowledge was far more advanced than the knowledge by 'common scientists and astronomers' of that time and for quite some time afterwards. And I wonder if the Antikythera mechanism could perhaps have belonged to the Parthenon researchers.

Conclusion

The Metonic cycle is represented in the Parthenon: its railings helped to demarcate the duration of the Tropical and Lunar year and show a Lunar year with an embolismic 13th month. The building elements of the Parthenon were already in place, and the railings provided the final touch in order to represent accurately the duration of their Tropical year average and the Lunar year average. In all, when it is considered that the Parthenon might even represent accurately the duration of 235 synodic Lunar months and 254 sidereal

Lunar months, with a Parthenon Tropical year of 365.242009 days, the Parthenon design shows a far more advanced knowledge about the duration of the Tropical year and Lunar months than Meton. It is not only probable that the Parthenon designers knew the duration of a Tropical year and a Lunar year in 438 BC, before the temple was finished, it is also possible that the Parthenon designers knew the Metonic cycle when designing the Parthenon.

Hence, Meton made public what the Parthenon designers already knew, and he turned it into a more general (somewhat inaccurate) formula, making 235 Lunar months equal to 19 Tropical years and equal to 6940 days in order to make a calendar to be used by 'sky gazers'. (Later, the Metonic cycle was improved by the calendar of Callippus (c. 370-300 BC) who subtracted one day after four cycles of 19 years: hence in the Callippic cycle of 76 Tropical years of exactly 365.25 days fitted 940 Lunar months.)

Similarities between Vedic architecture and the Parthenon?

Jay Kappraff saw similarities between Vedic altar architecture and the Parthenon, since in both structures, the measurements of 144 small units (which, according to Kappraff, form one large unit in the Vedic architecture) and 120 units (which form one large unit in Vedic architecture) are important. It is also noticed that some Indian scholars claim that the Vedic ceremonial altar uses a platform as base, of which the amount of building blocks, 324 blocks, is said to represent time in some way. The area of another platform on top of the base is then added to arrive at 372 tithis. From the total of 372, it is then needed to subtract 90/95 from it, in order to show the duration of a solar year since there is in the Vedic altar not one unit or not one square unit that directly relates to an accurate duration of a Tropical or Lunar year. Let's consider this some more.

It is quite possible that the Parthenon designers independently formed the idea to have a square unit represent time. And they then choose 120 Ds x 120 Ds as the 'Parthenon Day Unit' mainly since 120 x 120 is 14400, a number that easily agrees with hours, minutes and seconds. Moreover, 120 Ds also befit the musical scale they employed in their 'ideal' design proportions. The Parthenon was a temple steeped in Pythagoreanism, as I had explained thoroughly in my dissertation 'The Parthenon's Main Design Proportion and its Meaning'. The main Parthenon measurements follow the Pythagorean pentatonic musical scale, but these 'ideal' measurements had been altered somewhat for specific reasons.

Jay Kappraff brought the musical table of Nichomachus to my attention. Nichomachus (60 – 120 AD) wrote a musical table for expansions of the ratio 3:2 (as string lengths).

In my dissertation I showed that main measurements of the Parthenon are in Ds, within a large margin of 0.2 %, 640, 960, 1440, 2160 and 3240. Thus the main measurements of the Parthenon are in Ds a tenfold of the seventh column of this Pentatonic scale. (There is no 4860 D measurement, since the temple is not that large, but 486 D is the height of the columns, and 729 is the height altogether of the steps, the rise in curvature, the columns and the entablature.) In each column, the integers form a geometric sequence of the common ratio 3:2, the relative frequency of successive musical fifths announced in column 3. The Parthenon followed the geometric proportion 4/6 = 6/9, which means that for every square of 6 by 6, there is an accompanying rectangle of 4 by 9. Thus the 4/9 proportion has a base in this music, while Plato called the geometric proportion the most beautiful proportion. Hence, the 'ideal' stylobate area is 1440 D x 3240 D, which gives 324 days in Parthenon Day Units. Besides music, there is also another reason why the measurement of 1440 D is crucial in the Parthenon. This width is 1 arc second of the circumference of the earth measured along the meridians. In the book that I am writing I explain how the Parthenon designers could construct a meter by which this earth circumference had to measure 40 007 520 000 millimeters, so that their earth circumference is 343 mm (which is 7 x 7 x 7, symbolizing the virginity of Athena), altogether the Parthenon foot related to Athena, x 9, Poseidon's number (from whom Athena won the contest to become the

1	2	4 G	8	16 C	32	**64** …
	3	6 D	12	24 G	48	**96** …
		9 A	18	36 D	72	**144** …
			27	54 A	108	**216** …
				81 E	162	**324** …
					243	**486** …
						729

proportions of the Parthenon exhibit D measurements of which the numbers in Ds can be divided by many small integers (as can their earth's circumference since it consists of multiples of the first nine integers). That was intentional. It has been claimed that in Vedic architecture, they also employed numbers of building blocks that are divisible by many small integers. So, the fact that the base measurement in the Parthenon and the base measurement of the Vedic structure represent 324 time units, might be coincidence.

It is possible that representing time by an area had been an idea that already Pythagoras had picked up somewhere. Is it coincidence that both structures start with a base upon which to mount other areas (or other building blocks), so that finally one arrives at a larger area (or more building blocks) that can represent the duration of a Tropical year and of a Lunar year? And is it coincidence that the Metonic cycle has a span of 19 Tropical years, while Indian scholars claim that the Vedics considered a span of 95 Tropical years, which happens to be 5 Metonic cycles long? Scholars claim that the Vedic Indians used their fire altars for 95 years, and after 95 years they would use a new fire altar. There even is a scholar on Vedic architecture who claims that the Vedics followed this rule:

(372 – (90/95)) x duration of Lunar year / 360 = duration of solar year. Since 372 – (90/95) is equal to 7050/ 19, this formula is in fact 5 times the Metonic cycle. (And when I insert 354.367 days as one Lunar year, I have 354.367 x 7050/19 x 1/360 is 365.246688…days in a Tropical year.) Interestingly, some scholars claim that with five Metonic cycles, even if it is as inaccurate as Meton's calendar of 6940 days long, eclipses in series can be predicted for - at most - five consecutive times. After five Metonic cycles, the inaccuracy of this cycle has accrued too much to be able to predict the eclipses of a series any longer.

It has often been claimed by authors that Pythagoras came in contact with Brahmans from India, either directly in Persia or Egypt, or indirectly via Persia or Egypt or the Babylonians. One source[7] mentions that Meton based his 'Metonic cycle' on knowledge that he got from the Babylonians around 499 BC. The Babylonians were known to be able to predict eclipses. It is alleged that they knew the Metonic cycle, but they also knew of a much longer Lunisolar cycle that was far more accurate.

I am not familiar with Vedic architecture nor Vedic texts. All I can conclude is that 'Meton's Tropical year and Lunar months were portrayed more accurately in the measurements of the Parthenon, by carefully placing building elements at particular places and with specific measurements, and by putting railings in place, as is explained in this article. There might be some similarities between Vedic architecture and the Parthenon while there are also huge differences between the two methods to arrive at the length of a Tropical and Lunar year. No wonder that there are also huge differences in approach, since the Parthenon is a Greek temple, and the Vedic altar is Vedic Indian!

7 (See https://en.wikipedia.org/wiki/Babylonian_calendar.)

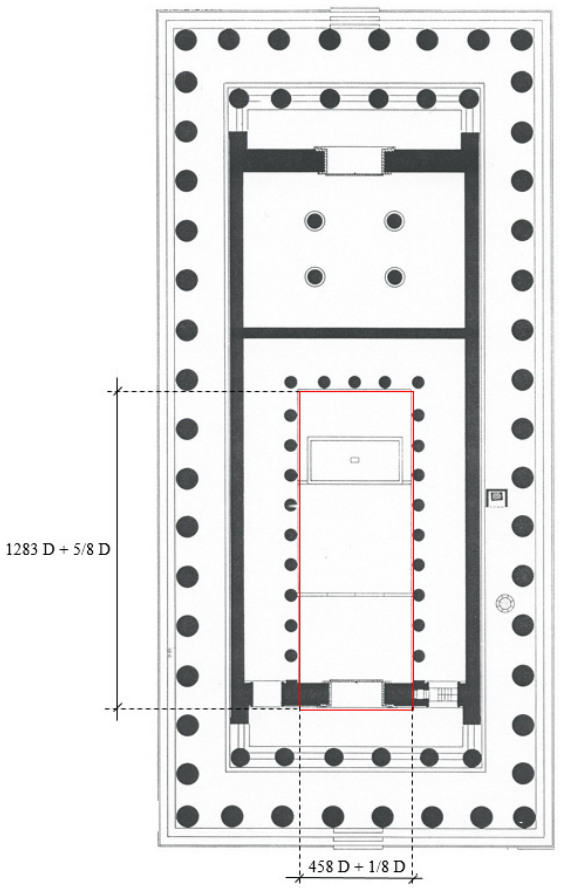

Figure 1. Parthenon plan, with a 458.125 D x 1283.625 D rectangle needed to calculate the first measurement of a Tropical year.

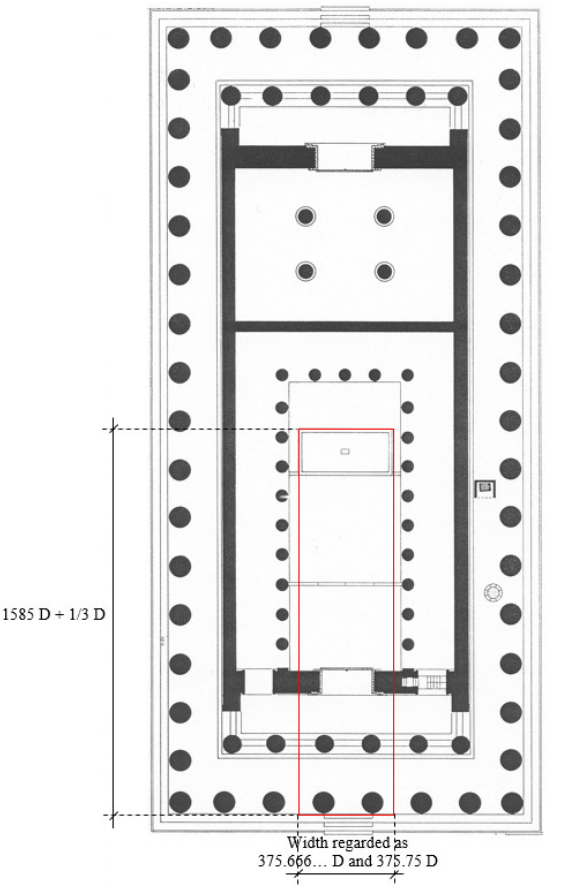

Figure 2. Parthenon plan, with a 375.666… D x (1585.333… D) rectangle needed to calculate the second measurement of the Tropical year.

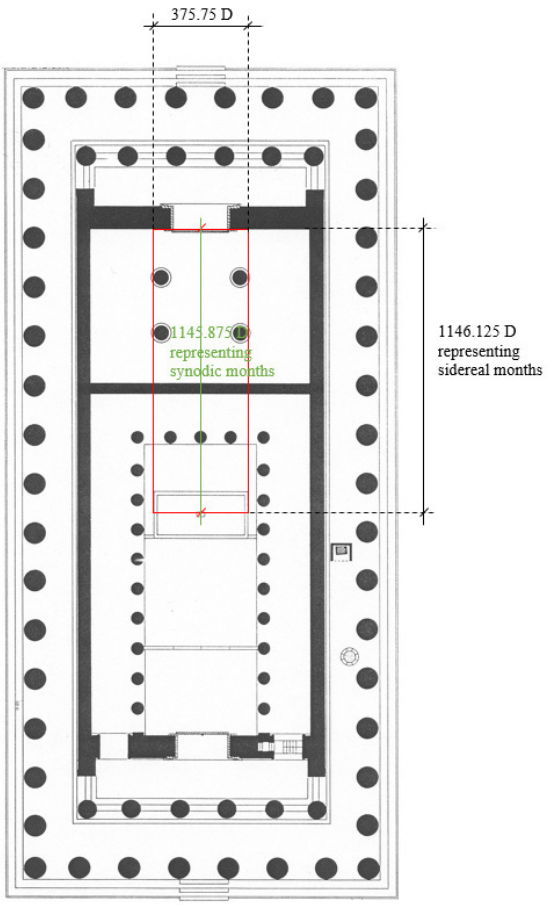

Figure 3. Parthenon plan, with a 375.75 D x 1145.875 D rectangle needed to calculate a synodic Lunar year of 12 months. Its east edge is covered by the statue of Athena.

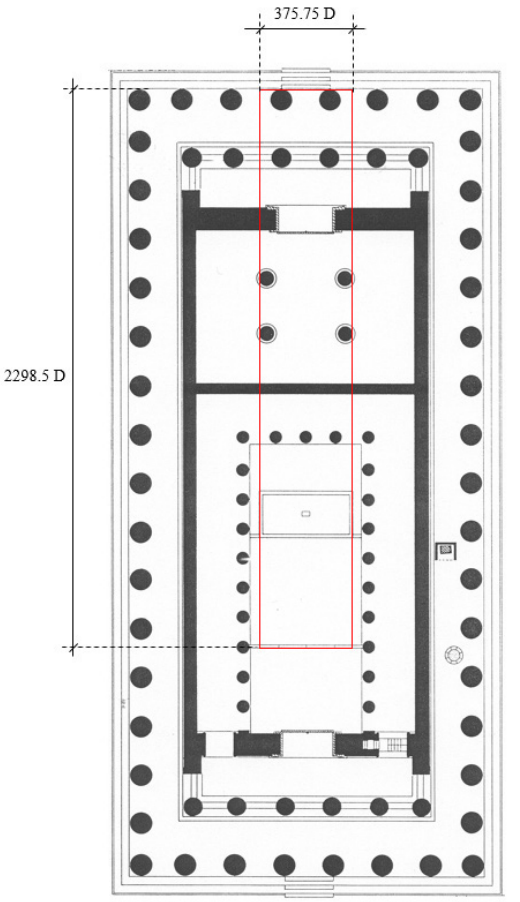

Figure 4. Parthenon plan, with a 2298.5 D x 375.75 D rectangle needed to calculate the second Lunar year (of 13 synodic months). The length goes from the west stylobate edge to the 3rd columns with the big railing in the naos.

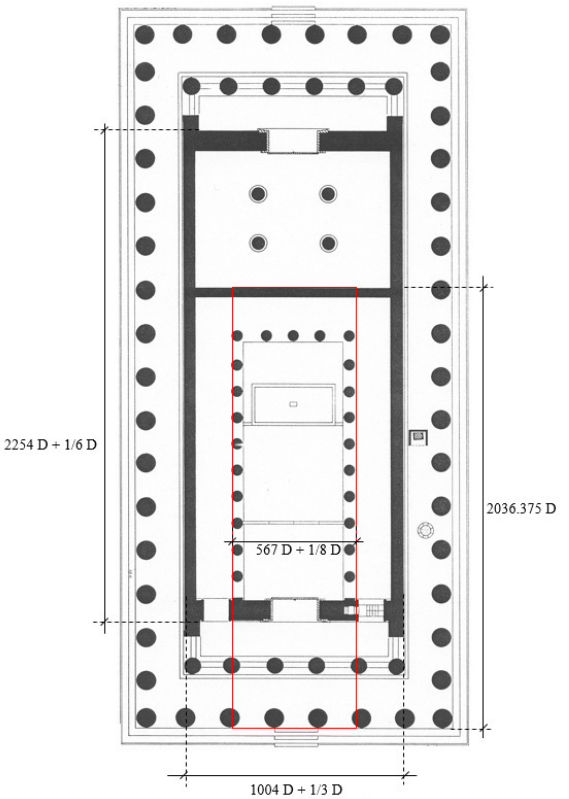

Figure 5. Parthenon plan, with a 567.125 D x 2036.375 D rectangle needed to calculate the 464 Lunar years.

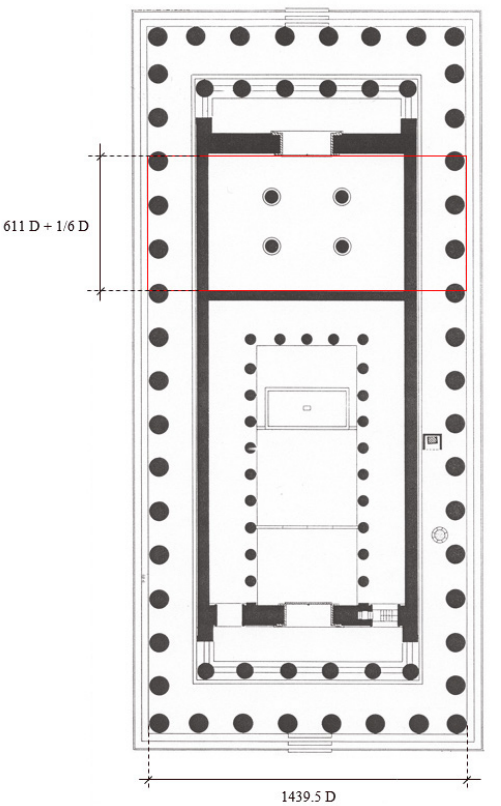

Figure 7. Parthenon plan, with a 1439.5 D x 611.1666… D rectangle, needed to calculate the volume of 343 Tropical years when Pericles gave his speech.

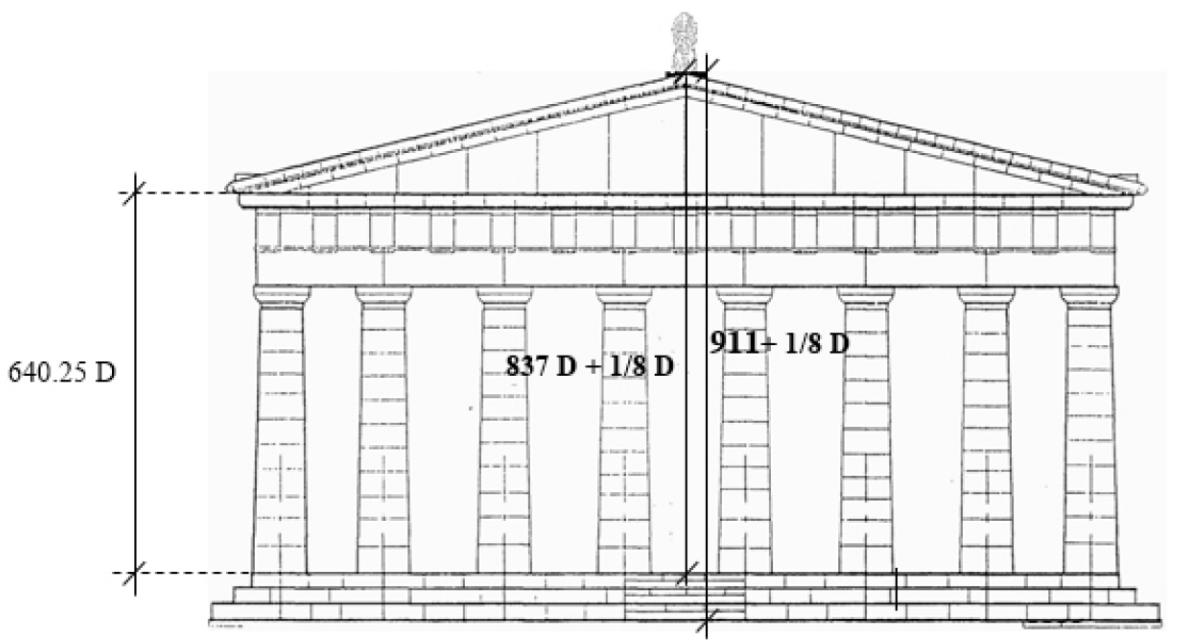

Figure 6. The temple height from the corner of the stylobare to the top of the entablature is 640.25 D. The general temple height from the bottom of the central stereobate to the top of the cymatium is 911.125 D. The general temple height from the bottom of the stylobate at the center of the west stylobate is 837.125 D.

81-95

THE PROPORTIONAL SYSTEM OF THE PARTHENON AND ITS CONNECTIONS WITH VEDIC INDIA

Jay KAPPRAFF and Ernest G. McCLAIN

Abstract

Anne Bulckens carried out research on the proportions of the Parthenon and discovered the module for the Parthenon and the smallest unit of measure called the Dactyl. As a result, and with the help of the ethnomusicologist, Ernest McClain, a strong case can be made that the lengths and widths of the inner temple, the cella, and the platform upon which the outer temple lies, the stylobate, and the temple height to the entablature correspond to relative frequencies of the ancient pentatonic musical scale. New work by the author shows that there may have been a connection between the proportions of the Parthenon and the dimensions of a Vedic ceremonial altar known as the Rathakatra Citi. We have also found evidence of Vedic measures of the length of the solar and lunar years in the proportions. Computation of the square root of two was a major theme in the Sulba Sutra, what may have been the first book of geometry associated with the Vedic world. Representations of $\sqrt{2}$ figure greatly in the proportions of the Parthenon.

1. Introduction

This article will explore the possibility that there was a connection between the proportions of the Parthenon built between 447 and 438 BCE and a Vedic ceremonial altar known as the Rathacakra Citi Chariot Wheel. The construction of Vedic Indian fire altars was described in the Sulba Sutra dating to between 600 and 800 BCE. Subhash Kak (1995, 2016) reports that ceremonial altars were also found in ancient Greece. As a result, two fundamental units are proposed that, when used, reveal measurements directly connected to the lengths of both the lunar and solar years. This work suggests a new approach to archaeology, namely, the careful study of mathematical proportions as a way to uncover aspects of a structure embedded ages ago in the architecture. We describe this as a kind of mathematical excavation. This article builds on previous articles of ours on the Parthenon which proposed that its structure was based on the Pythagorean and Just, pentatonic and heptatonic scales (Kappraff, 2002a, 2005).

2. The Parthenon

The Parthenon shown in figure 1 is a Doric temple with certain Ionic features. As in all Doric temples, the entablature, a platform above the columns, contains a line of triple-ridged triglyphs, a square stone block sitting atop the columns. These are surrounding by metopes, rectangular panels with carved sculptures by the great sculptor Phidias depicting scenes from Greek history and mythology. The outer temple sits on a platform called the stylobate, and encloses an inner temple called the cella, shown in figure 2. The cella includes two chambers separated by a wall. One chamber, known as the naos, housed a large statue of Athena: while the other chamber, the opisthodomos, served as the treasury of the Delian league. There was also a small shrine in the arcade between the cella and the columns to the South from Mycenean times dedicated to Athena. This shrine was also present in two previous Parthenons circa 566 and 490 BCE each of which were destroyed by the Persians, archenemies of the Greeks. The purpose of the shrine was to show that the Greeks had always occupied the land.

The current Parthenon also has several unusual features. Instead of the normative six columns to the East and West it has eight, while it is unique

among temples in having 17 columns along the north and south flanks. In addition, the stylobate is slightly curved in the north-south and east-west directions, and the columns are not perpendicular but slightly protracted inwards. More than any other temple, the Parthenon appears to have been crafted to extraordinarily close tolerances, in some cases less than a millimeter. These design idiosyncracies appear to have roots in the proportional system of the structure.

All ancient temples were based on a measurement known as the temple 'foot'. Each temple also had a most commonly encountered measurement known as its Module. Anne Bulckens (1999, 2001) studied the measurements of William Cranmer Penrose carried out in 1885 and she hypothesized the length of the Parthenon foot, as well as a Module that renders all of the important dimensions within the Parthenon as whole numbers, and a fundamental unit called a Dactyl, approximately the length of a finger joint. These units served as a kind of Rosetta stone enabling an exploration of the possible meaning behind the proportions. Of course every measurement has some degree of error, but Bulckens required measurements to be equal or less than 0.2% off from the actual measurements. For this reason, some measurements were taken from the corner of the stylobate and others from the midpoint of the sides taking advantage of the small differences at these locations due to curvature. Moreover, lengths were measured from diverse locations along the base of the Parthenon and they were taken from the inside or outside of a wall; or they included or excluded the antae wall or a small plinth. Bulckens was careful to indicate precisely where measurements were taken and one might wonder if this prejudices her arrival at a particular set of integers. By contrast, however, Penrose had suggested a foot length, which is referred to as the 'Penrose foot', from which almost no integer can be derived no matter which reasonable definition is used.

Now that a set of whole numbers were obtained for Parthenon measurements, with the help of the ethnomusicologist, Ernest McClain (1976, 1978), we shall show these numbers reproduced the tones of Pythagoras' musical scale, pentatonic, heptatonic, and Just.

The study of the Parthenon also suggests an infatuation with number with the recurrence of the numbers 4,6, and 7 associated with Athena while 9 referred to Poseidon who had been Athena's competitor for the designation as patron of Athens. The integers 4, 6, and 9 are found in many places within the Parthenon. For example, there are 46 columns along the perimeter of the temple. Inside the cella there are two stories of 23 Doric columns equaling 46. There are 4 Corinthian columns within the opisthodomos and 6 Ionic columns at its west and east porches and 9 columns along the side of the naos plus the antae wall. There are 444 coffered panels and 9000 marble tiles, etc., etc.

On the other hand, the temple could be seen as evidence of the ability of its architects to compute rational approximations to the square root and cube root of two. The number 7 is of particular importance. Not being divisible by any number other than itself or 1 it may have been associated with Athena as virgin not born of woman but from the head of Zeus. The number 7 was incorporated into the Parthenon proportions as described in a previous paper (Kappraff and McClain, 2005).

3. The Units

In 1982, the archaeologist Ernst Berger did a computer study of the Parthenon which showed that the temple had a recurring Module measuring 858 mm, rounded to the nearest millimeter. This Module was the size of the triglyph. The first century architecture historian Vitruvius also listed the length of a triglyph as the Module used for the construction of Greek temples and specified that the length of the metope should be in a ratio of 3:2 with the triglyph. Bulckens hypothesized that the Module should measure 2 ½ Parthenon feet, with the Module constituting a typical 'pace' of 2 ½ feet, making the Parthenon foot 343 mm and her Parthenon Module 857.5 mm. That 343=7x7x7 may be more than coincidental will be discussed in a forthcoming paper by Bulckens.

As in other Greek temples the 'foot' is divided into sixteen parts with each part called a dactyl D, or finger. This meant that the tryglyph would

measure 40 D while the metope would measure 60 D consistent with Vitruvius. The width of the stylobate is 36 Modules or 1440 D.

4. The Ancient Musical Scale of Pythagoras

The principal claim of this and previous articles suggests that the most prominent dimensions of the Parthenon are based on the pentatonic and, to a smaller extent, the heptatonic musical scales. We give a quick review of the nature of the Pythagorean musical scale and leave the reader to find more details in the references (Kappraff, 2002b, 2010, 2013). The relative frequency of the tones of the musical scale have been reduced to whole numbers in the sense expressed by Plato in the *Republic*:

> For surely you know the way of men who are clever in these things. If in the argument someone attempts to cut the one itself (i.e., use a fraction), they laugh and won't permit it. If you try to break it up into small coin, they multiply…
>
> *Republic* 524

The 12 tones of the equal tempered scale are shown on the tone circle in figure 3. Each of the twelve tones is referred to as a semitone. The tones are labeled by the letters A,B,C,D,E,F,G,A,… augmented by sharps and diminished by flats with frequencies of the tones increasing in a clockwise direction and doubling after one cycle. The tone at 12 o'clock is called the fundamental and assigned the relative frequency 1. D has been taken to be the fundamental and we will see that this results in the ancient Phrygian mode of the heptatonic scale. This was the preferred mode of Plato. Once around the tone circle is called an octave and assigned the relative frequency 2. After one cycle (octave), it is the miracle of music that tones sound identical to the ear so that each tone on the tone circle represents a pitch class of tones all having a ratio of frequencies a power of 2. On the equal tempered scale, where the tones are evenly spaced around the circle, the frequencies increase by approximately 6 % per semitone, doubling at the octave limit in a way similar to compound interest. The ancient Pythagorean scale approximates the values on the equal tempered scale using tones expressed as the ratio of integers.

In figure 4 the fundamental is represented by a length of string. If the bridge is placed at the midpoint of the string, the tone of the bowed string has a pitch an octave above the fundamental. Note that relative frequency is the inverse of relative string length. If the bridge is placed at the 1/3 position of the string and the remaining 2/3 of the string is bowed the result is the fifth. When 3/4 of the string is bowed the result is a fourth. So we see here the primacy of the numbers 1, 2, 3, 4 which were immortalized by the Greek tetractys of ten markers shown in the detail from Raphael's School of Athens in figure 5. The ratio 2:1 is the octave or diapason, 3:2 the fifth or diapente, while 4:3 is the fourth or diatesseron, and 3:1 a fifth above an octave. In this paper I will express tones in terms of relative frequency.

Consider the Nicomachus Table 1 below. He was a Syrian mathematician who lived in 150AD and was one of the last mathematicians to have direct knowledge of the musical system expressed in the works of Plato and Pythagoras.

1	2	4G	8	16C	32	64	...
	3	6D	12	24G	48	96	...
		9A	18	36D	72	144	...
			27	54A	108	216	...
				81E	162	324	...
					243	486	...
						729	

Table 1. Nicomachus' table for expansions of the ratio 3:2 (as relative frequencies)

Observe that the integers in each column form a geometric sequence the common ratio of which is 3:2, the relative frequency of successive musical fifths announced in column 3. For example the third column is 4,6,9 labeled G, D, A, i.e.,

G	D	A
4	6	9

The interval from G to D is a rising musical fifth (five tones: GABCD) while the interval from D to A is also a rising fifth (DEFGA). The ratio of 3:2 can alternatively represent string length in which case G and A would be inverted and we would have A,D,G. Mid-tone of this sequence D

is taken to be the fundamental and the other tones are placed in a single octave as follows: The largest value of the relative frequency is chosen in this case to be 9. The fundamental is then multiplied by powers of 2 to create a single octave enclosing 9, i.e., D is taken to be the 6/12 octave. The other tones are multiplied by a power of 2 to place them in the 6/12 octave, i.e., 4 is multiplied by 2 and this results in the following sequence:

D	G	A	D
6	8	9	12

This tetrachord can be found in the works of Plato (McClain, 1978). This tetrachord is also found in the arrangement of the statue of Athena and the ancient shrine. The distance from the edge of the Stylobate to the inside of the wall between the Naos and the Opisthodomos is 2000 D, the distance to the statue of Athena is 1500 D, and to the shrine is 1333.333… D or 1000:1333.333:1500:2000 = 6:8:9:12.

Figure 6 shows these three tones on the tone circle. If D is placed atop the tone circle, then the rising fifth at A, where a fifth amounts to seven semitones, and the falling fifth at G occur at 7 o'clock and 5 o'clock respectively on the tone circle. G can also be referred to as a rising fourth (DEFG).

Alternatively, 1 is reserved for the relative frequency of the fundamental. The number 2, the first female number according to Platonic mythology, results in the octave. The number 3, the first male number, is required to generate the other tones of the scale. For example, 3 is in the same pitch class as 3/2, the relative frequency of the rising fifth, whereas, 1/3 is in the pitch class of 2/3 a falling fifth. Therefore:

G	D	A
1/3	1	3
1	3	9
8	6/12	9

where we have multiplied the relative frequencies by 3 to remove fractions, then multiplied by powers of 2 to place the tetrachord in a single octave. When placed in scale order, this results again in the tetrachord:

6	8	9	12
D	G	A	D

Since the semitones are all the same size for the equal tempered scale, the tritone, the most dissonant interval, is located at 6 o'clock and has a relative frequency equal to $\sqrt{2}$ when the fundamental has the value 1. The tritone is located in the space between the rising and falling fifth at 9 and 8 respectively, and so it may be approximated by the average of 8 and 9. However, to avoid fractions, the tetrachord was doubled to: 12 16 18 24 where the average of 16 and 18 is 17, so that the square root of 2 can be approximated by: $\sqrt{2} \approx 17/12$. It was McClain's conjecture that 17 relates to the number of columns along the flank. We shall give more supporting evidence in Section 8. The Just scale was based on the ratio of integers factorable by primes 2,3 and 5.

For the integers in the fifth column, i.e.,

C	G	D	A	E
16	24	36	54	81

the tones represent five successive perfect fifths with the central tone D as the fundamental. Since the largest relative frequency is 81 it must be sealed in the 72/144 octave with the other tones multiplied by powers of 2 to place them in the 72/144 octave:

D	E	G	A	C	D
72	81	96	108	128	144

This is the pentatonic scale with fundamental D and shown in figure 7a on the tone circle. It is derived by counting 7 semitones twice in a clockwise (rising) direction and twice in a counterclockwise (falling) direction. Again we can generate the pentatonic scale using only the number 3 as we did for the tetrachord,

C	G	D	A	E
$1/3^2$	1/3	1	3	3^2
1	3	9	27	81
128	96	72	108	144

Placing the tones in scale order results again in the pentatonic scale as shown above.

E	D	C	B	A	G	F	E	D
384	432	486	512	576	648	729	768	864

The heptatonic scale is also derived by counting 7 semitones three times in a rising and falling direction. From D to D is the 432/864 octave shown on the tone circle in Fig. 7b. With an additional tone E, this is the tuning of the nine string lyre used during the time of Plato. The ratio between the frequencies of the tones at the beginning and end of these nine tones is 9:4, the most prominent proportion within the Parthenon.

5. The Pentatonic Scale in the Parthenon

Up to the time of this study, the proportions known for the Parthenon were the ratio of width to the height of the temple from the stylobate to the top of the entablature shown in figure 2 and the ratio of length to width of both stylobate and cella are 9:4. Bulckens has determined that the tones of the pentatonic scale: 16, 24, 36, 54, 81 relate to measurements within the Parthenon in terms of Module (M) as follows:

Height of the Parthenon from the stylobate to the top of the entablature = 16 M = 640 D
Width of the Cella = 24 M = 960 D
Width of the Stylobate = 36 M = 1440 D
Length of the Naos = 36 M = 1440 D
Length of the Cella = 54 M = 2160 D
Length of the Stylobate = 81 M = 3240 D
where, 36 : 16 = 81: 36 = 54:24 = 9: 4.

The 960 D width runs through the cella walls and is shown in figure 8a. It is the 5 interaxials of the perimeter columns that stand in front of the cella minus one radius of the perimeter columns. (In many temples of Greek antiquity there was a relation between the cella walls and the perimeter columns.)

Bulckens observed that the length of the naos fits as a missing 6 between the length and width of the cella:

54:36::36:24 = 9:6::6:4

It follows that 9x4 = 6x6 so that a square with the side equal to the length of the Naos has the same area as the rectangular area of the cella as shown in figure 8b. Furthermore the ratio of 3:2 enters in another way, width of stylobate: width of cella = length of stylobate: length of cella = 3:2

6. Pythagorean Triples in the Parthenon

Many Pythagorean triples were found on a cuneiform table, Plimpton 322, dating to about 1000 BCE. Three of these triples are to be found in the Parthenon:

a. 3,4,5-triangle
Area = 6
Perimeter = 12
Radius of the inscribed circle = 1

b. 5,12,13-triangle
Area = 30
Perimeter = 30
Radius of the inscribed triangle = 2

c. 8,15,17-triangle
Area = 60
Perimeter = 40
Radius of the inscribed circle = 3

We will see the first triple emerge in the proportions of the stylobate where the stylobate will be shown in Sec. 8.1 to be tiled by six 3,4,5-right triangles. The second triple will encompass the cella, and the third will relate directly to the connection between the Parthenon and a Vedic ceremonial altar described in the next section.

7. The Vedic Indian Fire Altars

The Sulba Sutras are part of the Vedic literature. They are Sanskrit texts written by the Vedic Hindu scholars before 600 BCE, but are thought to be compilations of oral wisdom which may go back to 2000 BC. They form part of the Kalpa Sutras which in turn are a part of the Vendantas. The meaning of sulba is 'string, cord, or rope'. The general format of the main Sulba Sutras are the same; each starts with sections of geometrical and arithmetical constructions and ends with details on how to build Citis which are ceremonial platforms or altars (Joseph, 1996; Kak, 1999). The measurements are performed by drawing arcs with different radii and centers using a cord or sulba. Following is John Price's description (Price, 2000):

> 'Each of the Citis is a low platform consisting of layers of carefully shaped and arranged bricks. Some are quite simple shapes such as a square or rhombus while others are much more involved, such as a falcon in flight with curved wings, a chariot wheel with spokes, or a tortoise with extended head and legs. These latter designs are particularly beautiful and elegant depictions of powerful and archetypal symbols, the falcon as the great bird that can soar to heaven, the wheel as the 'wheel of life'.

The Sulba Sutra may be the first recorded geometry book. I will state several of the geometrical constructions found in this book.

1. Given a rectangle, construct a square with the same area.
2. Given a two squares, construct a square that has area equal to the sum or difference of the two.
3. Given a circle, construct a square with the same area.
4. Construct a ratio of integers that approximates √2.

We have seen evidence of the first of these constructions in the Parthenon as shown in figure 8b. The second construction anticipates the Pythagorean theorem by more than one hundred years. The third cannot be carried out with compass and straightedge construction, however the Vedic mathematicians succeeded within 1.6%. To carry out the third construction, an accurate ratio was needed and the altar builders came up with an elaborate construction of the ratio, 577/408 which is in five decimal place agreement with √2 (Henderson, 2000). Information about the Pell's series and its application to expressing √2 may have been known to Vedic mathematicians.

The Pell's series are sequences of integers with the property:

$$a_n = 2a_{n-1} + a_{n-2}$$

Pell's sequences beginning with 1 2 and 1 3 yield,

	1	3	7	17	41	99	239	577
	1	2	5	12	29	70	229	408
n	1	2		4				8

We have found that the ratio of values are approximations to √ 2 for the values of n from the geometric series 1,2,4,8 to be the values most used in the Parthenon.

The Rathacakra Citi has the shape of a chariot wheel. Its construction was described in BSS III, 187-214 (i.e., Baudhayana Sulba Sutras). It requires seven types of bricks for the odd layers and nine types for the even layers. In the actual Citi, the bricks, although having different shapes, have all the same area. There seems to be some flexibility about the final design with one schematic shown in figure 9. This schematic illustrates the overall configuration although the accounting for the bricks does not completely correspond to the schematic. The actual Citi consists of a rim and a center connected by sixteen spokes. The space between the spokes is congruent in area to the spokes. It is recorded that the rim is made up of 145 bricks that have been properly subdivided. The spokes total 64 bricks in area and the center has an area of 16 bricks, leaving an area of 64 bricks for the spaces between the spokes. Therefore, the total built area of the Citi is,

$$145 + 64 + 16 = 225 \text{ bricks}$$

and the total wheel in which the spaces are filled in measures,

$$225 + 64 = 289 \quad \text{or} \quad 289 - 64 = 225$$

which can be rewritten as the Pythagorean triple,

$$17^2 - 8^2 = 15^2$$

This Pythagorean triple is found in the table of triples recorded on the cuneiform tablet Plimpton 322. The perimeter = 40, the area = 60, and the radius of the inscribed circle = 3 all numbers of significance for the Parthenon. The Module has length 40 D while the metope has length 60 D and, as we have seen, the musical scale of Pythagoras can be constructed, as we have shown, from the number 3. We suggest that this triple echoes the 17 columns to the North-South and the 8 columns to the East-West of the Parthenon. We will now make a case for 225 relating to the area of the stylobate.

Consider the area of the stylobate, 3240x1440 square D, and divide it by 225 bricks to obtain the area of a brick:

$$\frac{3240 \times 1440}{225} = 144^2$$

If the bricks are assumed to be square, the side

of a brick is 144 units.

Next we remove the rim from the chariot wheel and compute the filled in interior to be 144 units so that,

$144 + 145 = 289$ or $144 + 144 + 1 = 289$

As a result we have,

$$12^2 + 12^2 + 1 = 17^2$$

from which it follows that $17/12 \approx \sqrt{2}$ where the error is related to the left over 1. The square root of 2 was a key proportion in the Parthenon.

The geometry of the Rathacakra Citi used all of the Sulba Sutra constructions listed above. A square could be constructed with the same area as the chariot wheel. The area of the wheel itself, 225 Bricks, can be constructed as the difference between two squares, 289 and 64 square units. The basis of the square root of 2 calculation lies in the construction of a square, 289, equal to a double square rectangle, 144 + 144 with 1 unit error.

If we are to make a connection between the Rathacakra Citi and the Parthenon, then the unit of a Brick length B = 144D, is clearly important. Another length of 120 D arises from the Vedic India connection. The unit of a Purusha was used to construct the altars with 120 Angulas = 1 Purusha. The Purusha is exactly the length of a six foot man with his hand raised to 7.5 feet. Therefore, 1 Angula = 0.75 in. Compare this with the length of a dactyl where 1 D = 0.8439 in. or 1 angula = 0.888…D. So we see that the angula is comparable to the dactyl measurement. The unit of 120 D, which we shall refer to as a Parthenon Purusha or PP is a second unit inspired by Vedic sources where 8 dactyls = 9 Angulas or 8 PP = 9 Purushas.

The altar was created to have an area of exactly 7.5 square Purushas where,

7.5 x 120 x120 = 108,000 square Angulas.

The number 108,000 was important in Vedic culture representing the number of verses in the Bhagavad Gita. We will find that in the dimensions of the Parthenon and in Vedic lore variations on this number such as 108, 1080, 10800, 18 appear. For example, it was recognized by Vedic astronomers that there are very close to 108 lunar diameters reaching from the Earth to the Moon and 108 solar diameters reaching from the Earth to the Sun.

So we have derived two new units,

1 Brick = 144 D and 1 PP = 120 D

It should be noted that the integers 144 and 120 have an important musical meaning. McClain and Levarie (1994) have shown that the ancient Dorian heptatonic scale can be represented with smallest integers in a rising double octave, 36:72::72:144 and a falling double octave, 120:60::60:30 in which 144 and 120 provide the upper limits.

Bulckens observed that a day has 1440 minutes. She divided the day into 14400 = 120 x 120 parts with each part a duration of 6 seconds. In this way she is able to express an area of 120 D x 120 D as equivalent to one day so that area could be correlated with time. If I would do the same for the Vedic altar, for example, the area of the altar, 7.5 square Purushas x 120 D x 120 D is equivalent to 7.5 days.

8. Reflection on the Proportions of the Parthenon Through the Lens of Various Units

We shall now look at the proportions of the stylobate and cella through the lens of a unit of 1D, 40D (the Module), 360 D (the radius of the inscribed circle of the 3,4,5-triangles tiling the stylobate), the spacing of 200 D between the columns, 120 D (the Parthenon Purusha (PP)) and 144 D units (the Brick). Each unit will illustrate different aspects of the proportional system.

8.1 1 unit = 1 D and 1 Module = 40 D:

A schematic of the stylobate and the cella is shown in figure 10. Notice that the stylobate divides evenly into six 3, 4, 5 - triangles the areas of which are 777,600 again celebrating Athena while the radius of the inscribed circle is 360 D. The areas of the opisthodomos and the naos are 432 and 846 respectively, the octave limits of the heptatonic scale in the Phrygian mode. Since the dimensions of the Parthenon (McClain and Levarie) have been shown to relate to the musical scale, it is reasonable to imagine that the lengths might represent a class of integers differing by multiples of 2 so that 2160 could be interpreted as 1080 another instance of the Vedic number. Also the perimeters of the 3, 4, 5 - right triangles are 4320 D = 4 x 1080, the Vedic

number again making its appearance.

8.2 1 unit = 360 D:

This system (see Fig. 11) illustrates the 9:4 ratio of the stylobate and cella and also shows directly that the stylobate can be subdivided into six 3,4,5-right triangles. In Vedic India, 360 tithis was taken to be the length of the lunar year while in ancient Greece 360 days was the length of the canonical year, a compromise between solar and lunar years. We also see the areas of the cella and stylobate recapitulating the width of the stylobate and the height to the entablature, 36 and 16.

8.3 1 unit = 200D:

In the first planning stage of the Parthenon, the distance between the columns measured center line to center line was 200D. Later there were small variations on these measurements (Bulckens, 1999). In figure 12 you will notice that along the width of the cella there are six columns and five 200 D intervals between the columns on the East and West while along the length to the North and south there are 12 columns. As a result a 5,12,13 right triangle can be inscribed in the cella. This triangle has a perimeter of 6000D and a radius of the inscribed circle equal to 400D again illustrating Athena's numbers.

8.4 1 unit = 120 D:

In this system (see Fig. 13) it is clear that the inner spaces all have ratios of either 4:3 or 3:2, perfect fourth and fifth. Also the ratio of length of naos: length of opisthodomos = 2:1 while length of cella: length of opisthodomos = 3:1, all ratios of numbers from the tetractys. It should be noted that by the length of the cella we mean the length of the naos and opisthodomos combined. In figure 13 the lengths of the naos, opisthodomos, width of the cella, and 1/3 of the stylobate reproduce the tetrachord: 6 8 9 12. It is also clear now that six 3, 4, 5 - right triangles can also be inscribed in the cella. The stylobate has the proportions, 27:12 which suggests the Vedic computation of the solar year in terms of 12 months of 27 nakshatras per month of 324 tithis, where a tithis can be taken to be a Vedic day, slightly longer than our usual day, and where the area is expressed as units of time, according to Bulckens.

To this is added a correction factor of 48 Tithis equal to the area of the opisthodomos to get,

$$324 + 48 = 372 \text{ Tithis.}$$

This is the first approximation to the solar year in the Vedic system where 360 tithis was taken as the length of the lunar year. The fire altar builders established an elaborate procedure in which 95 levels were added to the altar to correct this value to close to the actual length of the solar year to $372 - 90/95 = 371.05\ldots$ Tithis which turns out to be within 0.1% of the value that we accept in terms of days (Joseph, 1996; Kak, 2005).

In figure 14 we see the stylobate divided into nine similar rectangles each with the same proportion as the whole with length 1080 D and width 480 D. The ratio,

$$\text{length: width} = 1080:480 = 9:4$$

This makes the area, A = 10,800 x 48 which corresponds to the 10,800 muhartas that sum to the number of minutes in a lunar year where one muharta equals 48 minutes. Dividing by 1440, the number of minutes in a day,

$$10800 \times 48/1440 = 360$$

which is the number of Vedic days or Tithis, in a lunar year.

We have now seen that both lunar and solar astronomy have been built into the Parthenon proportions. Anne Bulkens will provide in her article, *The Metonic Cycle of the Parthenon* in this edition, a more detailed analysis of the areas within the Parthenon directly correlated with the length of the solar and lunar years.

8.5 144 D = 1 Brick (B):

In this system (see Fig. 15) the width of the stylobate is 10 bricks while the area of the cella is 100 square bricks, and the length of the cella is 15 bricks while the area of the stylobate is 225 square bricks. The length of the stylobate is divided into thirds of 7.5 B, a number that duplicates the area of the fire altar.

9. From Rathacakra Citi to the Parthenon

In Section 7, we listed four geometric constructions carried out in Vedic times according to the Sulba Sutra. I would now like to show how, through several steps, these constructions enable the Rathacakra Citi ceremonial altar to be reorganized into the dimensions of the Parthenon.

1. The Rathacakra Citi has an area of 289 square units including the empty spokes which account for 64 square units. Construct a square with the same area as the Citi, i.e., a 17 x 17 square units as shown in figure 16 (step 3 from Sec. 7) .

2. Consider the 8, 15, 17 Pythagorean triple with a 17 x 17 square on the hypotenuse and a 15 x 15 square on a side as shown in figure 17 (step 2).

3. Construct a 22.5 x 10 rectangle with the same area as the 15 x 15 square, i.e., 225 square units as shown in Fig. 18 (see step 1). Note that:

$$10 : 15 :: 15 : 22.5$$

which replicates the principal Parthenon proportion:

$$4:6::6:9$$

4. Taking the 144 D length and width of the brick as the unit, the 10 x 22.5 rectangle now measures:

$$1440 \text{ D} \times 3240,$$

the length and width of the stylobate.

5. The Rathacakra Citi also presents a geometric construction of $\sqrt{2}$ according to Section 7 as shown in figure19 (see step 4 and Pells sequence).

10. Conclusion

This analysis has shown that the Parthenon represents an excellent expression of the Greek Quadrivium which states:

> Athenian youth should keep their eyes on Unity and study: music, astronomy, geometry, and number. *Epinomis*

As a result of the accuracy of Bulckens' measurements and choice of the basic unit of a Module, Parthenon foot and Dactyl, I am confidant that the proportions of the Parthenon were organized by the pentatonic and heptatonic scales of Pythagoras. The close fit between the Rathacakra Citi Chariot Wheel and the Parthenon reveal two natural units with which to measure the Parthenon, 144 D and 120 D. These units reveal in the proportions an expression of the lengths of the solar and lunar years to great accuracy pushing back the date of these discoveries.

Finally, I would like to convey to the reader my surprise as I, time and again, asked a question of the proportions and they responded with values significant to the analysis. For example how nice to have the radius of the inscribed circles within the 3, 4, 5 - triangles in the stylobate measure 360 D corresponding to the Vedic lunar or canonical Greek year. Or by dividing the area of the stylobate into 225 bricks we find the side of the Brick to be exactly 144 D. Or how often the sacred Vedic number 108,000 appears along with its variants.. Or that 120 D is a unit that reveals both solar and lunar years in the proportions. This suggests the possibility that the architects of the Parthenon may have had access to earlier mathematical and astronomical concepts from Vedic India.

Dedication

This article is dedicated to the memory of Ernest McClain who passed away in June 2014. For fifty years McClain was relentless in the pursuit of his vision of the treasures available in the ancient wisdom found in the sacred and philosophical works of all cultures still accessible if pursued in the proper way using music as the *lingua franca*.

Acknowledgements

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Figure Captions

1. The Parthenon
2. The façade and cella of the4 Parthenon
3. The equal tempered scale: the tone circle as a single-wheeled m chariot of the Sun
4. A sliding bridge on a monochord divides the4 string length representing tones intervals into segments corresponding to musical fifth (2:3); fourth (3:4), and octave (1:2).
5. Left: Detail from the school of Athens by Raphael, right: Pythagorean musical scale and tetractys. Detail from the painting hosted at the Vatican
6. Rising and falljng fifths. D, A, G are geometric, arithmetic and harmonic means in the octave.
7. The pentatoinc and heptatonic scales.
8. A) The proportions of the cella ; b) showing the 4 naos as a square equal to the area of the cella. 9:6::6:4..
9. The Rathacakra citi chariot wheels.
10. Unit = 1 D and 1 Modules (M).
11. Unit = 360 D
12. The 5, 12, 13 right triangle in the cella.
13. Unit = 120 D 27 Naksatras x 12 months = 324 tithis. Correction factor = 48 tithis, solar hear (approximation) = 372 tithis.
14. The stylobate divided into nine similar rectangles illustrating the lunar year as 10,800 muhartas where each muharta equals 48 minutes..
15. Unit = 144 D = 1 Brick (B).
16. Squaring the circle in terms of area for the Rathacakra Citi
17. The 8,15,17 Pythagorean triple
18. A 225 square unit rectangle with the same area as a 15 x 15 square.
19. A sequence of diagrams illustrating the approximation, 17/12, derived from the Rathacakra Citi altar.

Figure 1. The Parthenon.

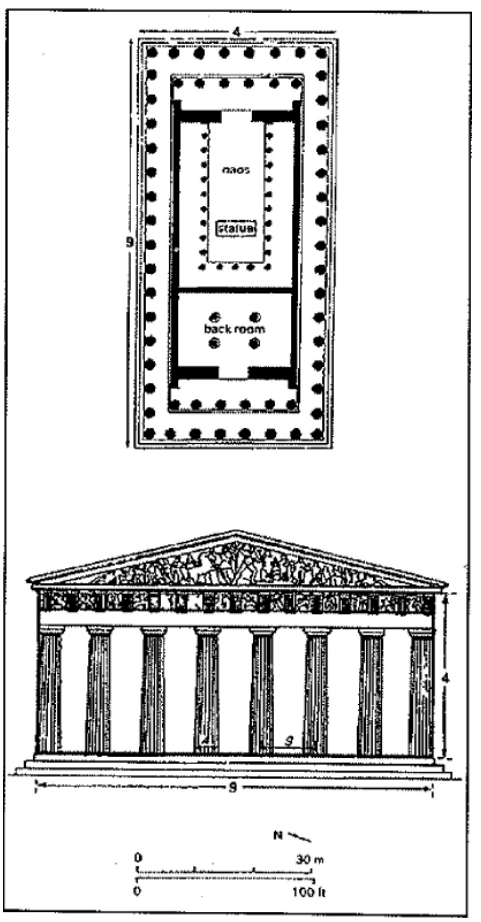

Figure 2. The façade and cella of the Parthenon.

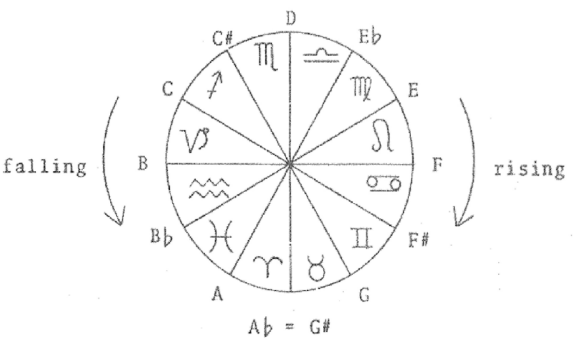

Figure 3. The equal tempered scale: the tone circle as a single-wheeled m chariot of the Sun.

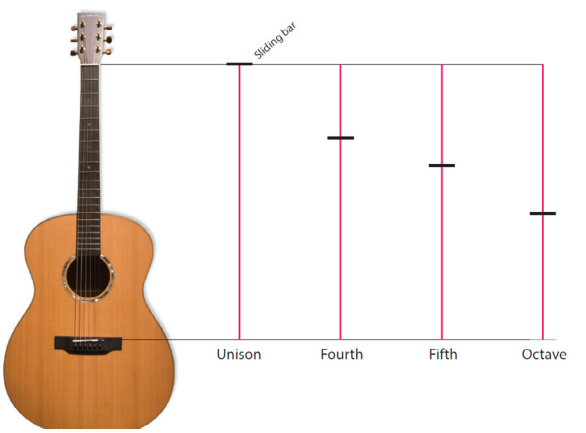

Figure 4. A sliding bridge on a monochord divides the string length representing tones intervals into segments corresponding to musical fifth (2:3); fourth (3:4), and octave (1:2).

Figure 5. Left: Detail from the school of Athens by Raphael, right: Pythagorean musical scale and tetractys. Detail from the painting hosted at the Vatican.

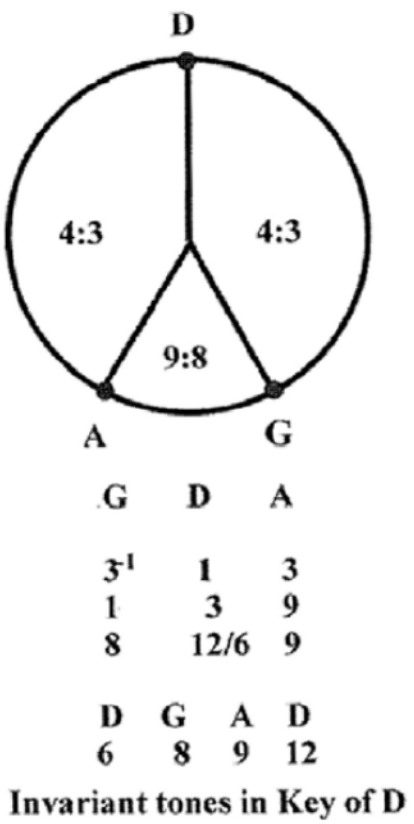

Figure 6. Rising and falling fifths. D, A, G are geometric, arithmetic and harmonic means in the octave.

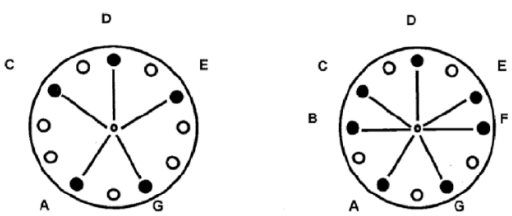

Figure 7. The pentatonic and heptatonic scales.

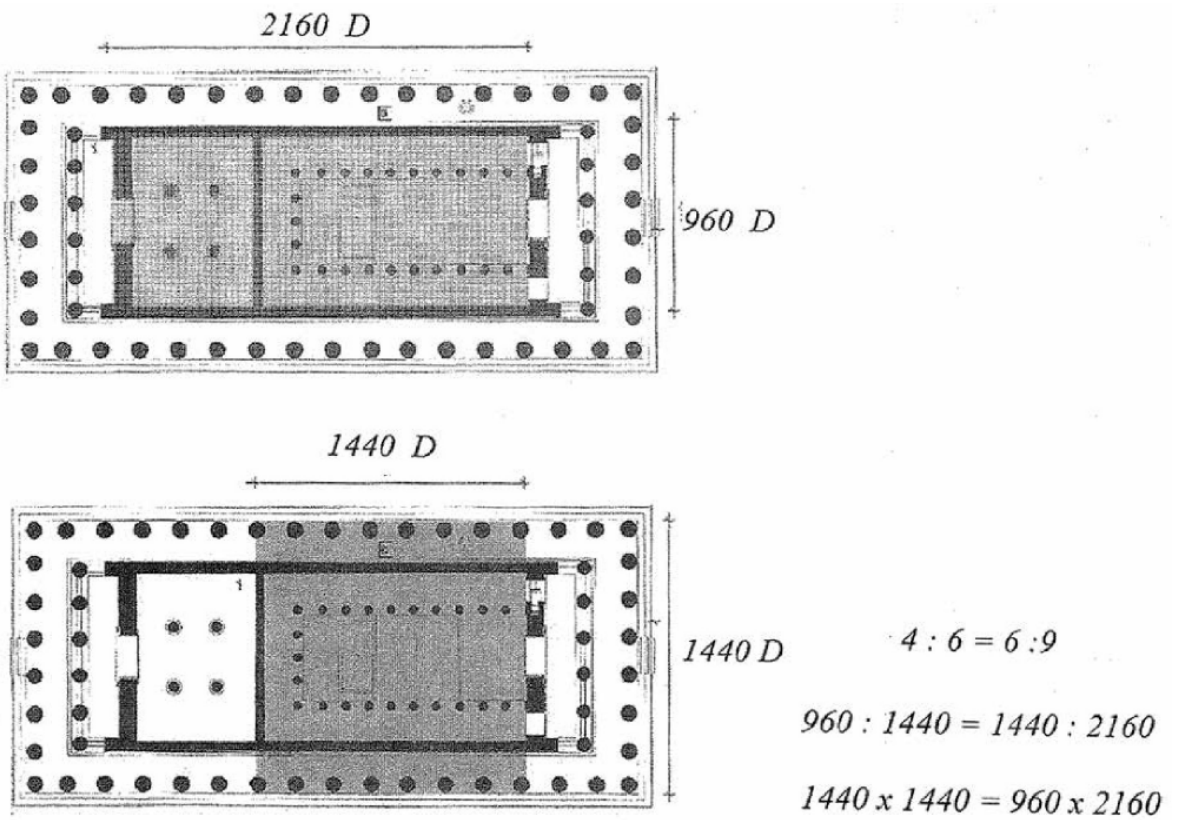

Figure 8. A) The proportions of the cella; b) showing the 4 naos as a square equal to the area of the cella. 9:6::6:4.

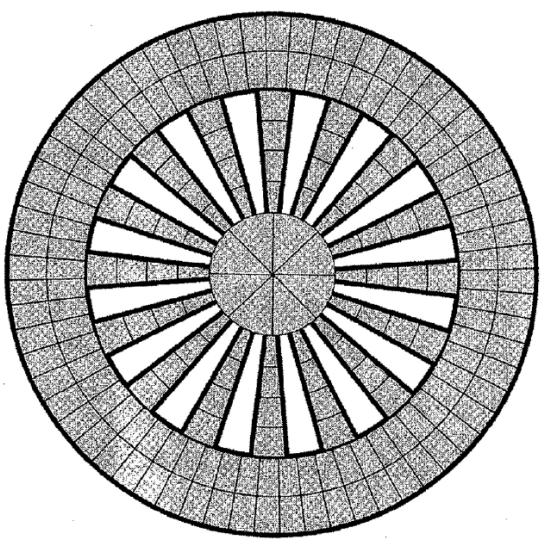

Figure 9.The Rathacakra citi chariot wheels.

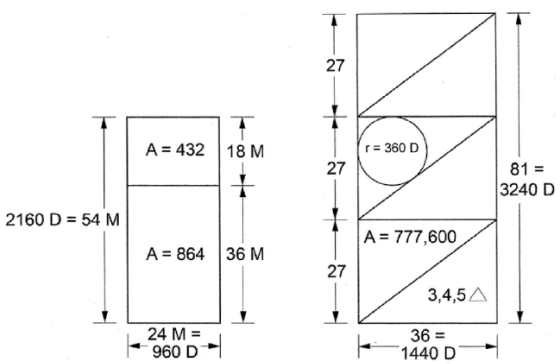

Figure 10. Unit = 1 D and 1 Modules (M).

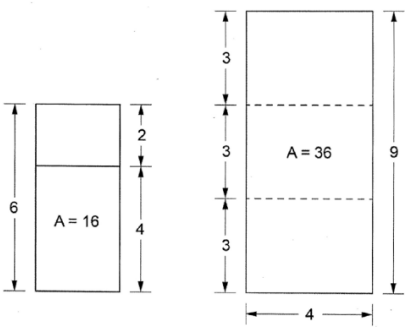

Figure 11. Unit = 360 D.

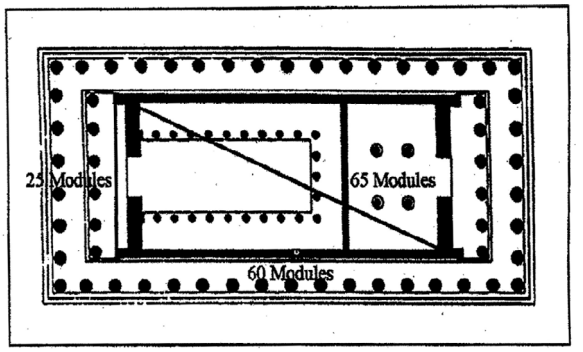

Figure 12. 5, 12, 13, right triangle in the cella.

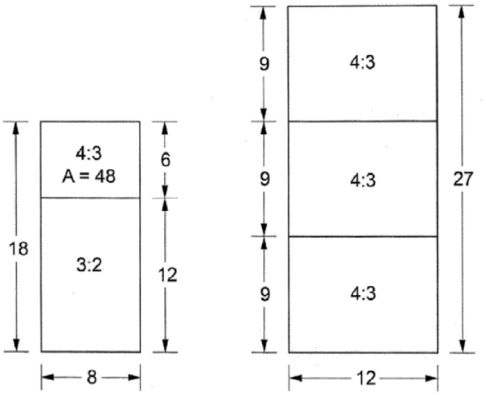

Figure 13. Unit = 120 D 27 Naksatras x 12 months = 324 Tithis. Correction factor = 48 Tithis, solar hear (approximation) = 372 Tithis.

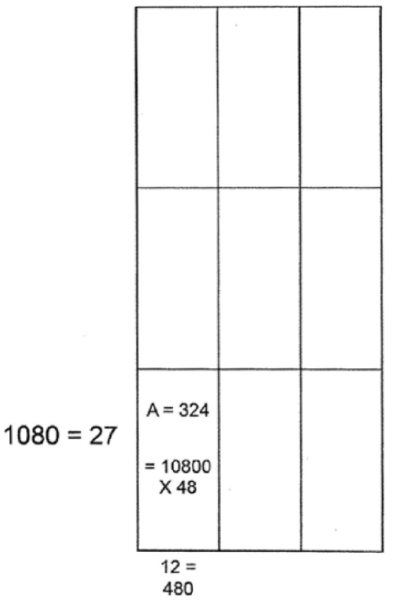

Figure 14. The stylobate divided into nine similar rectangles illustrating the lunar year as 10,800 muhartas where each muharta equals 48 minutes.

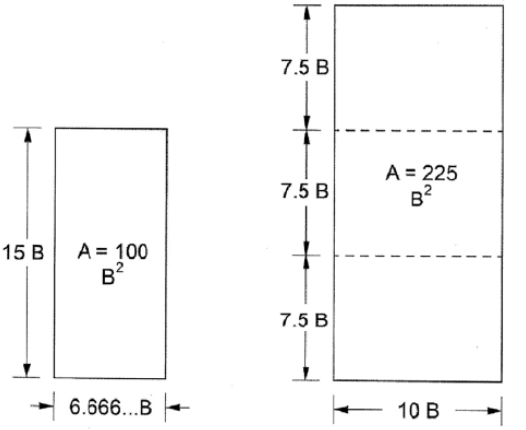

Figure 15. Unit = 144 D = 1 Brick (B).

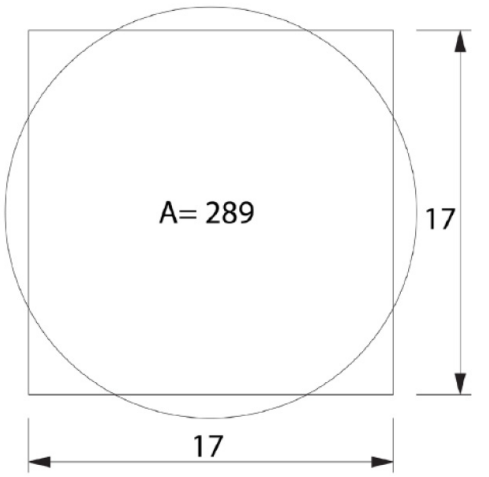

Figure 16. Squaring the circle in terms of area for the Rathacakra Citi.

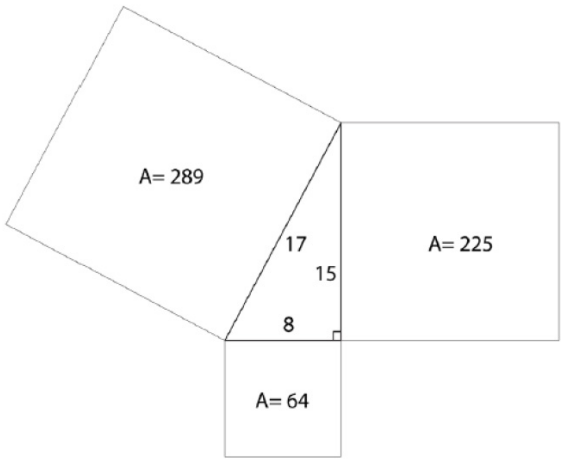

Figure 17. The 8,15,17 Pythagorean triple.

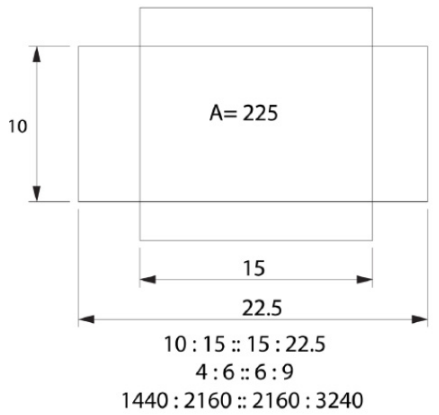

Figure 18. A 225 square unit rectangle with the same area as a 15 x 15 square.

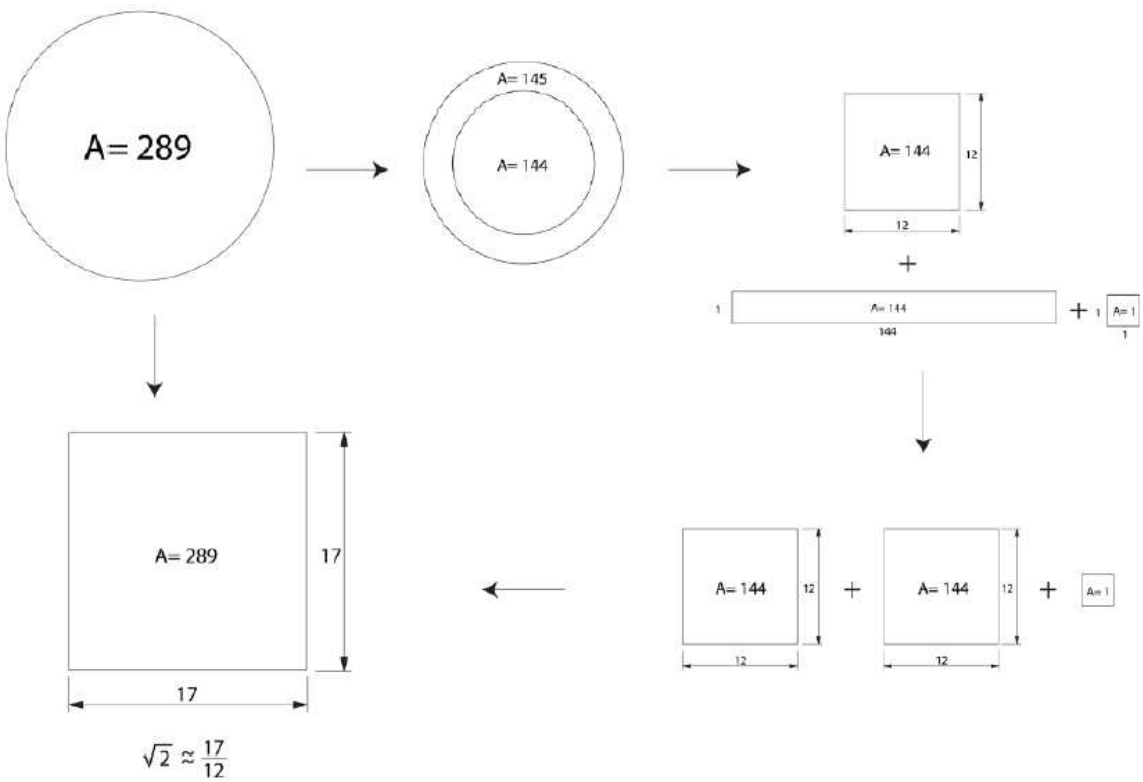

Figure 19. A sequence of diagrams illustrating the approximation, 17/12, derived from the Rathacakra Citi altar.

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THE GEODETIC AND MUSICOLOGICAL SIGNIFICANCE OF THE SHORTER SIDE LENGTH OF THE PARTHENON AS HEKATOMPEDON OR 'HUNDRED-FOOTER'

Richard HEATH

This note responds to Kapraff and McClain's preceding paper, in which they discover a many faceted musical symbolism in the Parthenon. Specifically, Ernst Berger's new measurements include the shorter side of the triple pedestal of the monument as an accurate length to represent one second of the double meridian of the earth. By applying a knowledge of ancient metrology, Anne Bulckens' doctoral derivations of a root foot can resolve to a pygme of 9/8 feet, of which one second of latitude would contain 90 such feet. However, as a 'hundred footer', the foot length should then be 81/80 (1.0125) feet, the ratio of the syntonic comma. This would indicate a replacement, by Classical times, of the geographical constant of 1.01376 feet within the model of the earth since the original model, by the late megalithic, assumed that the Meridian was exactly half of the mean circumference of the earth. These alternative geographical constants co-incidentally represent a ubiquitous theme in ancient musicology of the transition between Pythagorean and just tunings and their respective commas.

By Classical times the term hekatompedon or 'hundred-footer' had evolved, to describe the ideal dimensionality of Greek peristyle temples. One of the earliest, the Heraion of Samos, came to be 100 feet long by the end of the 8th century[1], in contrast to the surface width of the Parthenon's stylobyte which had been established as in the range 101.141 (Stuart, c.1750) to 101.341 (Penrose in 1888) feet[2].

Recent measurements in 1982 by Ernst Berger[3] found that the top surface of the stylobyte was just over 101.25 feet wide[4] and that the most frequently occurring length was 857.6 mm. Anne Bulckens'[5] corresponding foot measure for this would be a step of 2.5 feet, each of 9/8 (1.125) feet, to within one part in 2500; a foot length called a pygme within historical metrology, after the size of small men first mentioned when Herakles was travelling back from India[6]. The shorter ends of the Parthenon's stylobyte would then be 90 such feet across.

However, should the two ends be divided by 100, the required foot length of 101.25 feet becomes a microvariation of the English foot, namely 81/80 (1.0125) feet, a ratio identical with the syntonic comma. This is another ratio crucial to the history of ancient tuning theory; being found between pure Pythagorean tones (9/8) and their counterparts within just tuning (10/9); when string lengths are given specific whole number lengths to specify their pitches intellectually.

A recent article by Jay Kapraff and Ernest McClain[7] observes that the width of the Parthenon symbolically defined one second of latitude (taking surface lengths as linear fractions of latitude). This implies the double meridian length was

1 Hurwit, Jeffrey M., (1987), *The Art and Culture of Early Greece*, 1100-480 B.C., Cornell: Ithaca, 74-77.

2 Berriman, A.E., (1953) *Historical Metrology*, London: Dent. IX, 116-120.

3 Berger, E., ed. (1986) *Parthenon-Kongress Basel*, 2 Vols, Mainz: Philipp von Zabern.

4 an average noted by Berriman, 119.

5 Bulckens, A.M. (1999) *The Parthenon's Main Design Proportion and its Meaning*, [Ph.D. Dissertation], Geelong: Deakin University, 269 pp. ; (2001) The Parthenon's Symmetry in Symmetry: *Art and Science* (Fifth Interdisciplinary Symmetry Congress and Exhibition of the ISIS-Symmetry), (Sydney, 2001), no. 1-2, pp. 38-41.

6 Philostrates of Lemnos (c. 190 – c. 230 AD) Imagines Heracles among the Pygmies, see Loeb Classical Library.

7 The Proportional System of the Parthenon, in preparation for the In Memoriam volume for Ernest McClain (1918-2014)

known within 0.003% of its modern estimation. A geodetic symbolism was apparently given to shorter side length of the Parthenon, making it smaller than it would have been if modelled on the circumference of the earth as one 3,600th of one 360th part of the mean earth. If so, this geodetic meaning of the Parthenon can be compared with monuments built two thousand years earlier, such as Stonehenge and the Great Pyramid of Giza, within which the relationship of the mean earth was specified, relative to the polar radius, using the same metrological system.

The ancient model of the earth, recovered[8] by John Neal[9] and John Michell[10], used three different approximations of π to model the distortion of the rotating planet relative to its mean, or perfectly spherical, size. In that model, the Meridian was assumed to be half the circumference of the mean earth of 44 times 126 (131,383.296) feet or 24,883.2 miles. Had the Parthenon's builders used this model then its ends would be 101.376 feet in width and one hundredth of this would be a foot of 1.01376 feet, the foot known as the 'Standard Geographical Greek foot[11]'.

The mean circumference of the earth (24,883.2 miles) and the actual double meridian length (24,859.868 miles) are in the same ratio as the geographical foot of 1.01376 (3168/3125) and 1.0125 feet: the 81/80 foot measure that makes the Parthenon's 101.25 feet a 'hundred footer'. It is therefore reasonable to assume that, between the building of Stonehenge and Great Pyramid (by 2,500 B.C.) and the building of the Parthenon (designed by 447 B.C.), a more accurate measurement of the Meridian had superseded the previous assumption, within the old model, that the Meridian was half the length of the mean earth circumference.

Further to this, one can see how the transition from Pythagorean to just tuning systems[12] is strangely present in the relationship between the mean earth circumference and the actual meridian length, since the geographical constant of 1.01376 is near identical to the Pythagorean comma of 1.0136433 while the (chosen) ratio of 1.0125 is the syntonic comma and this, times 100, is near identical to the actual length of one second of latitude which would be 100 times 1.0128 feet[13], just one third of an inch different from a more modern result.

The Parthenon 'Hundred footer' was able to dimensionally reference one second of the Meridian by having its shorter sides one hundred feet of 1.0125 feet long. Aligned to north, this presented accurate Classical knowledge of the Meridian's length. The monument expresses other musicological features via its metrology: the 81/80 foot unit is 125/128 of the Athenian foot of 1.0368 feet, a musical interval called the minor diesis, also found within just intonation and equaling the deficiency of three major thirds to the octave.

8 Michell by 1980 and, fully formed, Neal by 2000.

9 Neal, John., (2000) *All Done With Mirrors*, Secret Academy: London.

10 Michell, John (1982) *Ancient Metrology*, Pentacle Books, Bristol, 1982; (2008 new ed.) *Dimensions of Paradise*, Inner Traditions: Rochester.

11 Using the terminology developed by John Michell and John Neal.

12 The latter prevalent in other aspects of the monument, see Kappraff, J. and McClain, E.G. (2005: Spring–Fall) The Proportions of the Parthenon: A work of musically inspired architecture, *Music in Art: International Journal for Music Iconography*, Vol. 30/1–2.

13 A non-harmonic 79/78 feet.

99-121

ERNEST McCLAIN'S MUSICOLOGICAL INTERPRETATION OF ANCIENT TEXTS: *Musicological Narrative Structures in Biblical Genesis*

Richard HEATH

Introduction

This paper attempts to interpret the first two books of the Bible, according to Ernest McClain's methods. It is contended that the compositions of ancient texts, as Plato insinuated[1], were both inspired and used for the science of numerical harmonics.

The invariant properties of harmonic numbers, and their evolution through limiting whole numbers, offer a large variety of distinctive scenarios which can be set into compatible narrative forms such as the Seven Days of Creation, the Garden of Eden, the Flood, the Patriarchal development of the Twelve Tribes and Moses meeting with YHWH. They are all set within a single harmonic frame of the 5th power of 60, and have features shared with other ancient texts. This paper takes numerical references, such as ages, numbers of objects, numeric values of names (as sequences, sums or as exponents), orders of the products of 3 and 5, and considers them as dimensions (and any other features) of integer matrices which McClain called 'holy mountains', and symbols such as the Star of David and other elements belonging to the octave tone-circle of a holy mountain. The resulting synthetic view points towards the presence of a secret component of harmony, in ancient texts, which supports harmonic parallelisms as a significant compositional technique shared with many scribal centres of ancient storytelling. The Bible starts with accounts of the Seven-Day Creation; of Adam and Eve in the Garden of Eden; of Noah's Ark and the Flood[2], also found in earlier literature and probably dated from the Old Babylonian *Epic of Atra-Hasis* (ca. 1900 B.C.). The first integral narrative cycle includes the complex journey of Patriarch Abraham's children, in the second half of *Genesis* spreading into *Exodus*. There, Abraham met with a 'Mountain God', (El Shaddai), a form which would later be known as YHWH. The purpose of this development was a covenant with YHWH in which Canaan was promised to Abram's descendants and sealed with the practice of male circumcision.

Ernest McClain pioneered a technique[3] in which these ancient stories would reveal a series of hidden musicological clues. However, his explanations for the underlying musicality of the Bible and other narratives, such as the formation of Twelve Tribes in the early *Pentateuch*, were incomplete. In this paper I shall apply McClain's methods and hypotheses where pure musical invariants, suggested by various means, were used within the writing of the Biblical narrative.

1. The Story of the Patriarchs

The journey of the Patriarchs ends with Moses, standing on the verge of the Promised Land (Fig. 7). Moses died at the age of 120 years. Musicologically, this would mark the completion

1 These insinuations are lifted from McClain's *Pythagorean Plato* where in turn, the author refers to Allen Bloom's *The Republic of Plato*, (New York, 1968); Trevor J. Saunders's translation of *Plato's Laws*, (London, 1970); Taylor's *Timaeus and Critias*, (London, 1929); Cornford's *Plato's Cosmology*, (London, 1937). Further insinuations in Godwin, 1993; McClain, 1976 and 1978 which contain synthetic references to Plato as alluding to a coherent system of thought, based upon harmony, geometry and astronomy having formed a numerical science. See also Heath *Aristarchus of Samos*; Godwin, *Harmony of the Spheres: A Sourcebook of the Pythagorean Tradition in Music*. (Rochester, 1993); McClain, *Myth of Invariance*, (New York, 1976) and *The Pythagorean Plato*, (York Beach, 1978).

2 Finkel, I., *The Ark before Noah: Decoding the Story of the Flood*, Hodder & Stoughton Ltd., (London, 2014).

3 McClain, op. cit. (1976).

of the Sumerian harmonic sexagesimal system, by the process of doubling. However, according to McClain, the story of the Children of Israel had already tripled 60 since Isaac, descended from Abram renamed as Abraham, who was 180 years at the time of his death. This set the musicological scene for the future Twelve Tribes, located in a fresh part of McClain's 'Holy Mountains', a topography based around powers of 3 and 5. Babylon's limit of 60 (Fig. 1) is located upon 3 times 5 whilst the Patriarchal limit moves to Adam's (45), whose letter-number formula adds up to an extra 3 relative to 60, the root of 15 which must be relocated to 45 to descend from Adam.

McClain was only able to form and interpret these harmonic mountains because, cryptically, Plato had alluded to their formation[4]. This revealed a scribal practice where harmonic numbers were inset into ancient texts, an ancient method connecting only important points of musical harmonic relevance. These advanced beyond aural performance and have been studied by taking musical string lengths as integer numbers forming musical intervals between them, thus producing rational fractions composed only of prime numbers 2, 3 and 5. Examples are found in the Bible, in Homer, and other texts of Heroic and Classical periods (c. 1,500B.C. to c. 500 B.C.)[5].

McClain's musicology reveals tonal patterns which are invariably contained within the dominance of a single octave limit. It binds the arising of its tones resulting from more than a simple doubling (the number 2) instead, as a consequence of the interaction of the next two primes: 3 and 5. With this ancient discipline, numbers 3 and 5 invariably populate any octave in a distinctive and successive developmental sequence, similar to patrilineality. Following Plato's guidance, a gender parallelism became natural, in which the mother reproduced the invariant tones within her 'belly', the octave, leading Plato to assume the feminity of the number 2[6].

3 and 5 were thought to be male numbers[7]. They impregnated the 'womb', (octave) with the father's limiting number. This made of primogeniture a perfect subject for a musicological parallel with the *Bible*, where Abram first saw an invisible god as an harmonic number field deity. This is how Patriarchs would have conceptualized *Genesis*, as a story based upon harmonic values (Fig. 1).

Abram comes from the base 60 region of the 'Chaldees' and expresses two Pythagorean children: Ishmael (G) and Isaac (A) within a pentatonic Covenant with YHWH, seen as the Just 'horizon[8]' over the land of Canaan, between pitches b and f within the Tone Circle for 60. Ishmael is dismissed when his wife Sarai, renamed Sarah, gives birth to Isaac. Abram is first renamed Abraham, meaning a 'father of many nations'[9], by YHWH, at the age of 99. Abraham laughs since he is 100 years old and Sarah 90 when their child is born. They called him Isaac since the name means 'he laughs' (Fig. 2). The joke arises through musicological necessity where 60 is transformed in 90 through multiplication by 3/2, giving Sarah's first born the prime root 9 x 5 = 45, rather than the 3 x 5 = root 15 which underlies the limit 60 as 4 x 15. In Mesopotamia, the goddess Ishtar was the principal female 'generatrix' (noting the 5 reciprocal synods to the 8 brick representing solar years). She is associated with the number 15 which, times four, becomes 60, the number for the sky god Anu. Babylonia was 'distanced' upon the form of the holy mountain, as D shifted further from the 'cornerstone', as a♭ (bottom left on the mountain).

Isaac died at 180 which was double the age of Sarah at his birth. This conforms with the octave as between birth at her 90th year and death at his 180th. Before his death, Isaac's wife Rachel gave birth to twin boys, G and A. The first born Esau, meaning 'hairy' looses his primacy to second born Jacob, meaning 'heel holder'. (Fig. 3)

The tonal framework of 180 is heptatonic but in an unconventional way relative to Pythagorean or Just norms: the 5 whole tones are just tones of 9/8 and 4 minor tones of just intonation of 10/9, whilst the semitones, which are large, have the value of 27/25[10]. Jacob is doubled by

4 See footnote 1.
5 See footnote 1.
6 See footnote 1.
7 See footnote 1.
8 See figure 6 of Moses looking over the horizon and into the promised land, at age 120, according to McClain.
9 Genesis 17:5
10 This type of Just heptatonism is exactly that

inheritance and then doubled again by means of name changing, where his new name, Israel, causes a revolutionary increase in the size and musicality of the holy mountain. 12 symmetrical tones divide the octave to parallel the fact that he will have 11 sons and 1 daughter within an octave spanning 360 to 720, rooted on the pure power of 32 x 5 = 45 (Fig. 4). Joseph is the special 13th tone which stands opposite Isaac's D of 180 in figure 3.

The role of the tone opposite D, previously diagrammed between 'sons' G and A, has been released by shifting D to its new root at 45 so that a true 12th note becomes possible, 16/15 greater than G and 64/45 greater than D = 360. This cornerstone role of opposition to D unblocks the development of twelveness, the cornerstone operating as a saviour to provide balanced tonal manifestation within a small number set. The primogeniture becomes magical through Joseph, its saviour, who moves the focus from Canaan to the great regional power of Egypt (Fig. 5).

A further doubling to 1,440 will achieve a limit in which Adam, the first man, can be seen within the new root of 45 since 1 + 4 + 40 = 45 and also 1.4.40, seen as decimal place notation, equals 1,440. The symmetrical tones in this doubling are unchanged but a new phenomenon appears in the top 2 rows: the 1st brick on the 4th row equals 1,000 which implies, alongside 1.4.40, an adoption of decimal notation, according to McClain, where the upper bricks are then readable as to their fractional ratios, as if they were in decimal notation, only times 1,000. For example, the value of the top brick, 1250, being 5/4 of 1,000, expresses the major third interval of 1.25 relative to the brick equal to 1,000.

The root of the word Adam √ADM =1.4.40, can be seen as a beginning of the story, as that starting with root 1+4+40 = 45 but also as the end of a story, in the transition to a 'decimal heaven' which can be found within the mountain for 1,440, where the great Tyrant rooted at the 6th power of 3 (= 729), appears but is exceeded by the cornerstone of 1,024. Joseph, behaves 'like' the cornerstone for the family, but Reuben, the first-born, is actually displaced by Benjamin, according to McClain. The tonal picture should be constrained by the Magen David pattern into 11 Just tones (see end of section 3), but even then the locations of tones as 11 plus the cornerstone hides another meaning of chromatism, as an important manifestation of twelveness involving a new organising idea.

McClain developed the view, as mentioned by Kapraff[11], that Joseph's siblings were organised by Jacob who died at 147. While not a harmonic number, this age of 147 can still function as a limit for the powers of 3 and 5, in the table of their products which inherently underlie the mountain itself, for 720 or 1,440 (or, in fact for any mountain greater than 144) (Fig. 6).

The first born is Reuben[12] and though 1st born, the patriarchal story always includes his displacement. As Jacob, Esau and Isaac displaced Ishmael[13], so Reuben is displaced. The cornerstone is the traditional position for a saviour and Reuben is shunted off into exile, to new and good lands 'to the east', whilst the 12th son, Benjamin, becomes the saviour for Joseph from the other brothers, and hence becomes the cornerstone instead, enabling Joseph to determine the whole family's history and prepare the way for the Twelve Tribes. But he is 'already' translated, as 11th son, into the 11th highest product of three and five, 5^3 = 125. This absolute location on any mountain, of the 11th cross-power of 3 and 5, gives Joseph, in the mountain for 1,440, a pseudo decimal value of 1,000, above the 1st 3 rows of practical Just intonation, and now beyond dependence on Babylonian roots and in the ancient land of Egypt; all this via a minor diesis jump from the cornerstone of 1st son, which always removes 27 = 128 and adds 5^3 = 125, a vertical take-off of 3 bricks (Fig. 5). His number, as 11th, is also a key number for the Magen David's harmonic limit

symbolised by the future Megan (Shield of) David symbol when it is laid over D = 720, which excluded Pythagorean C and E, already available in their Just counterparts c and e, within that star – all that remained was the cornerstone a♭ which becomes the 12th tone, see figure 8 of YHWH matrix overshadowing the Promised Land as a transformed Eden.

11 McClain, *The Lost Harmonic Law of the Bible*, (2006)

12 'Reuben is Jacob's oldest son. As such, he is assigned the 1st number, 1, of the Harmonic Matrix. Joseph is the 11th son. He is assigned the 11th largest number of the Matrix, 125.' McClain, *The Lost Harmonic Law of the Bible*, (2006), p. 485.

13 'Reuben as 'cornerstone' is exiled 'to the East' as if to 5th place in the 3rd row', the location symmetrical to the cornerstone which McClain names 'Star in the East'. 12 Sons versus 12 Tribes: *In Memoriam* (Draft for Bibal, 14/01/2013).

and also that of YHWH (Fig. 17). The completion of this decimal experience in Egypt will be the Exodus, set up by a Moses protected by a royal princess who adopted him and then arranges his education in the high Egyptian sacred sciences. While we started with an Abraham limit of 60, narrative doubling finds this Moses with an age of 120 at death, overlooking the Promised Land which he could never enter. (Fig. 7)

One may double 120 twice to get 480, whereupon Abram's letter-number value of 243 appears at the end of the bottom row. This brick is almost equal to low D as 240, being just 22 cents greater, and 243 has the same 5^{th} power of 3 as YHWH would have when interpreted as being the product of both the 5^{th} powers of 3 and 5. This could be why Abraham had to be given his new Hey = 5.

Exodus[14] tells us that Abram only knew the 'Lord God' as 'El Shaddai' (= 345) and not as YHWH, whose number-letter equivalence would only become known by MOSES (= 345), 345 being a likely code for numbers and exponents of (or powers of) 6 and 5: Y.H.W.H equals 6.5.10.5 which can be seen as 6 x 10 = 60, to the power of 5, which then equals 777,600,000 (Appendix One). The limiting D for YHWH would be on the 6^{th} row of a much larger mountain, but below YHWH, on the bottom row, lies the root of Abram = 243 = 3^5. YHWH is 5^5 above Abram but YHWH's root 2 cornerstone of ab is 5^4 above Abram, a cornerstone who will come to be seen as The Son of God through Joseph to whom Jesus has often been compared, through narrative similarities or direct references, found in the New Testament.

Abram probably defined, as the 5^{th} power of 3, the necessary limitation in powers of 3 to avoid the excessive numerocity and enharmonic excessiveness of the Pythagorean cycle of 5^{ths}, found when that cycle is taken to its natural consequence in which a Pythagorean comma arises, an audible comma expressing the difference between 12 5^{ths} and the 7 octaves they would span (Fig. 8). The chromatics of the Twelve Tribes, innovated by the Patriarchal journey to 720 and beyond, could occupy a Promised Land of Just tuning and mark a return from Adam's expulsion from Eden, for eating a fruit from the tree at the centre of what was probably, a musicologically conceived garden.

2. Harmonic Narratives before the Patriarchs

The Patriarchal story is preceded by 3 stories which set the scene for its dramas which, it appears, take place within the holy mountain of YHWH, at the left and base rows of Just tuning. This garden 'stage', outside the symmetrical tone set of YHWH, becomes the Garden of Eden.

a) Adam and Eve in the Garden of Eden

Before the Flood, there is a Creation in 7 days, referring to God as Elohiym = 86, and a subsequent creation of the first human called Adam = 45, and therefore keyed into the story of the Patriarchs whilst introducing the Lord God as El Shaddai. Adam lives in Eden = 126 (= 12^6?) and he is given Eve as companion, an as yet unnamed female made from the 'rib' of Adam, who as 45 is the basis of a future Just tuning framework 'in the image and likeness of God', in the story of the Patriarchs. If Adam's rib is Hebrew TAYT = 9 = ט, then ignoring 2s, Adam's Just tone of 9/10 less than 45 creates a future root value for Eve equal to 81 = 9^2, on the base of Eden. Beyond Eve, on the bottom row to the right, lie the roots 243 = Abram (3^5, and source of YHWH's powers of 3) and then 729 (3^6) (Fig. 9).

This is the serpent's 'head', which times 1024 (=2^{10}) makes D = 746,496 would symmetrically form 12 Pythagorean intervals about itself, leading to an enharmonic Pythagorean comma, between a 1^{st} note A♭ and a 13^{th} tone G♯ (and called enharmonic because Ab is then less than G♯.) The 2 trees described within the garden are therefore the Tree of Life, at the root on which stands Abram = 243, future Patriarch, and the Tree of Knowledge, at the root on which lies Plato's Tyrant number of 3 to the 6^{th} power and, as mentioned, a possible reference to EDEN = 126 (i.e. 12^6) in which tuning through the cycle of 5^{ths} obtains a chromatism of 12 notes but leads to an audible enharmonic comma (Figures 9 and 10).

The story of the Garden of Eden sets the

14 Exodus 6:2, see note 19.

scene for the patriarchal drama which will take place in 'the Creation' west of YHWH's lozenge of symmetrical tonality, an undeveloped area in which Adam, as male prototype, is raised to 45 by the male divine number 5^{15}, whilst his 'rib', the number 9 or (rib-shaped) alters 45 into 81 as a woman, the 5th tone on the base 'Pythagorean' row, called the earth. Adam will successively double to evolve the Patriarchs into Twelve Just Tribes as the mountain for 720 (Fig. 4) which having 18 tones fits inside the 19 tone area of non-symmetrical tones found in YHWH's mountain. Adam and his woman will be ejected from the Garden, and only then is Eve (=19) named, as they are cut off from the 19 Pythagorean tones in the base of YHWH's mountain by a cherubim with a flaming sword. As the mother of all living, Eve is evidently the Tree of Life above her as Plato's 'sovereign number' (= 60^4) and it is Adam's 'fall' which would be corrected by giving Abraham and Sarah their new Hay (=5).

It is not so difficult therefore to see the fruit of the tree of knowledge (of harmonic good and evil) causing a fall from grace when the first man eats its fruit. The subsequent naming of Eve[16](= 19) by Adam after his fall compares the 19 non-symmetrical tones of YHWH's mountain[17] with the 19 Pythagorean tones in the base of YHWH's mountain (of 605 = 777,600,000), whose symmetrical tones do not include the region including and descending from 729 (= Tyrant), to 243 (Abram), 81 (Eve), 27, 9, 3, 1, One being the ever-present 'cornerstone' of 0th power of 3 and 5, present in every mountain.

In summary: Eve, being 'named for all that lives', relates to the left hand non symmetrical area of 19 tones, within which the Garden of Eden was probably placed. This area of 19 tones has a base which forms 7 of the 19 tones forming the base of YHWH; The Tree of Knowledge as 729, was placed at the right hand limit of Eden since it is not a symmetrical tone on YHWH's mountain.

This is supported in that the mountain for 720 excludes just this tone (which exceeds its limit) whilst the mountain for Adam = 1,440 has a 19th tone equal to 729 and a cornerstone which is then equal to 1,024 = (2^{10}), the number required by the serpent's 'head' of 729 to form its body of 12 intervals, marking the redemption of Adam through Just Intonation's blending of powers of 3 and 5 and requiring only small numbers rather than 'giants' which can only be destroyed by having a Flood involving numbers even greater than YHWH or the fully developed serpent of 1,024 times 729.

b) The Story of Noah and the Flood.

The expulsion of Adam and Eve from Eden leads to evil results when the sons of God (knowledge of large harmonic numbers perhaps) have giant children[18] with the daughters of men. YHWH's creative imagination became hateful to Him[19]. Noah, a man rather than a god, is instructed to save his household and animals from the Flood by building an ark boat which can float above a 'flood' of large integers. This is symbolised differently in the Veda and other ancient texts, where the victor at the head of the flood is like Babylonia's Marduk or the Vedic Indra. But the important fact is that the Flood was adapted by the writers of the Bible, within their context of the fall of Adam and the Patriarchal story to come. It was an important insight for McClain to see that the Jewish mind was seeking to supersede Marduk with YHWH, who defines a harmonic firmament by standing, instead, at its far corner of 60^5, above Abram, as 3^5, rather than at the top of a mountain 5^{14} high, so as to kill the serpent's error. As McClain put it, 'In Judaism, god prefers a throne not at the top, but "in midst of his people."' And in number based Just intonation, the serpent created by tuning according to a cycle of 5ths, need never be encountered (see also the later section called Eve as the Mother of All Living).

15 Genesis 2:7 And the Lord God formed man of the dust of the ground, and breathed into his nostrils the breath of life; and man became a living soul.

16 Genesis 3:20: And Adam called his wife's name Eve; because she was the mother of all living.

17 This pattern of tones only appear, on their own, in the matrix for 1080 and the other area of non-symmetry to the right has 27 tones which is found in the matrix for 5400.

18 *Genesis* 6:4: There were giants in the earth in those days; and also after that, when the sons of God (powers of the divine male number three) came in unto the daughters of men, and they bare children to them, the same became mighty men which were of old, men of renown.

19 *Genesis* 6:5: And God saw that the wickedness of man was great in the earth, and that every imagination of the thoughts of his heart was only evil continually.

The actual limit required to kill the serpent is only alluded to directly in the Vedic account as both a low D of 4,320,000,000[20] and high D of 8.640,000,000[21], these numbers evidently containing 57. The singular top tone of this mountain, has the hero god Indra equal to 5^{14}, so cutting up the serpent Vrita below, whose head, containing 3^6 = 729, is directly below, on the bottom row of the mountain (Fig. 11).

The flood generates a very accurate 12th note, Indra (602 cents) and Vritra (598 cents) differing from the ideal tone (600 cents) by 2 cents either side, and therefore the Flood defines *the most accurate Diophantine approximation to the geometric mean of D*, as being the square root of two. This achievement is irrelevant to what YHWH has in mind through Just intonation and the Flood is another type of giantism. The flood has a root of 3^3 = 27, limiting it to a heptatonic relationship of 3 symmetrical notes either side of D, a phenomenon first encountered on the base of the mountain D = 864 (32 times 27), the 'lowest' of Pythagorean diatonic octaves. The use of decimal notation is then important to register the seven 10s (as powers of 5 times 2) which raise this Ark up to the maximum extent of the flood equal to ten to the 7th power.

This story of Noah was still needed for the transition between Adam, and Abram = 243 since it gives the context for rejecting the serpent's fruit, the root value of 243 introducing the fifth root of three: the power of three belonging to YHWH = 777,600,000. Noting that the letter H equals 5, YHWH threw Adam out of Eden because of a serpent's wisdom and this Flood is to deal with this serpent without introducing the heptatonic

Marduk, whose ark would be the diatonic number of 864, raised up by 10 to form a ship-like Just boat, but then a boat not fitting with the Garden story which fits an Ark based upon 720 into the Bible's new 'story space' of the garden of Eden and a god with 11 not 7 principle tones. While the flood number was omitted other details of Marduk's flood, the extreme number of levels (14 + 1 = 15) are referenced in the 15 cubits height of the flood[22]. Noah is the human hero, who escapes this deluge and begins a new story associated with the human male number 5, in concert with and as a corrective to the divine male number 3.

The serpent's sin was offensive to YHWH (as the god of tonality) who innovates the balanced alternative called Just tuning, which is more intelligible, requires smaller numbers in its generation, and is best handled in a decimal frame. As Pythagorean chromatics are formed out of a tuning process, unbeknownst to the tuner the string lengths develop 12 powers of 3 which can never cancel to form octave completion. In contrast, Just chromatics can be produced as string lengths using small powers of 3 and 5, such as the root of Adam = 45, shared by Sarah at 90, and Isaac at 180, forming a 12 tribe tonal set to the cornerstone, of Just chromatics around D=720, many generations after Noah.

YHWH replaces the Flood with His regularisation upon the base sixty[23] and its easy access to 'both "paired reciprocal multiplication tables" and "floating place value"'[24]. Abram 'from the Chaldees', only saw the god as El Shaddai (KJV: Almighty = 345) rather than YHWH (6.5.10.5 = 26 or $6^5.10.^5 = 60^5$). It is Moses = 345, trained in Egypt, who comes to know the Lord as YHWH[25] and Moses has a name of the same number as El Shaddai and it is that number which Ernest McClain interprets as being a reference to apparently shared knowledge, existing in the ancient Near East, but made somewhat explicit by Plato,

20 *RgVeda* 4:58:2-3: Four horns, three feet, 2 heads and seven (=107) hands, McClain 1976, p. 80.

21 *RgVeda* 1.126.3-5: Eight cows, sixty thousand kine, forty bay horses and ten (= 1010) cars to equal 8.6.40.(0000)0000000 the tenth power giving the scope of the whole decimal notation, overwritten by the 'head' number 8640, [McClain 1976, p. 81].

22 Gen 7:20: Fifteen cubits upward did the waters prevail; and the mountains were covered.

Ernest G. McClain: Thus gods 10, 15, 20, 30, 40, 50 and 60 in the early Sumerian pantheon taken over lock, stock and barrel by Babylon and Assyria with new names all preced[ing] YHWH as regularization of BASE 60 with the standard tables by Hammurabi's time. BIBAL message 28594

23 BIBAL, message 28229

24 *Exodus* 6:2. And God spake unto Moses, and said unto him, I am the Lord: And I appeared unto Abraham, unto Isaac, and unto Jacob, by the name of God Almighty (Shaddai), but by my name Jehovah [YHWH] was I not known to them.

25 [Christensen, 2009], p. 25.

the secrets of '3:4 mated with 5', already mentioned and indicative of using 3 x 4 x 5 = 60 as the ideal symmetrical mean when dealing with the harmonic god's domain of holy mountains. (See Appendix 1)

c) The Seven Days of Creation.

The garden of Eden seen within the holy mountain of YHWH (= 6.5.10.5 = 60^5 = 777,600,000) and Adam = 45, the first male human, address similar issues as may be found in Plato's harmonic allegories. Are the Seven Days of Creation, which start the Bible, also part of this musical composition or is it an appendage grafted on to the Pentateuch?

The story can be summarised by the achievements of each day (See figure 12):

The story is probably creating the framework within which the later stories will be couched and so is concerned with the domain of musical harmony, ready to be used within stories in which harmonic parallelism will be employed. In a theological sense, what is being proposed is the creation of an Eternity or Heaven which can coincide with Existence and within which, harmonic coincidence is good.

1. Light was divided from darkness.

> '1 In the beginning God created the heaven and the earth.
> 2 And the earth was without form, and void; and darkness was upon the face of the deep. And the Spirit of God moved upon the face of the waters.
> 3 And God said, Let there be light: and there was light.
> 4 And God saw the light, that it was good and God divided the light from the darkness.
> 5 And God called the light Day, and the darkness he called Night. And the evening and the morning were the first day.'

The light here is AUR as we would have 'aurora' meaning the light of a new dawn. This most fundamental act of division into two parts is the same as the creation of the cornerstone of unity, from which all ratios must derive. The word 'unity' itself stresses a unit, like 'the stone the builders rejected' (which haunts McClain's presentations.) But the marking of oneness is an act of division in that no other can function as 'the one' thereafter which defines not only interval and tone ratios (the parts of the coming firmament) but also the idea of the harmonic system (the whole). The number one is therefore the creation of a framework god, in which creation is based upon harmonic relationships between positive integers and their reciprocals.

2. A Firmament, called Heaven.

> '6 And God said, Let there be a firmament in the midst of the waters, and let it divide the waters from the waters.
> 7 And God made the firmament, and divided the waters which were under the firmament from the waters which were above the firmament: and it was so.
> 8 And God called the firmament Heaven. And the evening and the morning were the second day.'

This firmament =רקיצ = 380 = 4 x 5 x 19 can also be seen as 3 x (3 times 2 =8) and (2 times 5 = 10), giving a close formula for Plato's '4/3 mated with 5'. If this firmament is made up of products of three and five, multiplied by a number of twos, and these between octave doubling, 380 suggests [3,4,5] whilst also indicating the 19 tones on row one of YHWH's kingdom of 777,600,000. Therefore the generalised formula for composing a harmonic field using the 2 numbers, 3 and 5, as 2 dimensions within which their products define the locations within that space, and then the factor 19 to indicate that this creation is that of YHWH, though specified by God as elohiym, those declaring this framework.

3. Dry land formed, and seas, growing from below.

> '9 And God said, Let the waters under the heaven be gathered together unto one place, and let the dry land appear: and it was so.
> 10 And God called the dry land Earth; and the gathering together of the waters called he Seas: and God saw that it was good.
> 11 And God said, Let the earth bring forth grass, the herb yielding seed, and the fruit tree yielding fruit after his kind, whose seed is in itself, upon the earth: and it was so.
> 12 And the earth brought forth grass, and herb yielding seed after his kind, and the tree yielding fruit, whose seed was in itself, after his kind: and God saw that it was good.
> 13 And the evening and the morning were the third day.'

The Elohim command the waters below this firmament (reciprocals of 2, 3 and 5) to gather up and form dry land. Such holy mountains, within the firmament, are accompanied by seas, which are the non-symmetrical tone areas within a holy mountain, YHWH having three of these, one at each corner. This implies that mountains were being pivoted around D to overlap with their reciprocals and form a dry land of symmetrical tones (Plato's twins or paired male warriors) and seas of nonsymmetrical tones.

4. Two lights, which illuminated Day and Night.

> '14 And God said, Let there be lights in the firmament of the heaven to divide the day from the night; and let them be for signs, and for seasons, and for days, and years:
> 15 And let them be for lights in the firmament of the heaven to give light upon the earth: and it was so.
> 16 And God made two great lights; the greater light to rule the day, and the lesser light to rule the night: he made the stars also.
> 17 And God set them in the firmament of the heaven to give light upon the earth,
> 18 And to rule over the day and over the night, and to divide the light from the darkness: and God saw that it was good.
> 19 And the evening and the morning were the fourth day.'

This central part of the Creation concerns the Tone Circle within which the octave places tones in a very special setting, so that invariant intervals cumulatively populate the octave as limits increase. The symmetrical tones (day 3) occupy the opposite halves of the tone circle in which the principle axis is D, the letter-name used by modern notation for (a) the geometrical mean of the 12 semitones used in our staff and keyboard. When it appears the sun and moon are being introduced, the sun perhaps stands for D, the god of a given mountain, defined by a harmonic number. Standing opposite in the tone circle is (b) the geometrical mean of the octave, then a moon or lesser luminary, of reflection, overseeing the integrity of the octave from the perspective of the point opposite D on the base of the axis of symmetry, to rule the 'night'.

5. The Location and Multiplication of Moving Creatures.

> '20 And God said, Let the waters bring forth abundantly the moving creature that hath life, and fowl that may fly above the earth in the open firmament of heaven.
> 21 And God created great whales, and every living creature that moveth, which the waters brought forth abundantly, after their kind, and every winged fowl after his kind: and God saw that it was good.
> 22 And God blessed them, saying, Be fruitful, and multiply, and fill the waters in the seas, and let fowl multiply in the earth.
> 23 And the evening and the morning were the fifth day.'

The waters are the multiplications by 2. The moving creatures, fowls of the air, and great whales, are the actual combinatorials of 3 and 5 which occupy the Firmament (day 2) and constitute its identities. All of the potentials of this Firmament, held within these products of 3 and 5, must be brought into the octave through multiplication by 2. If they are in the symmetrical area of tones around D, they belong to the earth. Otherwise they belong in the air above. Beside or below the earth they belong to the sea.

6. Human beings and their pastoral fecundity.

> '24 And God said, Let the earth bring forth the living creature after his kind, cattle, and creeping thing, and beast of the earth after his kind: and it was so.
> 25 And God made the beast of the earth after his kind, and cattle after their kind, and every thing that creepeth upon the earth after his kind: and God saw that it was good.
> 26 And God said, Let us make man in our image, after our likeness: and let them have dominion over the fish of the sea, and over the fowl of the air, and over the cattle, and over all the earth, and over every creeping thing that creepeth upon the earth.
> 27 So God created man in his own image, in the image of God created he him; male and female created he them.
> 28 And God blessed them, and God said unto them, Be fruitful, and multiply, and replenish the earth, and subdue it: and have dominion over the fish of the sea, and over the fowl of the air, and over every living thing that moveth upon the earth.
> 29 And God said, Behold, I have given you every herb bearing seed, which is upon the face of all the earth, and every tree, in the which is the fruit of a tree yielding seed; to you it shall

be for meat.
30 And to every beast of the earth, and to every fowl of the air, and to every thing that creepeth upon the earth, wherein there is life, I have given every green herb for meat: and it was so.
31 And God saw every thing that he had made, and, behold, it was very good. And the evening and the morning were the sixth day.'

This culmination of the creation is an image of the Promised Land and of humanity built into the cornerstone area of YHWH as 60^5, the one God of the creation, yet in the image and likeness of the Elohim. The image and likeness is presumably the ability to form mountains as seen later in the Bible, based upon 60 or 720 or anywhere in the firmament, as the harmonic specialists come close to being the Elohim creating the world of musical harmony within which gods, demons, creatures, herbs and human characters can develop within the frameworks of different holy mountains.

7. Rest and reflection upon all that had been achieved.

What was to become a distinctive number symbolism of seven enters into a harmonic canon quite naturally through the fact that only the numbers 1:2:3:4:5:6 participate in musical harmony, in the sense that the Bible would be talking about.

8. Eve as 'Mother of all which lives, beneath 81.

The fact that the first 6 numbers contribute the most harmonious intervals means that these numbers and the first 3 primes, produce harmony out of numerical simplicity. That is, ratios are harmonious by virtue of their relative simplicity. When something is a noise there is no tonal simplicity, as with 'white noise' which expresses a continuum of all frequencies. The 7^{th} day caps this fertile period of simplicity with a number which does not combine easily with the earlier prime number, 2, 3 and 5. This is why tradition calls 7 a virginal number, associated with purity, genetic reluctance and of a sacred and reserved character such as the Sabbath on which nothing should be done.

Beyond 7 are 8:9:10, all derived from [2, 3, 5], and these give rise to 2 types of tone interval, one Pythagorean as 9/8 and the other, a minor tone of just intonation of 10/9. 11 and 13 are primes too high to be harmonious and 12 on its own merely repeats 6/5, the minor 3rd, with 10, and 4/3 with 9 and 3/2 with 8 – and these are nothing other than 'similarities to the already arisen' since the Senarius has already achieved these intervals. The conditions of adjacent harmonious numbers are only met, below the number 20, by 15 and 16 which generate the Just semitone of 16/15.

This separation of numbers into those made up of only the first 3 prime numbers [2,3,5] and those 'not harmonious' can be seen in the Nippur Tables [2,200 BC] which only lists the harmonious numbers. This suggest a use for the Nippur numbers with regard to investigating musical harmony and possibly generating holy mountains or their equivalent. It is also the case that the strings of fourth millennium instruments appear to be operating upon the least possible string number lengths for the scale they present, in scribal records. This implies some natural means to establish what those numbers, as lengths, should have been to achieve intervals between strings when struck or plucked (Fig. 15).

What is most remarkable is the fact that the last 2 numbers, 80 and 81 (our deduced root for Eve), form the syntonic comma of 81/80 which is the amount by which the Pythagorean tone differs from the minor tone of just intonation (204 - 182 = 22, and that 22 cents = 81/80).

If we then look at the holy mountain for the Senarius as 6! = 720 (of Israel), we see that the mountain shape for that limit (based upon a pure table of multiplication between numbers 3 and 5), when raised into the same octave 360:720, appears defined as a series of fifths (3/2) running left to right and a series of major thirds (5/4) running the positive diagonal. This highlights the duality often found within instruments in respect of their design and tuning. Strings must have an physical length but the pitch must often be tuned through tension, relative to the other stings. The mountain shown below can represent string lengths in any units, such as a millimetre, and if only the 'wetted', darker bricks were realised as strings, then 11 of these tones would be available as rising or falling

equivalents and the Pythagorean tones are in capitals whilst their Just equivalents are lower case.

In the figure below, Just tones are employed where possible, above and below, which eliminates (c & e). McClain calls this traditional Megen David 'throne' more strict than Marduk's throne in which the outline of the whole mountain for 720 is used, and hence all of its tones symmetrical when raised up to D = 8,640,000,000, like an Ark but not a shield.

This may seem a strange point until one looks at the claim that man should be in the image and likeness of god and then look at YHWH in the light of the restriction to 11 tones presented by the Magen David, since YHWH has 5 ambidextrous Pythagorean tones either side of his D, that is 11 Pythagorean tones relative to D.

Figure 17 shows the relationship of man to God as YHWH, in which the mountain for 720 occupies the region of both the Garden of Eden and the Promised Land whilst being developed through the evolution of Adam as 45 who doubles to 720 to create the Twelve Tribes within a Just intonation, which defeats the enharmonious Chromatism of the Serpent's Pythagoreanism, by including the Cornerstone and its inverse 12th tone and Savior. In contrast with the Serpent, YHWH has 11 Pythagorean tones including D, allowing the Magen David to represent God's image and likeness. But then YHWH's 12th has to be made symmetrical, 'on earth as it is in heaven', in that YHWH can make a useable 12th tone either end of his 11 Pythagorean tones using the 'non-symmetrical' G♯ within the mountain as his descending 12th tone and its reciprocal, then A♭, in the inverted mountain as his ascending12th tone. The two will never be heard together because they only arise in the mountain or its inverse.

The situation for YHWH, concerning his means of obtaining a 12th Pythagorean note without audible comma is therefore reflected within the harmonic restriction of eleven Just notes where the Cornerstone (ab) 12th tone exists within the mountain and so also must its inverse (g#) 12th tone, enabling the mountain for 720, restricted to eleven symmetrical tones, to achieve twelve tones in a kind of chromatism. Another similarity between YHWH and the Magen David centred on D, is that the upper and lower arms of the star shape correspond with the top of the YHWH's mountain and its inverse, equated to Sheol by McClain.

Conclusions

Perhaps the best statement on Biblical musicology published by Ernest McClain was that written with the late Bible scholar Duane Christensen in his 2009 Anchor Yale Bible volume on Nahum. One finds a rare discussion of the likely context for biblical musicology:

> 'Within ancient Israel, scribes adapted established principles to their own ends in the numerical (and musical) composition of the biblical text… Ernest McClain argues that the protoscience of ancient harmonics developed initially as "Sumerian grain piles" and eventually became Mesopotamian "holy mountains" limited to multiplicative products of 2, 3 and 5; it was acquired by Jewish scribes in Babylon in highly sophisticated forms. … Pythagorean thought has its origins in ancient Mesopotamia and Egypt, particularly in Babylon of the sixth century BCE, where it shaped biblical thought within Jewish circles in a different direction.'

It is hoped that the above interpretation of early biblical story-telling, using McClain's methods and some of his published number insights, such as exponents, decimals, gematria and numerical and structural features of specific holy mountains; will give further credence to his hypothesis: that the (otherwise) anomalous appearance of harmonic numbers within ancient texts came about due to a widespread and active interest in musicology during the Heroic to the Classical ages, as being expressive of spiritual values. The writing of the Bible and Plato's writings appear closely linked in both time and their musicological theories and techniques, the former being a demonstration of the latter.

Appendix: Plato's '4/3 mated with 5' and the role of Powers of 60.

It is in the nature of holy mountains to be triangular due to the 2 dimensions being underlaid by the primes 3 horizontally and 5 (figure 18). The other prime number involved in such mountains is 2, but in terms of practical filling of what is a semi-ordered lattice of products, the size of 2 relative to 3 and 5 is brought into balance by doubling 2 to 4. This usage is rewarded by noticing that (3,4,5) generates 60, which appears as 4 times 3 times 5 as the Babylonian base number, deified as Anu = 60. If the root of a holy mountain is its cornerstone of the 0^{th} power of all 3 numbers, then the square of 60 = 3600 is the same distance from 60 as 60 is from the cornerstone. From this one sees the median nature of 60 within such mountains, a kind of backbone to which Plato refers in his Republic (See McClain, (1978), Chapter 2, for an integrated summary in which a series of powers of 60 are developed in 3 steps, leading to the 'sovereign number' of 60^4 =12,960,000).

Obviously Plato's 3 steps now invite a 4^{th} since YHWH = 6.5.10.5 is given the limit 60^5 = 777,600,000 by McClain through his insight that exponent notation was meant, that is 6^5 times 10^5.

The same number has been inferred for Apollo within the text of Plato's Ion and the work of the late Platonist Plutarch, as follows in John Bremer's Plato's *Ion: Philosophy as Performance*, (2005), chapter 13:

> 'Plato composed his dialogue Ion around a joke and a riddle… Plutarch (c.46 to 120 A.D.), affords us some help. He was, for the last twenty or thirty years of his long life, a priest at Delphi. He had taken an oath of secrecy, and so he wrote with knowledge of much that he could not reveal but could not refrain from hinting at. The most relevant hints are to be found, first, in Isis and Osiris (381C) where he states that sixty 'is the first of measures for such as concern themselves with the heavenly bodies,' that is, with science - astronomy and harmonics. Second, in The E at Delphi (388-9), Plutarch tells us that the Pythagoreans called five the marriage number and adds that five is an attribute of the god, Apollo, which is confirmed by the importance of the number in music. The right-angled triangle 3, 4, 5 is used by Plato in the Republic, in formulating his 'marriage number,' and Plutarch points out that 3 is the male number, 4 the female number, and 5 is 'in some ways like its father and in some ways like its mother, being made up of 3 and 2.' This makes 5 the human number. Thus, there is the number sixty which is the base of all scientific work (derived, no doubt, from the ancient Babylonian use of sexagesimals; this usage spread not only to Hellas but also, later, to India where the yugas or 'great periods' are all based on some power of 60). Together with sixty, there is the number five, an attribute of Apollo, important in music; it is the human number, and, moreover, it is designated by the letter E in the Greek alphabetized system of numerals (epsilon being the fifth letter) and was carved in wood (and stone or metal?) at Delphi, in Apollo's temple. One combination of sixty and five produces 60 to the fifth power, 60^5, or 777,600,000, which may be regarded as Apollo's number, as Plutarch intimates; it is not certain, but this number may well have been known at Delphi. The number is of the utmost importance in tuning theory and is the least number necessary for tuning with the spiral of fifths and reconciling the sexagesimal and decimal expression of the tones involved, as will be explained below. It is thus worthy of the title of Apollo's number. Since, according to ancient practice, the zeros can be omitted [Neugebauer The Exact Sciences in Antiquity, Chapter II] we are left with 6 to the fifth power, 6^5 or 7776. Plato deliberately used only the head digits. Plato's joke is that the Ion has 7776 syllables - and thus Apollo does not need to be named in the dialogue since he is the whole of it. The joke is in the form of an unstated riddle, an (enigma): Why is Apollo not in the Ion? Because the Ion is in Apollo.'

The similarity between the Greek and Hebrew languages comes from their proximity to the development of the alphabetic system of phonemes, evolved near Canaan and possibly at Ugarit. The semitic languages, being Trilateral rather than Indo-European, caused Hebrew Hay =5 to be Greek E = 5 but the musicality and characterisation of the Deity as YHWH or Apollo equal to 60^5 leaves little doubt that only the usage of harmonic parallelism differed between different language groups while the harmonic facts are invariant. Thus McClain's proposed harmonic parallelism seems reasonable though quite hard to express to an audience needing to understand the texts, the harmonic facts and the imaginative way in which harmonic allusion was employed within texts, including those of Plato.

This diagram (Fig.19) was constructed to illustrate how the balanced metrical triangles of each of the 5 earliest powers of 60, of 1, 2, 3, 4, 5, climb the 'middle path' within the holy mountain of YHWH. It illustrates a commonality of approach by the musicology inherited by the Classical Greeks and Jewish compilers of the Bible, each from a similar, if not the same, tradition having a long but uncertain history lasting at least one, and possibly two, thousand years.

ILLUSTRATIONS

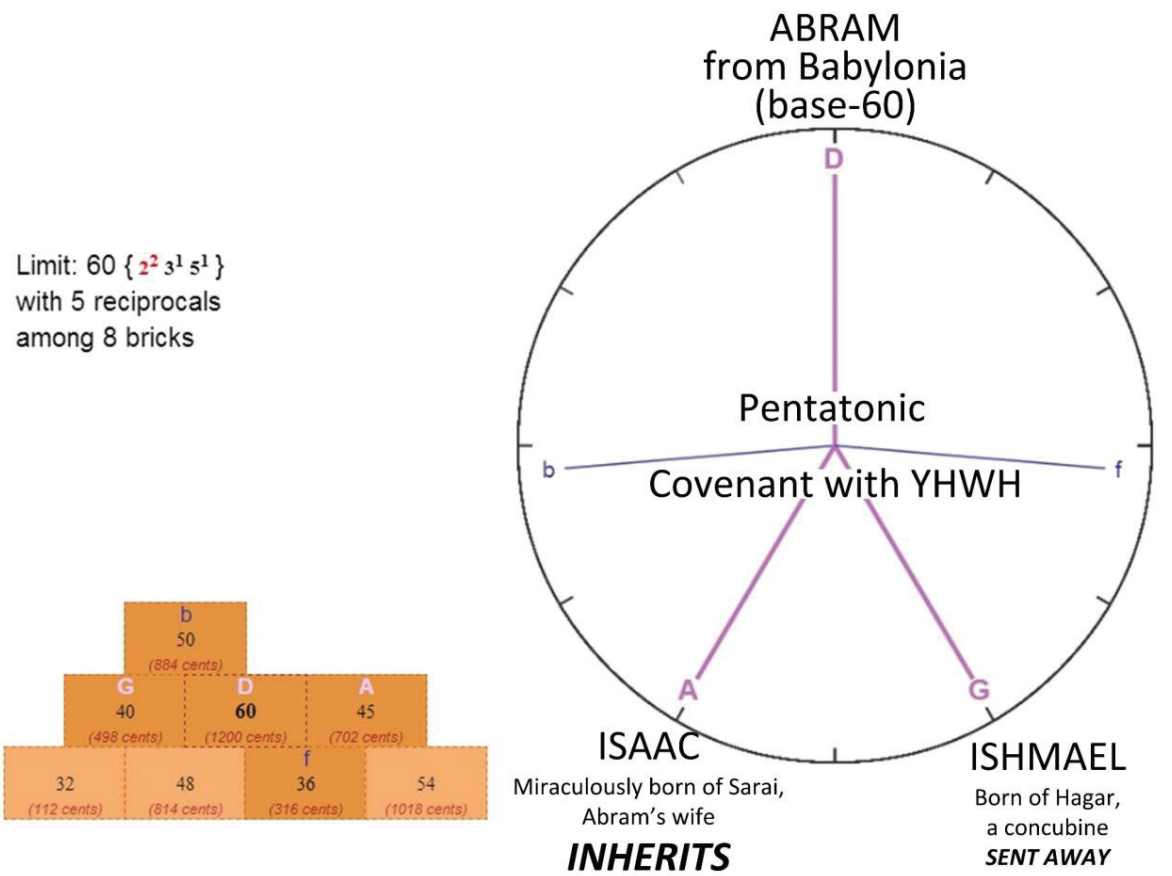


Figure 1. Abram comes from the base 60 region of the 'Chaldees' and expresses 2 Pythagorean children, Ishmael (G) and Isaac (A) within a pentatonic covenant with YHWH, seen as the Just 'horizon' over the land of Canaan, between pitches b and f within the Tone-Circle for 60.

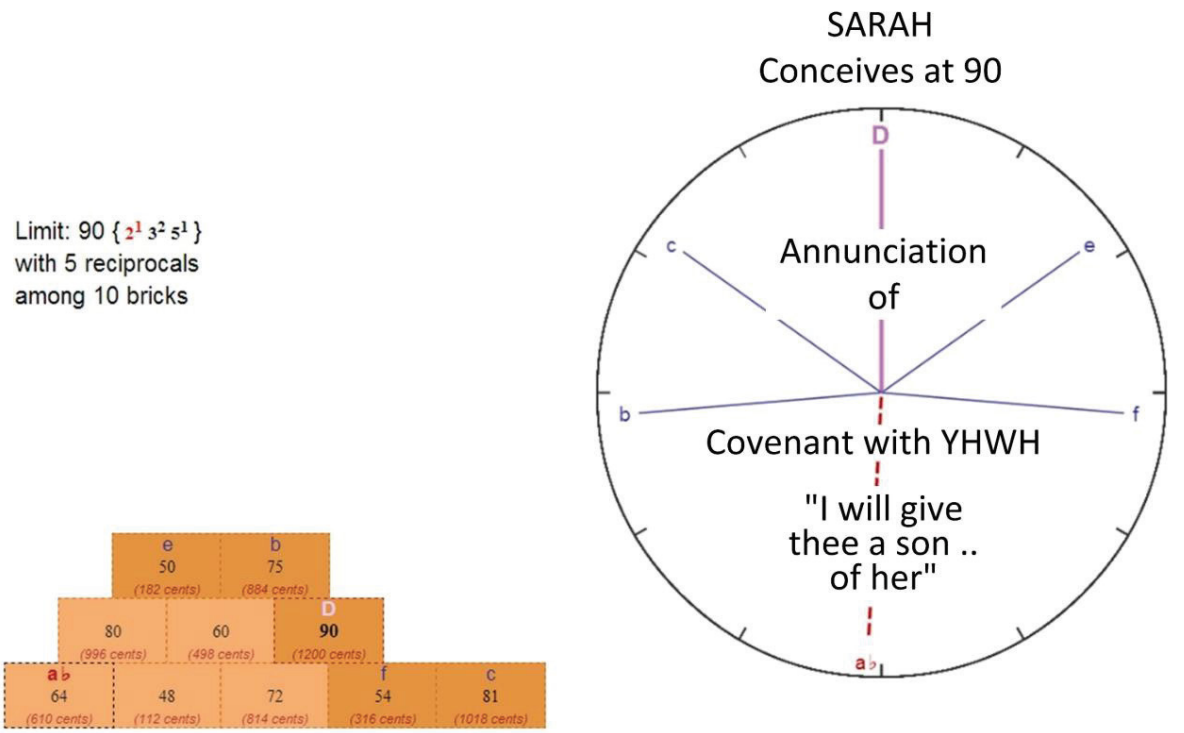


Figure 2. Sarai is renamed Sarah when she gives birth, at 90, with 2 3s and 1 new 5 and indicating a change in the location of D within the narrative. All the components of Isaac's matrix are then in place except for Isaac's twin sons Esau and Jacob. Sarah, like Abraham, have both received a new Hey (H = 5), lifting her and Abram (243), one step towards YHWH through the creative male number 5 which places tones onto row 2 of the mountain, the row the bricks of which generating Just intonation.

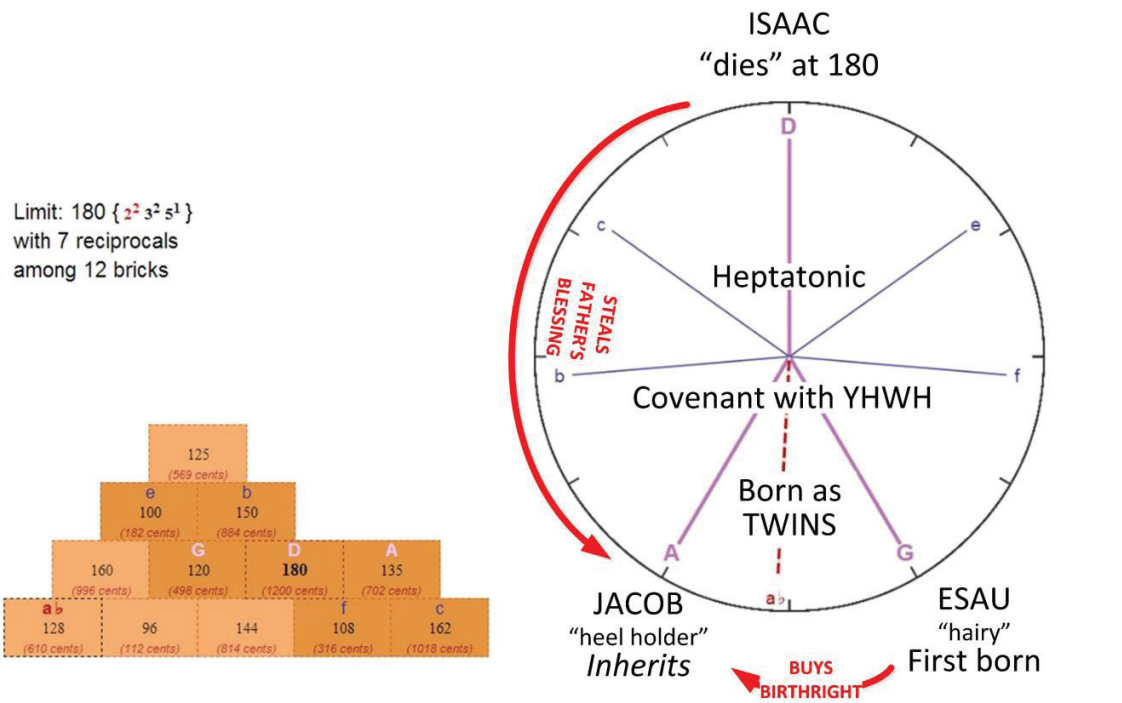


Figure 3. Conceived when Sarah is 90 and dead at an age of 180, Isaac represents the shift out of root 15 into the richer territory rooted in 45, two powers of 3 instead of merely 1. This will 'set the stage' on the holy mountain for Jacob to give birth to the (chromatic) Twelve Tribes when Jacob is renamed Israel.

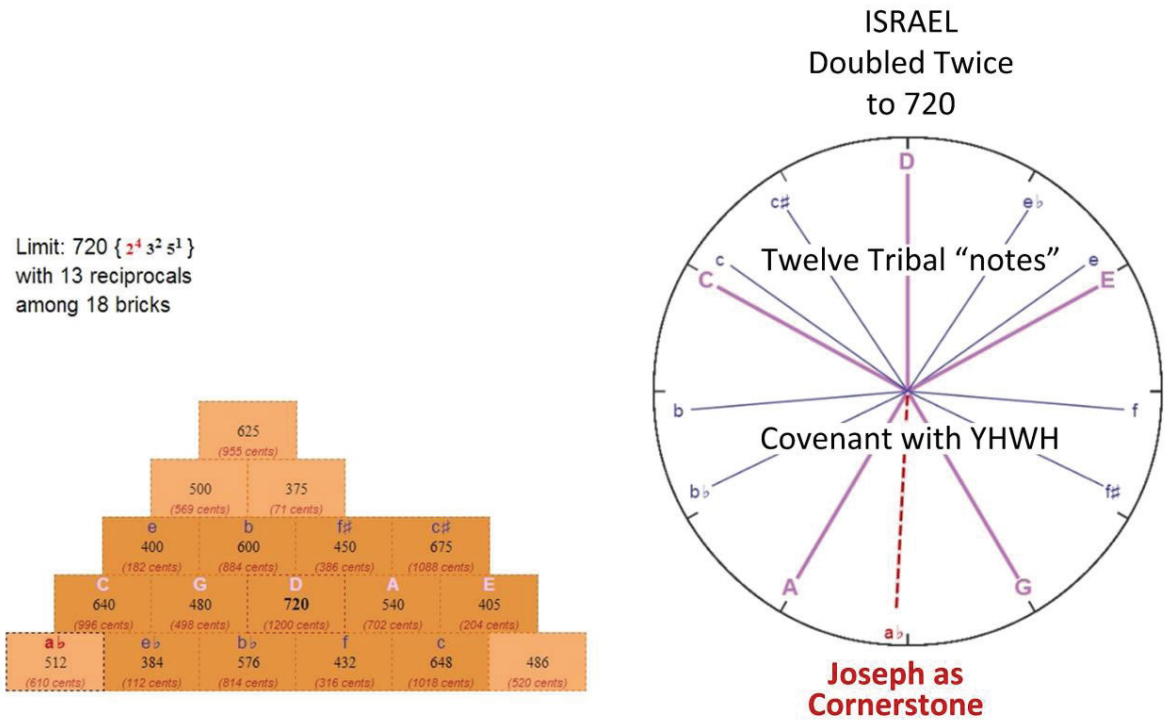


Figure 4. Jacob is doubled twice through being renamed after wrestling with the angel of the Lord, YHWH, to then form the octave 360:720 in which 12 symmetrical pitches surround Israel. The 11 sons and 1 daughter, as 12 tones, are found wanting but Joseph magnifies the family potential as Cornerstone, triggering an epic transition to Egypt.

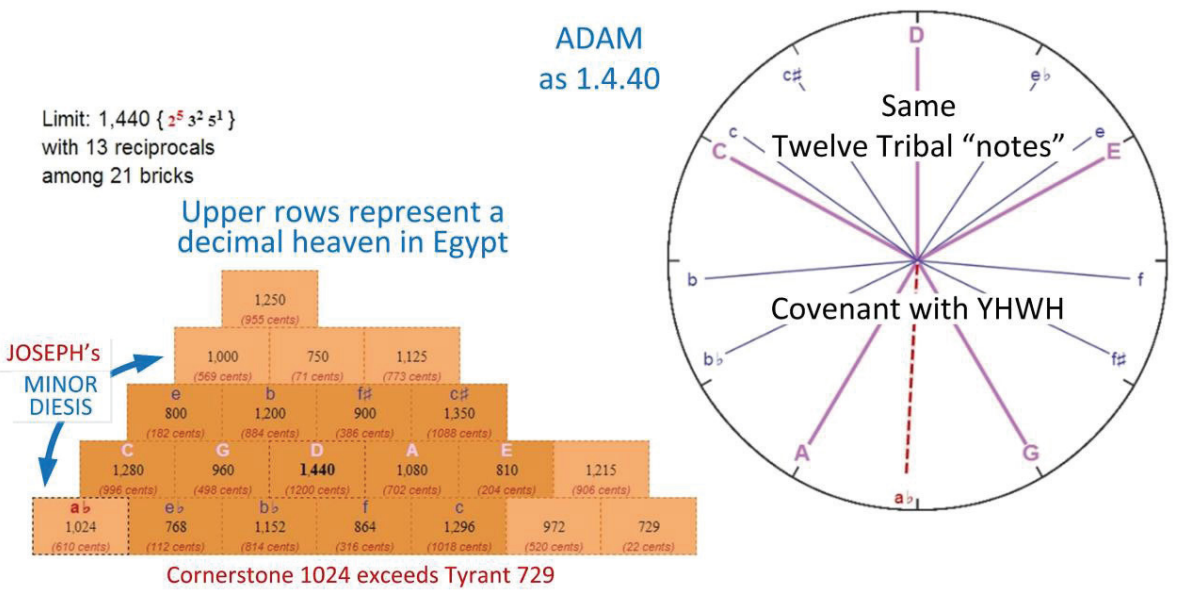


Figure 5. Adam's decimal completion as 1.4.40 expressed as mountain and 12 tribe tone circle.

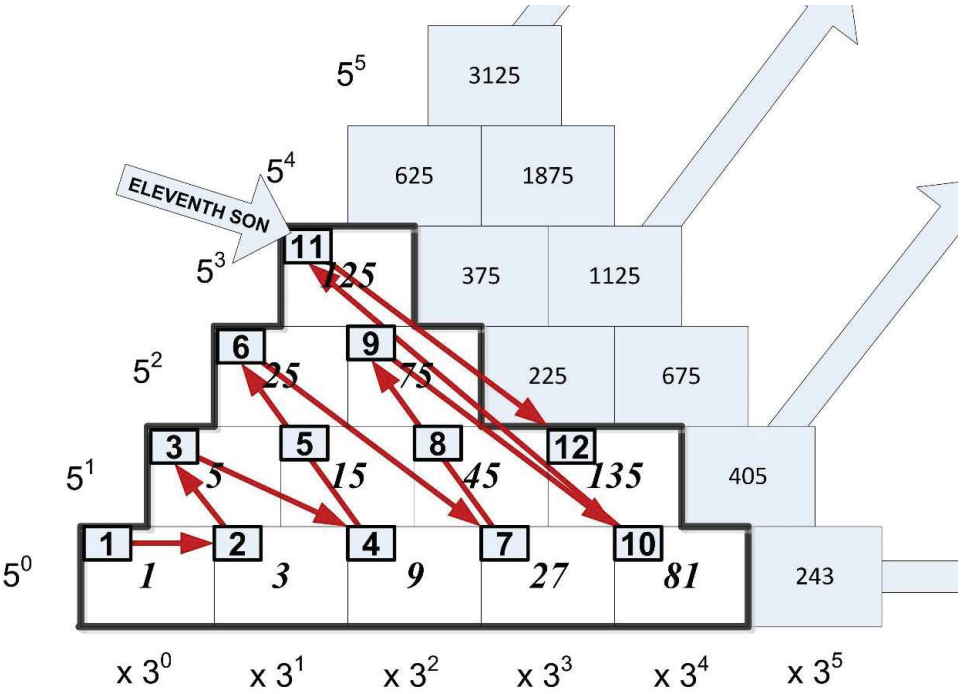


Figure 6. Counting the 12 sons of Jacob as to the numerical order in which the root of the mountain values arise as products of 3 and 5: [1, 3, 5, 9, 15, 25, 27, 45, 75, 81, 183 125, 135]. This outline of 'bricks' does exist as the earliest pentatonic mountain, with the limit 144 = 12^2.

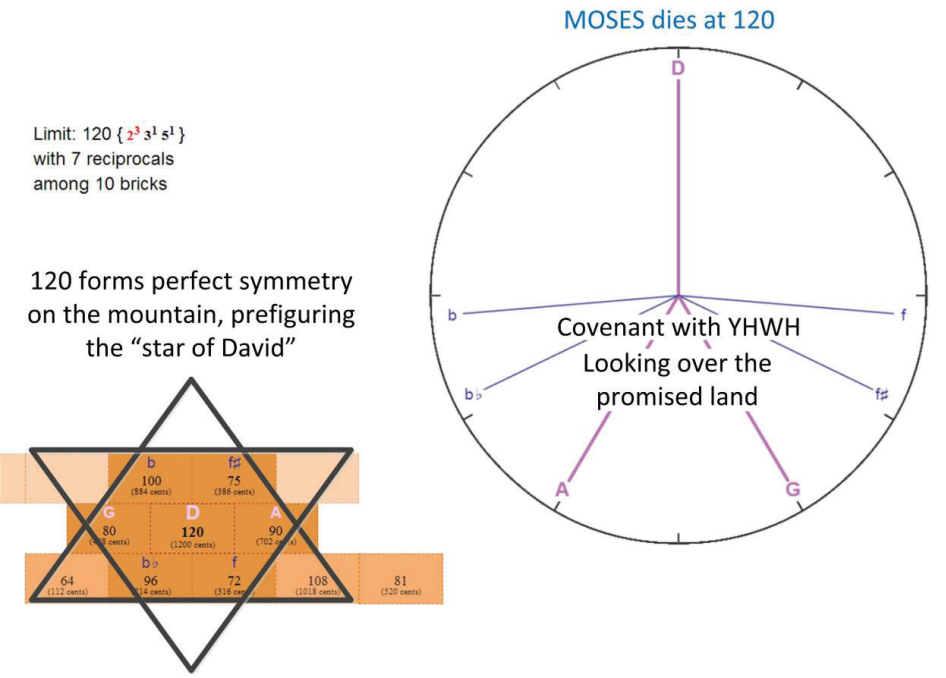


Figure 7. Moses appears coded at 120 to complete the Journey of the Patriarchs, leading back to Canaan. The limit encodes 12 in the essence of the decimal notation as 12 times 10 (rather than 2 times 60.) The form of the Magen David symbol appears latent on the mountain due to the perfect hexagon at its core of symmetrical tones generated by Moses' age of 120.

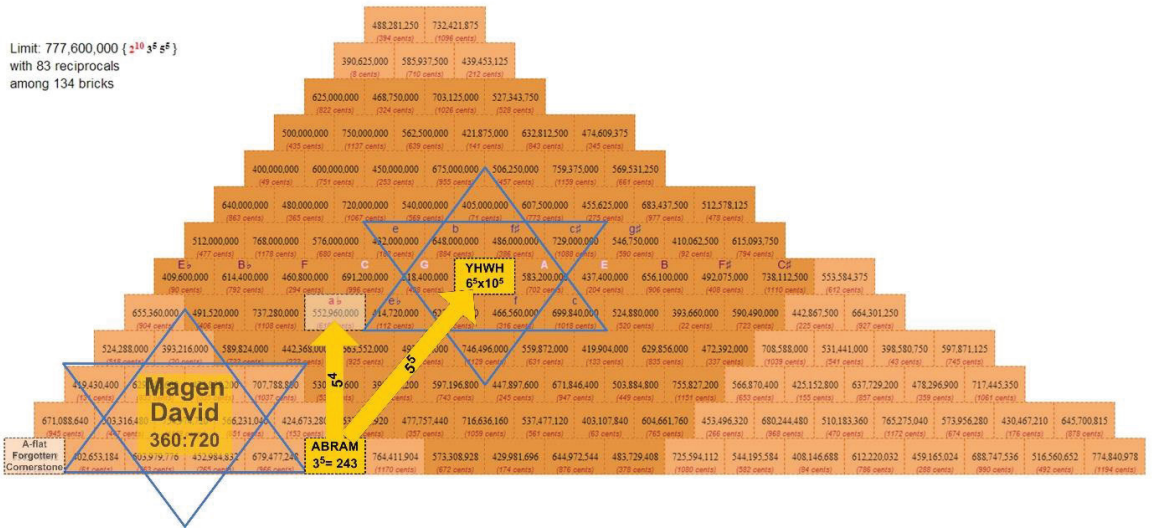


Figure 8. YHWH has Abram = 243 as its root within the powers of 3, being 3,200,000 230 larger and creating the characteristic Twin Peak, when powers of 5 are introduced.

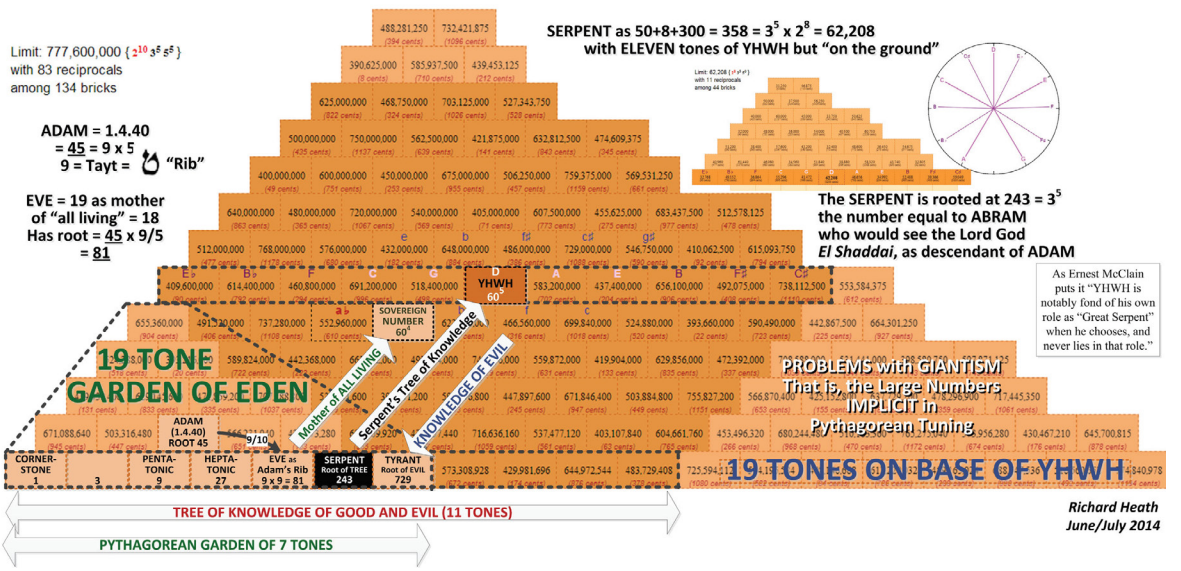


Figure 9. The harmonic mountain of YHWH with the Garden of Eden set within the non symmetrical region of 19 tones. Only Adam has a power of 5, the other actors 'on the earth'. The cornerstone is on the left and Serpent's Tree of Knowledge on the right. See also figure 9, for closeup of Garden.

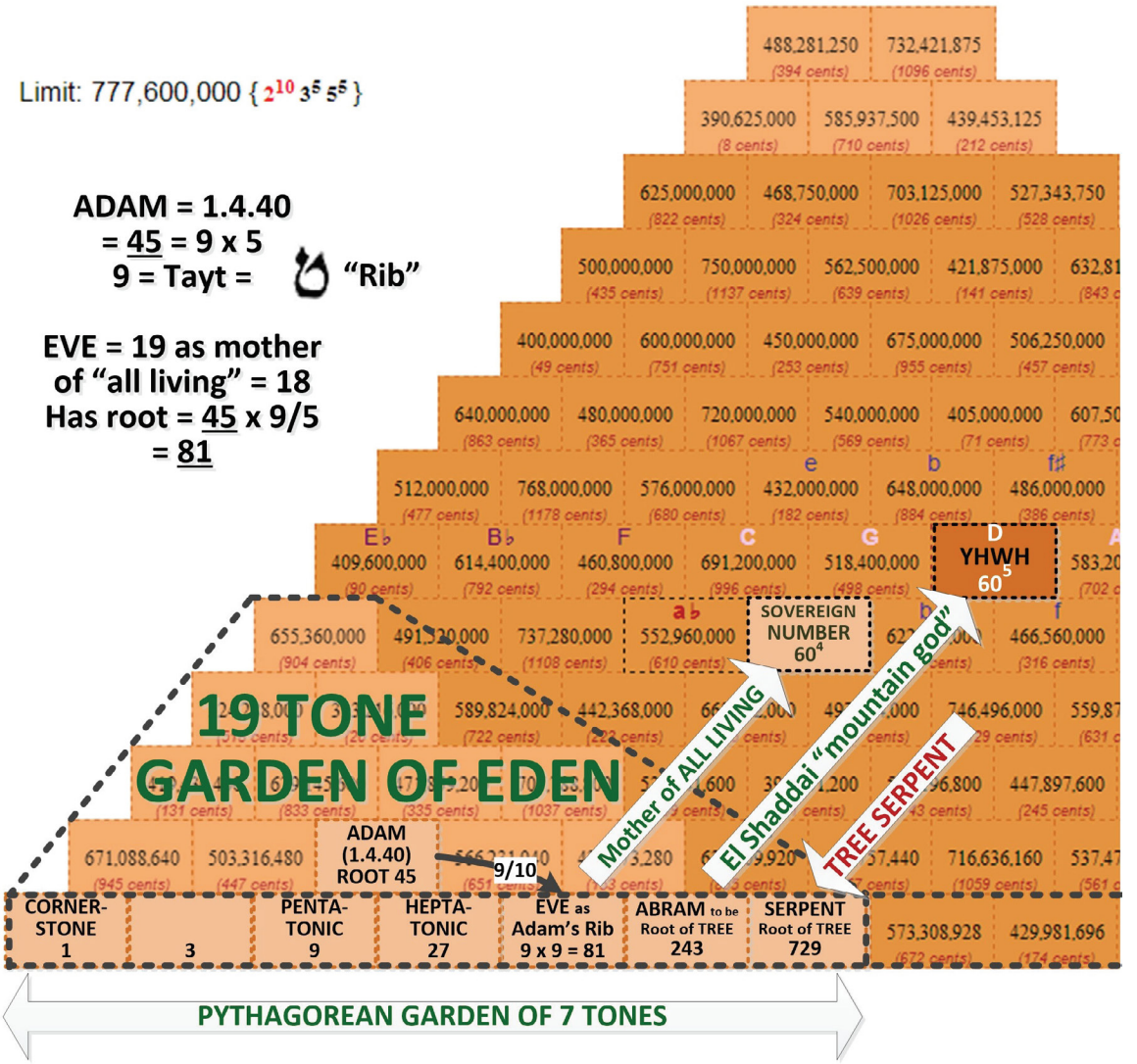


Figure 10. reference figure 9

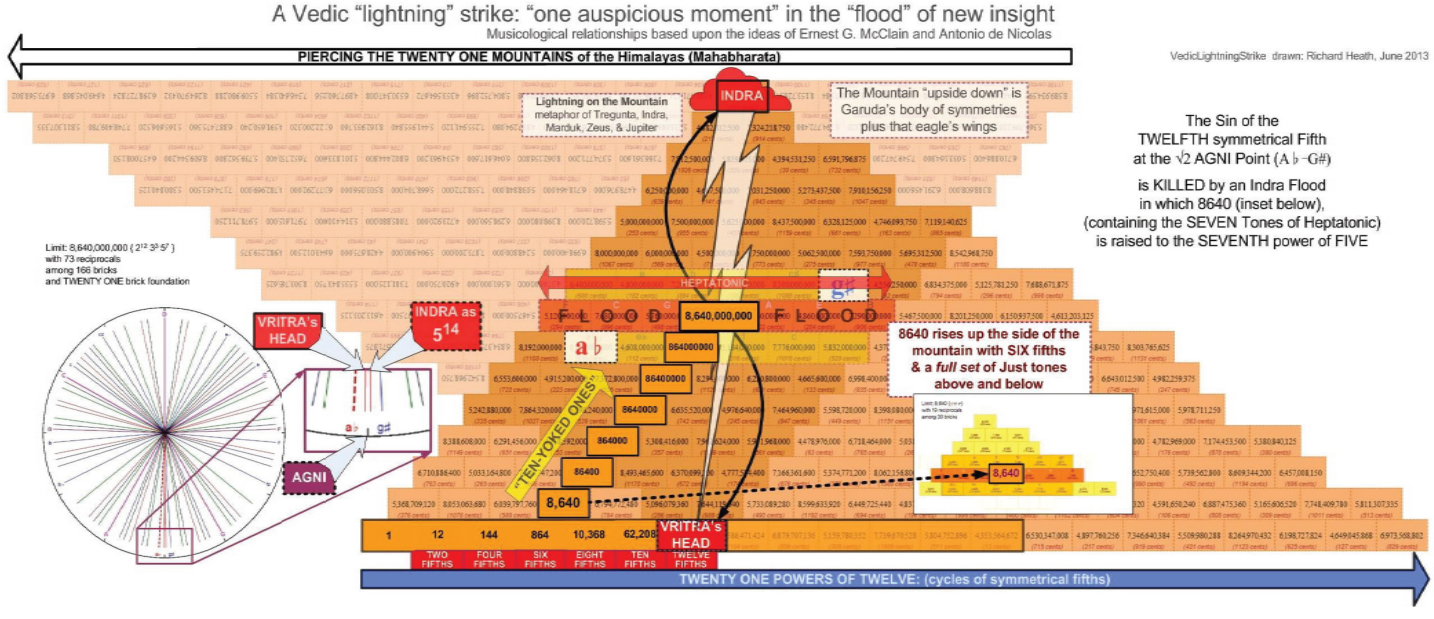
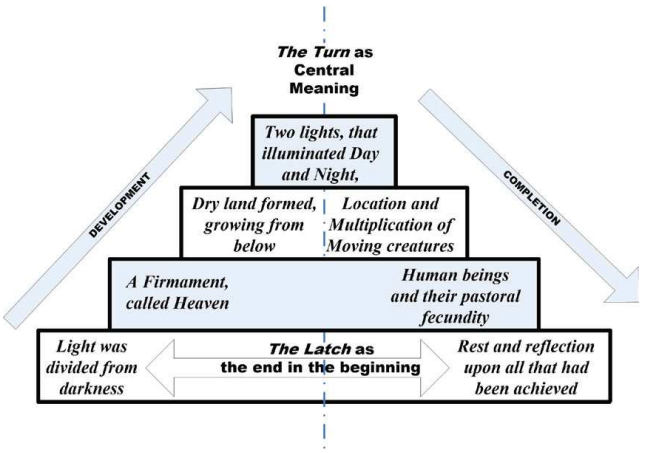


Figure 11. The Flood Dynamics



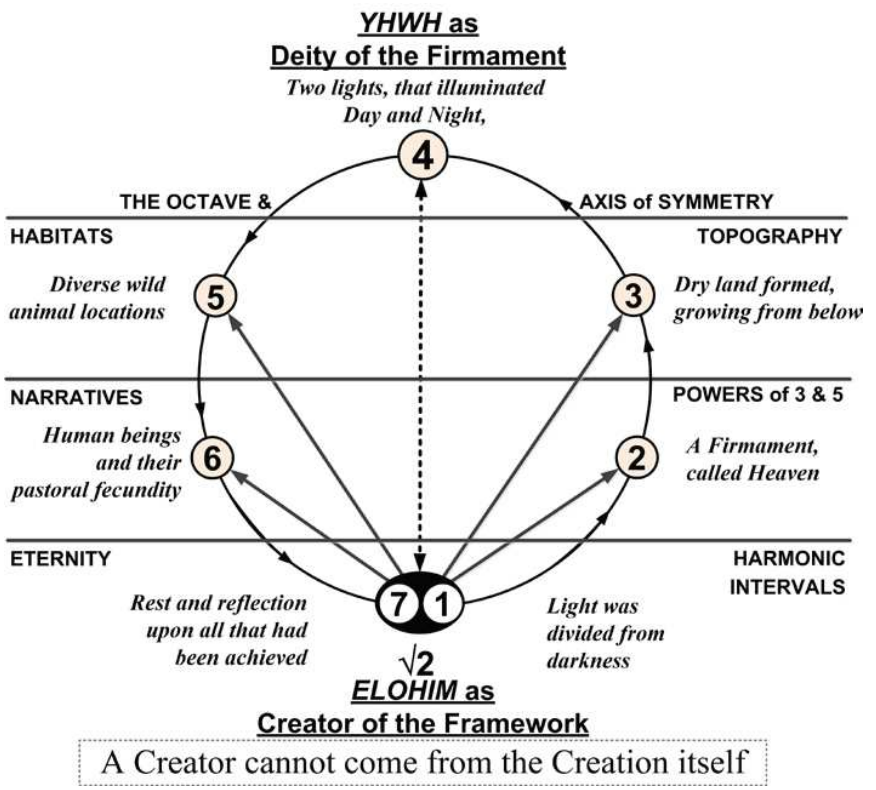


Figure 12. (above) Narrative structure of Seven Days in "pedestal" format. 12 (below) Seven Days of creation as Ring composition: then existing text

Figure 13. Generation of all the significant intervals from the first 6 numbers, sometimes called Senarius, the octave, fifth, fourth, major third and minor third. These are the essential intervals required for generating McClain's holy mountains and doubtless figured in the teachings of the Pythagoreans by the time of Plato's Academy.

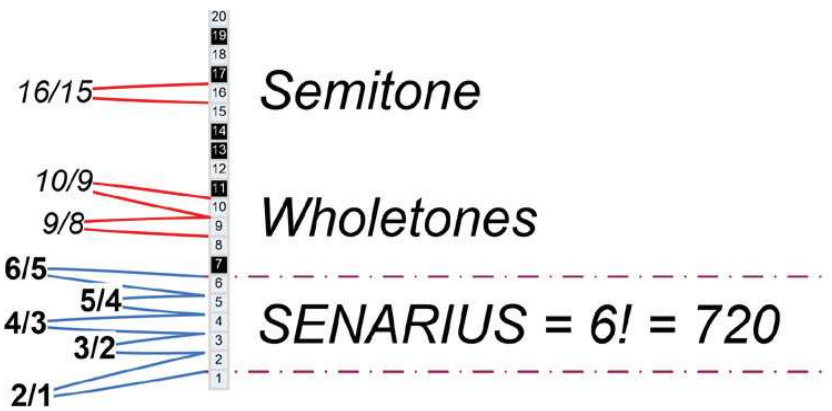


Figure 14. The wholetones 9/8 and 10/9 and the Just semitone of 16/15 are new musical intervals below 20. (Non-harmonious Prime numbers seven and above are being shown in black.)

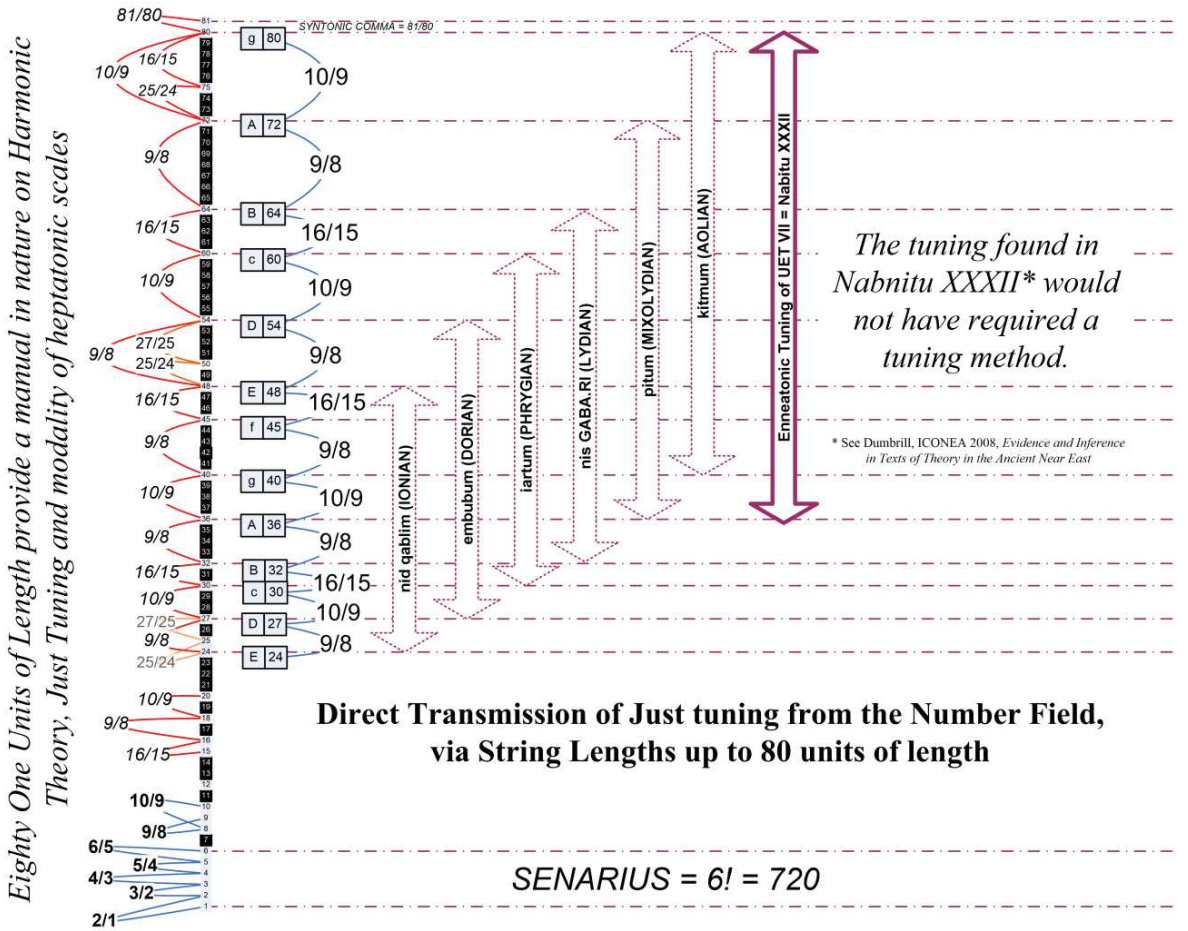


Figure 15. The field of numbers within 81 describe all of the Just heptatonic modal scales employed in Babylonia, Egypt, India and Greece. The white numbers were on the Nippur harmonic numbers list and these punctuate the black number lengths which contain factors other than (2,3,5)

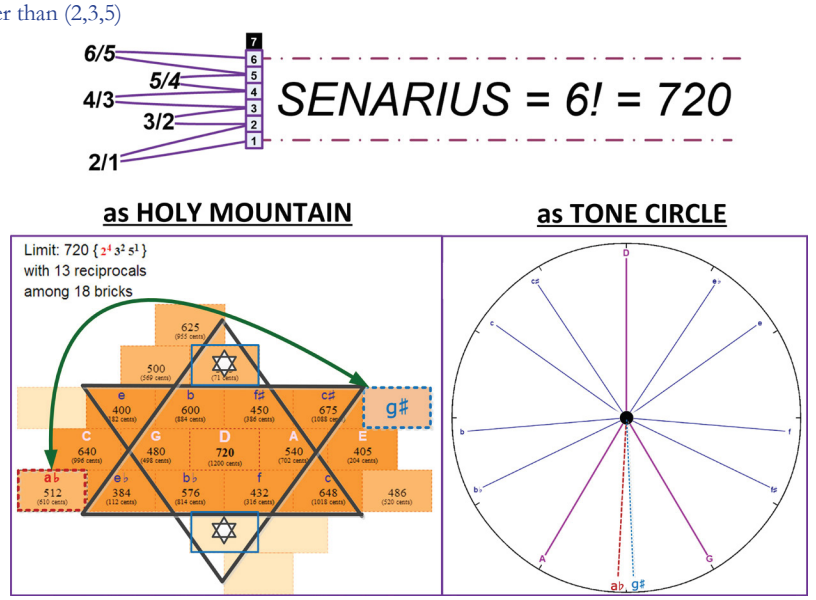


Figure 16. The Senarius 6! = 720 as a Pentatonic instrument restricted to only three Pythagorean tones (G,D,A) by applying the shield of Magen (Shield) of David. The 12th tone can be provided from the mountain (ab on descent) and its inverse (g# on ascent).

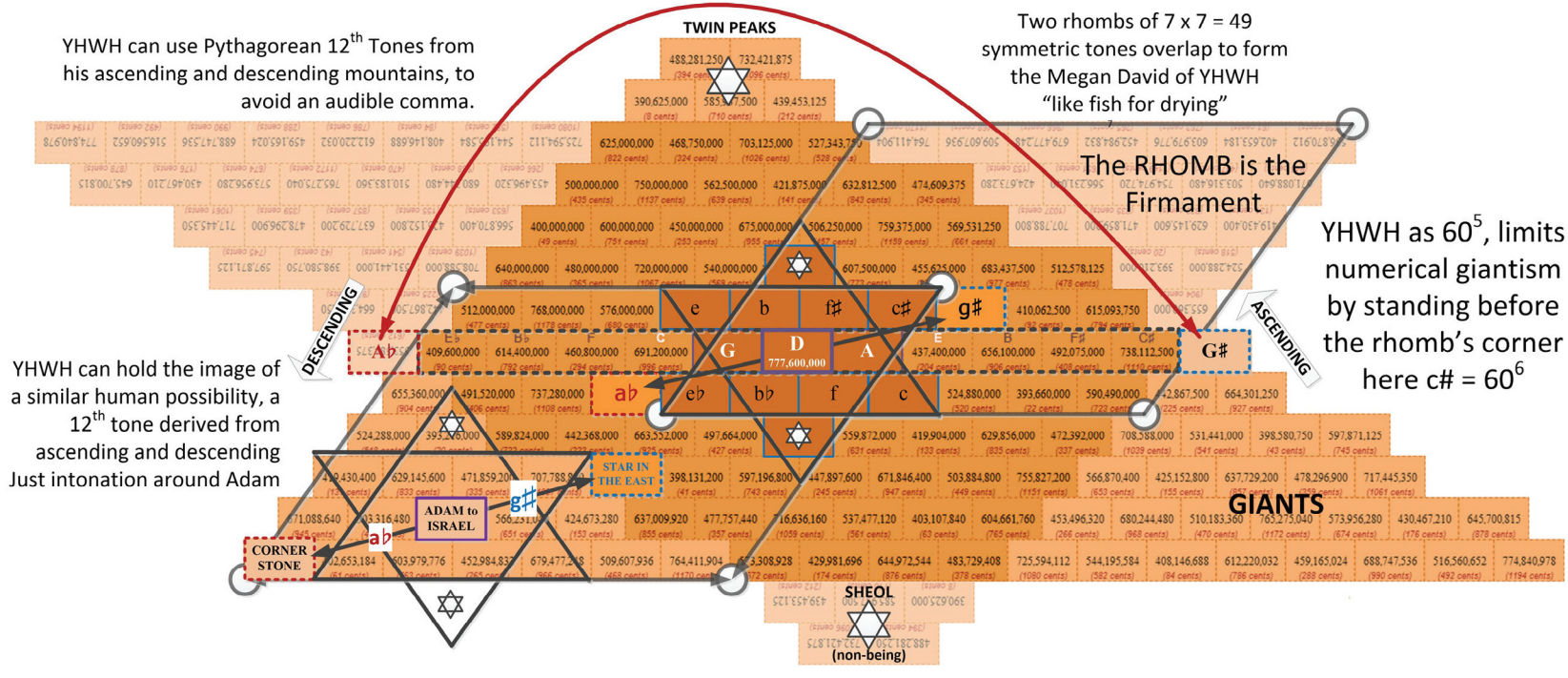


Figure 17. YHWH matrix has a set of 11 symmetrical tones, matching the 11 tones selected out by the Magen David. YHWH can select an ascending or descending 12th tone 605 free from any Pythagorean comma.

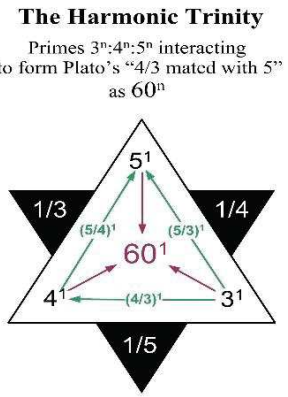


Figure 18. The Mountainous nature of 60; the natural archetype for the Biblical Holy Mountains is Plato's 4:3 'mated with 5', showing the triangular form as emerging from their product of 60 and its powers.

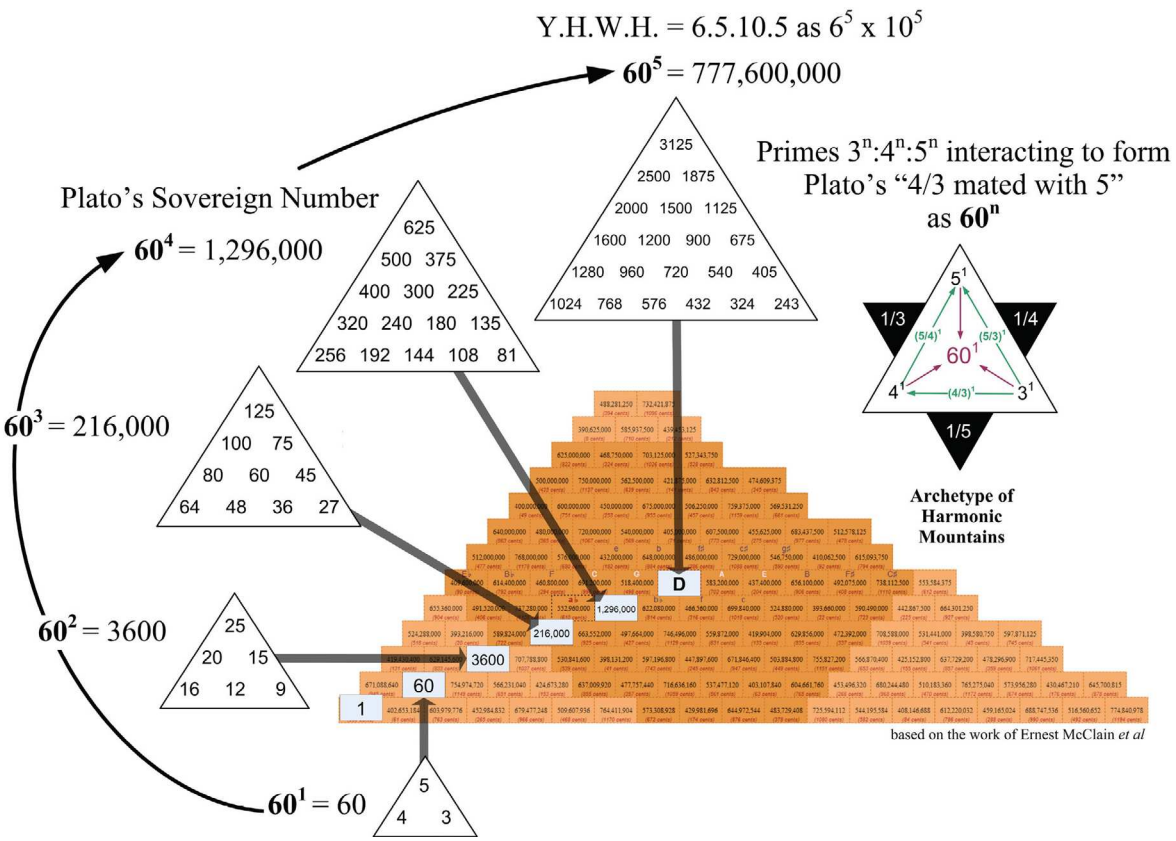


Figure 19. Plato's Three Steps plus that of Apollo/ YHWH

123-137

THE OPENING OF PLATO'S *POLITY*

Κατέβην χθὲς εἰς Πειραιᾶ μετὰ Γλαύκωνος τοῦ ᾽Αρίστωνος

John BREMER

To open Plato's masterpiece is to make accessible a comprehensive view of the cosmos, including and especially of ourselves in it.

Unfortunately, the dialogue is known, at least in the English-speaking world, as the *Republic*. This is not the title that Plato himself gave it. He called it Πολιτεία, which is best translated or transliterated as Polity, a somewhat old-fashioned English word meaning 'constitution'. But *of what* is it the constitution?

The conventional title of *Republic* has encouraged the misleading supposition that the dialogue is essentially a political treatise, a treatise on government. In a way it is, but not in the sense in which it is customarily understood. The *polity* or constitution that the dialogue describes and embodies is that of the human soul, or of human nature, and the government with which it is concerned is self-government

The πόλις or city - state or society - does not have a soul or a nature of its own, but takes its character from the souls of those who govern and control it, and from custom and habit. It is derivative - just as the city 'in speech' (τῷ λόγῳ, *St.* 369c) is derivative, made (ποιῶμεν) from the beginning (ἐξ ἀρχῆς). This only reflects Plato's insistence on the responsibility of each one of us, of individual humans, an insistence that lay behind Socrates' questioning of individual Athenians and Sophists.

This essay regards the literal opening of the *Polity* as an introduction to the dialogue as a whole. In the first part, it examines the beginning words from an historical and literary point of view; in the second part, it examines the philosophical import, the meaning, of that beginning; and in the third part, offers some reflections.

Diogenes Laertius, in his Life of Plato (III.37), says:

> 'Euphorion and Panaitios relate that the beginning of the *Polity* was found revised and re-written several times. . .'
>
> 'Εὐφορίων δὲ καὶ Παναίτιος εἰρήκασι πολλάκις ἐστραμμένην εὑρῆσθαι τὴν ἀρχὴν τῆς Πολιτείας.'

Diogenes is a shadowy figure. Born perhaps in Caria or Cilicia around 200 C.E., he died around 250 C.E., but little reliance should be placed on these suppositions. It is certain that he wrote before 500 C.E. (since he is quoted by Sopater [fl.500 C.E.]) and he refers to no philosophers after the third century C.E. But *The Lives of Eminent Philosophers* by Diogenes Laertius is not the first reference to the opening words of Plato's *Polity*. He cites Euphorion of Chalkis (3rd century B.C.E.) and Panaitios of Rhodes (2nd century B.C.E.), and there are two other sources. One is Dionysios of Halikarnassos (fl. 40 B.C.E.), and the other is Quintilian (fl. 70 C.E.).

Adam (in his edition of the *Republic*) quotes Dionysios (*De Comp. verb.*, pp. 225-7. Loeb):

> 'ὁ δὲ Πλάτων, τοὺς ἑαυτοῦ διαλόγους κτενίζων καὶ βοστρυχίζων, καὶ πάντα τρόπον ἀναπλέκων, οὐ διέλιπεν ὀγδοήκοντα γεγονὼς ἔτη. ἅσι γὰρ δή που τοῖς φιλολόγοις γνώρισμα τὰ περὶ τῆς φιλοπονίας ἀνδρος στορούμενα, τά τ ᾽ ἄλλα, καὶ δὴ καὶ τὰ περὶ τὴν δέλτον ἥν ελευτήσαντος αὐτοῦ λέγουσιν εὑρεθῆναι ποικίλως μετακειμένην τὴν ἀρχην τῆς πολιτείας ἔχουσαν τήνδε 'κατέβην χθὲς εἰς Πειραιᾶ μετὰ Γλαύκωνος τοῦ ᾽Αρίστωνος.'
>
> 'Plato did not stop, even at the age of eighty years, combing and curling his dialogues, braiding them in every way. Of course, the stories about his love of labor are known to all lovers of learning, and especially the one about the writing tablet, which, they say was found at his death among his possessions with the beginning of the Republic in a variety of ways: κατέβην χθὲς εἰς Πειραιᾶ μετὰ Γλαύκωνος τοῦ ᾽Αρίστωνος.'

Adam also cites Quintilian (*Inst.*VIII 6.64). He adds that, since Cicero (fl. c. 60 B.C.E.) was tolerably familiar with the writings of Panaitios, it is possible that he too knows the same story, in view of de *Senectute*, V 13, where he says of Plato 'uno et octogesimo anno scribens est mortuus'. The complete passage is as follows:

> 'Est etiam quiete et pure atque eleganter actae aetatis placida ac lenis senectus, qualem accepimus Platonis, qui uno et octogesimo anno scribens est mortuum.'
>
> 'But there is also the tranquil and serene old age of a life spent quietly, amid pure and refining pursuits - such an old age, for example, as we are told was that of Plato, who died, writing, in his eighty-first year.'

The phrase 'died, writing' need not be taken literally, but probably indicates only that Plato was still writing in his old age.

Quintilian says that the opening words were found written 'plurimis modis' as an example of the rhythm imparted to the written word by 'opportuna ordinis permutatio'.

> 'Nec aliud potest sermonem facere numerosum quam opportuna ordinis per mutatio; neque alio ceris Platonis inventa sunt quattuor ilia verba quibus in illo pulcherrimo operum in Piraeum se descendisse significat, plurimis modis scripta, [quam] quod eum quoque maxime [numerosum] facere experiretur.'
>
> 'Further, it is impossible to make our prose rhythmical except by artistic alterations in the order of words, and the reason why those four words in which Plato in the noblest of his works states that he had gone down to the Peiraeus were found written in a number of different orders upon his wax tablets, was simply that he desired to make the rhythm, as perfect as possible.'

All the references seem to be in works (or at least, in discussions) of style. Dionysius uses the story to confirm and illustrate the extreme attention that Plato paid to style and stylistic details. Riginos (in *Platonica*, p. 186) suggests that the originator of this anecdote, which 'illustrates the careful and deliberate manner of composition cultivated by the writers of the Hellenistic period', may be Euphorion who was librarian to Antiochos the Great (reigned 223 - 187 B.C.E.).

T. Herbert Warren, in his edition of The *Republic of Plato*, Books I-V, writes that the story, true or not, is fairly used by Dionysius to indicate that Plato's excellent style was not attained without conscious trouble 'but that he was a most laborious and fastidious composer'. He continues:

> 'That the ancients, masters as they were of style, did not believe in felicitous scribbling, is confirmed by many such stories, both in Greek and Latin, e.g., Sophocles's accounts of his attainment of his own third period of 'golden mediocrity' (Plut. de Prof. Virt. Sent., p. 79B); and the well-known story of Demosthenes transcribing Thucydides eight times (Lucian adv. Indoct., c.4); in Latin, Horace's criticism upon Lucilius, Satt. I. 4.9. et seqq., I. 10.9 etc.; and his own precepts in the Ars Poet., v. 388 etc; or the account of Virgil's laborious method in the Suetonian life c.22. ed. Nettleship. Cp. Qintil. x. 3.8 and Aul. Gell 17.10. In modern days we have the instances of Pope and Gray, to mention no others, in our own language, and we may remember, with Carlyle, Goethe's remark about himself, that he 'had nothing sent him in his sleep, no page of his but he knew well how it came there'; or Sheridan's famous, if unparliamentary, dictum about easy writing.'

Warren also points out that the story is improved by Seneca (fl. c. 50 C.E.), *Ep. SB*, 3 1 and Plato is made to die on his eighty-first birthday.

> 'Plato ipse ad senectutem se diligentia protulit. Erat quidem corpus validum ac forte sortitus et ille nomen latitude pectoris fecerat, sed navigationes ac pericula multum detraxerant viribus; parsimonia tamen et eorum, quae aviditatem evocant, modus et diligens sui tutela perduxit ilium ad senectutem multis prohibentibus causis. Nam hoc scis, puto, Platoni diligentiae suae beneficio contigisse, quod natali suo decessit et annum unum atque octogensimum inplevit sine ulla deductione. Ideo magi, qui forte Athenis erant, immolaverunt defuncto, amplioris fuisse sortis quam humanae rati, quia consummasset perfectissimum numerum, quern novem novies multiplicata conponunt...'
>
> 'Plato, himself, by taking pains, advanced to old age. To be sure, he was the fortunate possessor of a strong and sound body (his very name was given him because of his broad chest); but his strength was much impaired by sea voyages and desperate adventures. Nevertheless, by frugal living, by setting a limit upon all that rouses the appetites, and by painstaking attention to himself, he reached

> that advanced age in spite of many hindrances. You know, I am sure, that Plato had the good fortune, thanks to his careful living, to die on his birthday, after exactly completing his eighty-first year. For this reason wise men of the East, who happened to be in Athens at that time, sacrificed to him after his death, believing that his length of days was too full for a mortal man, since he had rounded out the perfect number of nine times nine.'

It is not known what evidence Seneca had for this version of Plato's death. According to Diogenes Laertius, who cites Hermippus, he died in his eighty-first year in 347 B.C.E. while attending a wedding feast. King Philip of Macedon is said to have paid him honors at his tomb. Also according to Diogenes Laertius, the name Plato was a nickname, his real name being Aristocles, the name of his grandfather; the nickname was given because of the breadth (πλατύς) of his shoulders, of his forehead, or of his style.

Sheridan's dictum in *Clio*'s *Protest* may have been 'unparliamentary' when Warren wrote (1901) but it will not cause any blush these days:

> 'You write with ease to show your breeding,
> but easy writing's curst hard reading.'

In *Greek Prose Style* (p. 41), J.D. Denniston writes:

> 'It is said that Plato meditated long before he arrived at the order in which the first eight words of the Republic stand in our text: μετὰ Γλαύκωνος τοῦ Ἀρίστωνος. The considerations which ultimately led him to the choice of this wholly satisfactory arrangement have not been recorded, but may be guessed. The disposition of the colon into two commata of approximately equal length, with a barely perceptible pause between them, makes for ease and grace. That means that μετὰ Γλαύκωνος τοῦ Ἀρίστωνος must come at the end, and it only remains to settle the order of the first four words. The placing of the two monosyllables in the middle, flanked by the two trisyllables, gives a pleasing variety of long and short words. Unstudied as this opening appears, the art that goes to the making of it is yet susceptible to analysis. Similar considerations of what writers on prose rhythm style 'typology' underlie much that is beautiful in prose and verse . . .[There follows an example from Herodotus] . . . Here the words are of the simplest, their arrangement perfectly natural. The effect may seem due to accident. But such accidents do not befall inferior writers.'

It is not known who, if anyone, claimed that Plato 'meditated long'.

In *Greek Word Order*, K.J. Dover discusses the logical determinants affecting Greek word order:

> 'The relation between theme and predicate has played a larger part in discussions of word order than it deserves, for the statement 'the theme precedes the predicate' is tautologous, and 'the predicate precedes the theme' is self-contradictory. One can no more predicate something of a theme not yet expressed than one can 'contribute to' a discussion not yet proposed or begun. In the case of English utterances of the type 'as for x,y' and 'the one who x was y' we have formal linguistic grounds for saying that x is the theme. But where we have only the order x, y to guide us, we cannot infer that the speaker necessarily conceives x as theme; we can only say that y is not the theme. Most actual utterances have neither theme nor predicate. Dionysius's story about Plato's attempts to find a pleasing arrangement of Rep.327 A, κατέβην χθὲς εἰς Πειραιᾶ μετὰ Γλαύκωνος τοῦ Ἀρίστωνος is 'bien trouvé'.'
>
> 'No element in this sentence imposes itself as the 'theme'; and however determined we might be to analyse it as saying something about something else, we could never find cogent grounds for deciding whether it says something about yesterday, or about Socrates going to the Piraeus, or about Socrates being with Glaucon.'

Dover later adds that the opening eight words contain:

> '. . . five elements no one of which is predictable and no one of which is dispensable in the sense that if it were omitted it could be understood from the context. If we are to retain the terms 'logical subject' and 'logical predicate' we must say that the utterance consists simply of five logical predicates; which would be a bizarre use of the word 'predicate."

Another reference to the *Polity*'s opening sentence can be found in Demetrius' *On Style*, 1.21, probably written around 70 C.E.:

> 'διαλογικὴ δέ ἐστι περίοδος ἡ ἔτι ἀνειμένη καὶ ἀπλουστέρα τῆς ἱστορικῆ˜ς, καὶ μόλις ἐμφαίνουσα, ὅτι περίοδός ἐστιν, ὥσπερ ἡ τοιάδε, κατέβην χθὲς εἰς τὸν Πειραιᾶ μέχρι τοῦ ἄτε νῦν πρῶτον ἄγοντες. ἐπέρριπται γὰρ ἀλλήλοις τὰ κῶλα ἐφ ' ἑτέρῳ ἕτερον, ὥσπερ ἐν τοῖς διαλελυμένοις λόγοις, καὶ ἀπολήζαντες μόλις ἂν ἐννοηθεῖμεν κατὰ τὸτέλος, ὅτι τὸ λεγόμενον περίοδος ἦν, δεῖ γὰρ μεταξὺ διῃρημένης τεκαὶ κατεστραμμένης

> λέξεως τὴν διαλογικὴν περίοδον γράφεσθαι, καὶμεμιγμένην ὁμοίαν ἀμφοτέροις.'
>
> 'The period of the dialogue is one which remains lax, and is simpler than that of narrative. It scarcely betrays the fact that it is a period. For instance, 'I went down to the Peiraeus' as far as the words 'since they were now celebrating it for the first time'. The members are flung one upon the other as in the disjointed style, and when we reach the end we can hardly realize that the words formed a period. For the period used in dialogue should be a form of writing midway between the resolved and the compacted style, compounded of both and resembling both.'

It is hard to understand what Demetrius is saying here, or to understand its correctness. The testimony of the ancients and of our own reading of the opening eight words makes it hard to see that the 'members' are 'flung together' and in 'the disjointed style'. Moreover, Demetrius misquotes the opening words, inserting the definite article τὸν before Πειρειᾶ. The situation is further complicated because although the words appear in a dialogue, in fact they are part of an accompanying narrative. Should they be referred to criteria appropriate to the periods of a dialogue or of a narrative?

The opening eight words of the *Republic* are undoubtedly metrical and, as Denniston points out, they fall neatly into two commata. The first of these appears to be made up of an ἀνάπαιστος, an ἴαμβος, and a μολοττός, according to the nomenclature of Dionysius of Halikarnassos, who gives them the characters of stately, not lacking in nobility, and dignified and far-striding, respectively. The second comma appears to be made up of an ἴαμβος and a βακχεῖος, repeated. The latter is given the character of virile and grave. Although rhythmical, the eight words do not seem to fall into any recognizable verse form, so they must be taken as prose. But they also have a musical quality, and Dionysius reminds us (XI.20) that:

> 'μουσικὴ γὰρ τις ἦ καὶ ἡ τῶν πολιτικῶν λόγων ἐπιστήμη τῷ ποσῷ διαλλάττουσα τῆς ἐν ὠδῇ καὶ ὀργάνοις, οὐχι τῷ ποιῷ.'
>
> 'The science of public oratory was a musical science, differing from vocal and instrumental music in degree, not in kind.'

Although stories about the opening eight words of the *Polity* were freely circulated, as evidenced above, there is no explanation as to their importance, except as a tribute to Plato's style and his care in writing. If, as indicated in more than one story, the opening words were written on Plato's tablet in several different ways, the only conclusion that may be drawn is that the way they were 'published' must have been the right way, the best way. Before discussing their rightness, the subject of the second part of this essay, it should be observed that the eight words contain eighteen syllables. Now, a standard unit of composition in the Hellenic world was the Homeric line, the hexameter, which theoretically was made up of six feet, each of which could have a maximum of three syllables - producing a theoretical line of eighteen syllables. In practice - in Homer - the hexameter line is either of fifteen or sixteen syllables, and so Plato could be seen as improving on Homer by maximizing the standard line, while remaining within its limits. It suggests that the *Polity* rivals and replaces the *Iliad*, that Plato replaces Homer, that philosophy replaces poetry.

A further consequence is that since the *Polity* has 180,000 syllables, it may be regarded as composed of 10,000 lines, each of eighteen syllables. Given public reading as the normal mode of 'publication', at a reading rate of 250 syllables a minute, the *Polity* takes twelve hours to recite. Since Socrates, in his final speech, refers to midnight, it suggests that the *Polity* conversation would have begun at noon.

> 'Κατέβην χθὲς εἰς Πειραιᾶ μετὰ Γλαύκωνος τοῦ Ἀρίστωνος,'

The opening eight words may be regarded as a proper, an appropriate introduction to the *Polity* as a whole.

The first word of the dialogue, Κατέβην, tells a great deal. It is a verb and it signifies an action, a movement. Moreover, the verb is a compound verb and the prefixed preposition κατά or 'down' indicates a direction of movement. When the first-time hearer or reader learns that the root verb is 'to go' in a past tense, and that the ending indicates the first person singular, the sense becomes 'down-went-I.' It is a pity that this would be such a forced use of English as to render it unacceptable. But it

is nevertheless true that the choice of the aorist, involving both tense and aspect, indicates an action in the past which is to be regarded as complete and as a whole. It should also be noted that given the fact that the weight of a Greek sentence is usually at the beginning, the word that tells of a descent also marks the beginning of a decline within the sentence, and what follows.

Although the verb signifies an action, it does not identify who carries out the action, who the 'I' is. The hearer must wait until the 134th word to discover that the speaker is Socrates, and even then the hearer is not told directly who is speaking; he learns it only because a character in the dialogue, Polemarchus, addresses him by name. If we, as hearers, had been present when the dialogue was first held, then we could have seen that it was Socrates . . . except, of course, that there was no such first occasion. Leaving the speaker un-named allows us to silently recognize the writer - Plato himself - and to distance what is written from the historical Socrates. It must be added that for Plato to write the *Polity*, he must himself have gone down.

Identifying the speaker as Socrates is premature for us but convenient, however, and on recalling the first word of the dialogue as 'down-went-I' it becomes apparent that if the speaker, later named as Socrates, reports that he went down, then he must now be 'up'. Otherwise he would have said 'I came down . . . ' But where is 'down' and where is 'up'? Or is Socrates, like 'the brave old Duke of York', in some third place, neither 'up' nor 'down'?

But Polemarchus, while being the first to name the speaker, also indicates the direction of his motion, although Socrates had, in any case, told the hearer ἀπῆ`μεν πρὸς τὸ ἄστυ - that they were on their way to the city, to Athens, to their home. In a simple sense, 'down' is the Peiraeus, and 'up' is Athens - and Socrates is now speaking from 'up', that is, from Athens.

The second word of the dialogue intervenes. It is χθὲς, meaning 'yesterday'. It is impossible to answer the question when 'yesterday' was, unless today's date is established. At a certain level of discourse, 'yesterday' is the feast-day of Bendis, but that is not known by the hearer until the end of *Polity*, Book I (St. 354) when the goddess is identified. Consequently, as the dialogue reading is commenced, it is only known that the reading takes place 'today' but, in a curious way, the quotation marks are no longer needed. The dialogue - whenever it is read - is always read today and by the simple use of the word 'yesterday', both Plato and his Socrates speak as eternal contemporaries. The time of the dialogue is ever-present.

If the time is an eternal present, then the journey 'I went down' is an eternal journey, and if, in the third and fourth words of the dialogue, the information is offered that the journey was 'to the Peiraeus' there is the growing suspicion that this is no ordinary Peiraeus. The fact that the words are really 'to Peiraeus' - without the definite article - and not 'to the Peiraeus', leads gently towards the idea that for 'Peiraeus' the hearer is to understand 'the beyond', the land beyond the limits. Charles Heald Weller in *Athens and its Monuments*, p. 383, says:

> 'Ancient literature preserved the tradition of a geological era when Peiraeus was an island, and ascribed the very name to association with the word 'peran', 'beyond' the coast. But before the earliest historical period the Kephissos had brought down the silt which thereafter connected the former island with the mainland.'

The associated words are: πέραν, meaning on the other side, beyond, across; Πειραιεύς, the name of the most noted harbor of Athens; πειραϊκός, meaning over the border, the border-country; and, finally, πειράω, to endeavor or to try.

Everybody knew where the Peiraeus was - and what it was, a sea-port town, with all its wild and turbulent opinions and actions, the home of strange, foreign sailors and merchants (Bendis is a Thracian goddess of the moon), as well as of the Athenian fleet of triremes and their crews. It was the bastion of democracy in its more extravagant form but it was also where Cephalus lived, a fact to be explored later. In a sense the Peiraeus was 'beyond' the more settled, the more domesticated Athens, but Socrates invites the hearer to go beyond the beyond, to venture into a realm that exceeds - or transcends - the conventions of the city. It adumbrates the famous passage at the beginning of *Polity*, Book 7, the image of the

Cave, although obviously this could not be known on a first reading. But a comparison of the two topologies is instructive. (See illustrations)

A general rule may not be out of place. One cannot understand many - perhaps any - of the dialogues on a single reading. It is only when one has come to the end of the dialogue - in this case the *Polity* - that one comes to understand the beginning, and all that has gone before, in a new way, at a higher level. There is a curious rhetoric at work - a circular rhetoric, it might be called - that guides the hearer or reader back to the beginning of the work supposedly just finished. The dialogues are self-generating, it seems, and, moreover, self-regenerating. This will be illustrated in the final section of this essay.

No more need be said at this point about the first four words of the *Polity*, the first of the commata. The second comma, however, demands knowledge of Plato's family - a knowledge that would be freely available in Athens. Everybody would know that Glaucon was an older brother of Plato; his other, oldest brother, Adeimantus, is named even before Socrates. Plutarch remarks in his essay 'On Brotherly Love' that Plato memorializes his two brothers in, of course, the *Polity*, and there can be no doubt that he is showing his affection and pride by making them the main interlocutors of the dialogue.

> 'Just so did Plato make his brothers famous by introducing them into the fairest of his writings, Glaucon and Adeimantus into the *Polity*' (484)

It must be observed, however, that, of the two, it is Glaucon who is preferred, for he begins the dialogue, with Socrates, and is explicitly and exclusively addressed in the last sixth of it. It may be assumed that Adeimantus is still present - there is no indication that he has left - but the whole dialogue is really for the sake of Glaucon.

We know - what all Athens would have known - that Glaucon was extravagantly ambitious and, indeed, Xenophon tells us (*Memorabilia*, III, vi.2) that Socrates once asked him 'Have you made up your mind, Glaucon, to be our chief man in the city?' and Glaucon immodestly replied 'I have, Socrates.' Immediately prior to this passage, Xenophon says:

> 'Ariston's son, Glaucon, was attempting to become an orator and striving for headship in the city, though he was less than twenty years old; and none of his friends or relations could check him, though he would get himself dragged from the platform and make himself a laughing-stock. Only Socrates, who took an interest in him for the sake of Plato and [his uncle] Charmides, managed to check him.'

Because of his youth, it was illegal for Glaucon to try to address the assembly. But only Socrates could control him and there is the suggestion - borne out by the dialogue as a whole - that the *Polity* may be seen as the education of Glaucon, mirrored by Odysseus in the story of Er, for he gave up his search for honor, his ambition, and chose a quiet private life. We hear nothing more of Glaucon, historically, and so it appears that the Polity tells of Socrates' successful education of him. Plato tells of his conversion.

It is worth considering the meaning of the names. 'Socrates' is made up of the two words, σως (safe) and κρατης (power), and it would be hard to find a more fitting name for him. 'Ariston' is from ἄριστος, the best, the noblest, the finest. 'Adeimantus' is from ἀδείμαντος, fearless, dauntless. And 'Glaucon' is derived from γλαύξ, the owl, which identifies him as an Athenian, as the Athenian, as standing for Athens herself; the owl, of course, was the sacred bird of Athena, γλαυκῶπις, bright-eyed. And since he is the son of Ariston, the best, he is the best that Athens can offer.

All these names add an aura of excellence and importance to whatever is said and done. As a side note, it has already been observed that the name of Plato (appearing only twice in all the dialogues) was held to be a nickname, his real name being that of his grandfather, Aristocles, meaning 'famed as the best'. It is tempting to suppose that Plato does reveal himself in the *Gorgias*, coyly disguised as Kallikles (an otherwise unknown historical character), substituting καλλι, beautiful, for ἄριστος, which served as the superlative of ἀγαθος, good. Certainly, Kallikles has an intellect comparable to that of Plato.

The last four of the eight opening words

do two things. Firstly, by using actual historical characters, it is made clear that the dialogue is about real, concrete people; it is not about abstractions. Moreover, the people are not all Athenians, for Cephalus and his family are metics, that is, resident aliens; this suggests that the topic of conversation, justice, is of more than local interest, indeed, is universal.

Secondly, by using his own famous family, Plato exhibits the connection of politics and education. The *Polity* may be the first published suggestion that rulers need to be educated, that they need to be educated as rulers. That power is not enough. The same theme is taken up by Xenophon in his *Education of Cyrus*, written probably as a response and implicit criticism of Plato's *Polity*. The cure of a young man's excessive ambition - a cure which actually took place, apparently - is condensed into one comprehensive and stimulating dialogue.

This possibility may be strengthened by the meaning attached to the fifth word, the simple preposition, μετά. With the genitive (as here) it means 'together with', so that Socrates and Glaucon go together, without any suggestion that one of them has a kind of priority, such as an initiative. But μετά, in composition, also means 'after' and there is a faint suspicion that Socrates goes to the Peiraeus at the invitation, as it were, of Glaucon, that he goes because of and for the sake of Glaucon. That Glaucon is the origin of his descent, that Socrates is, in a certain sense, following his lead.

To give meaning to the details provided - and not to regard them as isolated bits of casual and accidental information - 'merely corroborative detail, intended to give artistic verisimilitude to an otherwise bald an unconvincing narrative' - it is only necessary to realize that Glaucon and Adeimantus, between them, and with the help of their friends, have contrived a plan to 'capture' Socrates, to manipulate him into a circumstance not of his choosing, without his knowledge and agreement.

The bare outline of the first part of the plan is simple. Glaucon - because of his special relation to Socrates - will bring him down from Athens (which he virtually never leaves) to the Peiraeus on the pretext of attending the new celebration of Bendis (an almost totally irrelevant foreign goddess). Pious Socrates will offer his prayers and also satisfy his curiosity about the conduct of the festivities. After the ceremonies and before Socrates and Glaucon can leave for Athens, Polemarchus sends his slave to prevent them from going. The slave - a usual sign of the rich - conveys his master's message, but more in accordance with his slavish nature, he takes physical action - grasping Socrates' cloak, as if he did not believe in the efficacy of words.

Socrates merely asks where Polemarchus is, and it is Glaucon who says that they would wait - speaking for Socrates as well as himself. Obviously, Polemarchus has been on the look-out for Socrates and Glaucon, knowing that, by prior arrangement, they would be there. He soon arrives with Adeimantus and Niceratus and a few nameless others. It is important that Polemarchus issues the 'order' to wait, for a reason to be explored in the second part of the plan; had Adeimantus done it, there would have been a more obvious connection and collusion with his brother, Glaucon.

The presence of Niceratus is explained by two factors. The first is that he is the son of Nicias, a superstitious general in the disastrous Sicilian expedition who did not know how to use power, and who made his son learn the whole of Homer, both *Iliad* and *Odyssey*, by heart 'so that he would become a good man'. In the whole dialogue he is, apparently, present but is speechless; he can contribute nothing to the discussion. And, by implication, neither can Homer.

Polemarchus jokingly tells Socrates - but not Glaucon - that he can only leave for Athens if he can show himself stronger - and he is outnumbered. Socrates responds - not to the possible threat of force - but by offering persuasion, of what ought to happen, instead. Polemarchus' reply is not to repeat the humorous threat of force, but to say that persuasion is not possible if they won't listen. (Persuasive power, apparently, is in the words and the only defense against them is not to hear them.) But it is Glaucon - not Socrates - who admits that persuasion is only possible if they listen. And Polemarchus asserts that they would not listen.

This creates an impasse that is only broken

by Adeimantus taking another tack - a tack of persuasion, although the persuasion is not of words but of visual enticements. He holds out the possibility of seeing a novelty horseback race with torches. It is clearly no novelty for Socrates who already knows how it will be conducted, but Polemarchus - taking his cue from Adeimantus - adds the further incentives of an all-night festival, dinner, and young men with whom to talk. And he changes his tone by asking and not demanding (κελεύω).

It is Glaucon who then agrees, saying that 'it looks as if we should stay'. And Socrates, agreeing with neither Polemarchus nor Adeimantus, does allow himself to follow Glaucon by using the legal formula of resolutions in the Athenian assembly. Thus he stays, not because of any individual, not even Glaucon, but because it is 'lawful', passed by 'the Assembly'. Socrates obeys the law but is controlled by no person; he maintains his personal responsibility.

This concludes the first part of the plot - the successful 'capture' of Socrates, who must have realized that something was going on, but who chose to allow it to happen, without compromising his freedom. It was only as the second part of the plot began to unfold that he could see, in detail, the purpose of the conspiracy. But this part, acting as a preface to the whole dialogue, brings into focus the contrast between two fundamental modes of political action, indeed of all human interactions, force and persuasion, and the *Polity* is an act of persuasion.

The second part of the plan begins with the whole party going to the house of Polemarchus. And there it begins to unfold. Socrates is still in a narrative mode, as it were, and he says:

> 'We went with them to the home of Polemarchus, and there we found Lysias and Euthydemus, the brothers of Polemarchus, and also Thrasymachus of Chalcedon, and Charmantides of the deme of Paeania, and Cleitophon the son of Aristonymus. Cephalus, the father of Polemarchus, was also in the house.'

The hearer or reader is immediately alerted to the group which is 'found'. It would appear that it was not serendipitous, an accidental discovery; the group was there, awaiting the arrival of Socrates, brought by Polemarchus and Adeimantus, for a pre-arranged purpose which is never stated, but which we - like Socrates - may divine.

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The composition of the group is instructive. Lysias and Euthydemus are naturally there as brothers of Polemarchus, as family members, but it is noteworthy that Cephalus has three sons, just as Ariston has three sons, and that invites us to make a comparison of the two families. But it is the presence of Thrasymachus that reveals the purpose, explaining the deceitful actions of the sons of Ariston. That purpose was to bring about a confrontation between Socrates and Thrasymachus.

Thrasymachus is not a family member, nor - being from Chalcedon - is he of their social circle. He is a well-known sophist, a teacher of rhetoric, whose views are made clear in the discussion of the first book of the *Polity*, where he exhibits himself as a rude and aggressive proponent of 'justice is the interest of the stronger', of 'might makes right', of an ethics of power. His name, like his character, of course, reflects this, being derived from θρασύς, meaning (in its bad sense) over-bold, and μάχη, meaning battle: he is, indeed, a rash fighter. Presumably, he knows why he has been invited and no doubt expects to be paid; at St.337, he will not accept Socrates' thanks for his anticipated teaching, but insists on getting money. And Glaucon volunteers that they will all 'contribute for Socrates'.

The suggestion that this is the clandestine plan of Glaucon and Adeimantus is further strengthened by the fact that, as Socrates reports, there were a number of seats arranged in a circle. It may be supposed that such an arrangement was already made, in anticipation of a 'group discussion'.

Although this is only speculation, it would appear that Glaucon's life has reached a crisis

point. He is intelligent and intellectually honest - he wants to know how a man should live, how he himself should live (see *St.* 344) and sees only two alternatives, represented by Socrates and Thrasymachus. The two extreme alternatives are not unconnected to his political ambitions, and although Socrates describes himself as 'the only true politician' (as in the *Gorgias*, *St.* 521) the fact is that he never sought office, he never held power - except over himself. This flouts the conventional opinion, and, attractive as it seems in some ways, it appears unrealistic. Thrasymachus, on the other hand, takes the conventional view to its extreme and advocates the seizure of power, which would enable its holder to do as he liked; this was a notion of politics that any young person would find attractive. But Glaucon is not convinced that it is right - a tribute to his fine nature, for he still clings to the vague notion that there is 'a right way', that there is a 'good', and that maybe power is not self-justifying.

The dilemma of Glaucon is that of every young person, and it is a very profound and very disturbing dilemma, but it requires an intellectual ability and an intellectual honesty to face it. Most people, especially most young people, glimpse it and then avoid it, but the finest natures, the sons of 'the best', like Glaucon (and Plato), wrestle with it. Glaucon seeks the help of his older brother and between them they devise a scheme to help resolve it.

It might be asked why they do not approach Socrates directly, why they do not explain to him what they intend, their plan. The answer is that it is contrary to Socrates' way. He does not seek out 'wise men' for himself, but only on behalf of a young friend, like Hippocrates in the *Protagoras*. Neither would he be willing to challenge Thrasymachus, or anyone else, to what would amount to a duel; nor would Thrasymchus agree to such a combat without being paid, and Socrates has no money (and wouldn't use it if he had). Socrates is not manipulative - but he will discuss with anyone he happens, by chance, to meet.

What Glaucon and Adeimantus do is wrong - they set out to deceive Socrates, to trick him into doing something that he would not freely choose - but they see no alternative. And it becomes clear in Book II of the *Polity* that their fine and noble natures allow Socrates to accept them and to do what they want and need. Conventionally, it might be said that Socrates forgives them; but Socrates does not 'forgive', for that implies a judgment and an inequality of relationship which is foreign to him.

The comparison of the two opposing views of justice is not to be an abstract, academic exercise, witnessed and assessed by Glaucon and the others. They are to be presented dramatically, by their living proponents, in the lives of living men, for the characters of the presenters gives meaning to what is said, just as what is said gives meaning to the speakers. One powerful rhetorical element in the *Polity* is the nature of the speakers: would we prefer to associate with Thrasymachus or with Socrates?

The character - and presence - of Cephalus is crucial to the discussion. He welcomes Socrates and expresses his delight in good talk, but that delight comes in large part from the weakening of his body and is not self-justifying as it is for Socrates. He has a moral view of life, of his own life, but faithfully relies on sacrifices to the gods to make amends for any wrongdoing. But it is hard to forget that he made his wealth as an armaments manufacturer, as a shield-maker - albeit it is a defensive weapon. Socrates accords him much respect and invites him to share his wisdom on old age - something he is experiencing - but skillfully brings the discussion round to the topic that Glaucon and the others have been waiting for, justice, for 'how a man should live'. Cephalus uses the word first and is immediately invited by Socrates to affirm its meaning, its definition. Polemarchus, as an impatient but respectful son, has waited his opportunity and now leaps in, quoting a poem by Simonides as a definition of justice. This enables the discussion that Glaucon and Adeimantus had always wanted to begin, and it also gives Cephalus the opportunity to leave in order to attend to the sacrifices which, to him, are more important than conversation.

The 'very old' Cephalus (his name from

κεφαλή, means 'head' - of a body, or of a family or group), really is not much more than a 'head', for his bodily appetites (like those of Sophocles) have vanished, not mastered but simply dead. He has to be present at the beginning so that it can be absolutely clear that he must leave before the conversation can take place. He departs and is never heard of again. This indicates that a discussion of such terms as 'justice' cannot take place in the presence of what has been called 'the ancestral'; the ways of our forefathers were not determined by rational discourse and are not subject to examination or criticism by the intellect. Their authority is prescriptive; they are to be obeyed not discussed.

The whole group consists of the following eleven named people. At the house, awaiting the advent of Polemarchus and his group, there are six - Cephalus, Lysias, Euthydemus, Thrasymachus, Charmantides, and Cleitophon. They are joined by five - Polemarchus, Adeimantus, Niceratus, and, of course, by Glaucon and Socrates. They total eleven, a number not without its significance for Socrates. And if Socrates is excluded, the total will be ten.

When Socrates was condemned in 399 B.C.E., he was put under the authority of the group, like our police commissioners, known as The Eleven, who were responsible both for his imprisonment and for administering the hemlock, as described in the *Phaedo*. Plato, in his Letter VII, provides this account:

> 'In the days of my youth, my experience was the same as that of many others. I thought that as soon as I should become my own master I would immediately enter into public life. But it so happened, I found, that the following changes occurred in the affairs of the city.'

This - and what follows - is a dim reflection of Glaucon's experience, without his driving ambition and his supreme self-assurance. When Sparta had defeated Athens, it set up in 404/403 a brutal and repressive oligarchic government known as The Thirty Tyrants or just The Thirty. Discredited by their military incompetence and their cruelty, The Thirty were soon overthrown and a reconciliation was effected by the Spartans. Amongst the conditions of reconciliation it was said that 'each man could return to his own possessions, except for the Thirty and the Eleven and the Ten who ruled in Peiraeus'. Plato continues:

> 'In the government then existing, reviled as it was by many, a revolution took place; and the revolution was headed by fifty-one leaders, of whom Eleven were in the city, and Ten in the Peiraeus - each of these sections dealing with the market and with all municipal matters requiring management - and Thirty were established as irresponsible rulers of all. Now of these some were actually relatives and acquaintances of mine; and indeed they invited me at once to join their administration, thinking it would be congenial. The feelings that I then experienced, owing to my youth, were in no way surprising: for I imagined that they would administer the city by leading it out of an unjust way of life into a just way, and consequently I gave my mind to them very diligently, to see what they would do. And indeed, I saw how these men within a short time caused men to look back on the former government as a golden age; and above all how they treated my aged friend Socrates, whom I would hardly scruple to call the most just of men then living, when they tried to send him, along with others, after one of the citizens [Leon of Salamis], to fetch him by force that he might be put to death—their object being that Socrates, whether he wished or no, might be made to share in their political actions; he, however, refused to obey and risked the uttermost penalties rather than be a partaker in their unholy deeds.'
>
> 'So when I beheld all these actions and others of a similar grave kind, I was indignant, and I withdrew myself from the evil practices then going on. But in no long time the power of the Thirty was overthrown [by the democracy] together with the whole of the government which then existed.
>
> Then once again I was really, though less urgently, impelled with a desire to take part in public and political affairs. Many deplorable events, however, were still happening in those times, troublous as they were, and it was not surprising that in some instances, during these revolutions, men were avenging themselves on their foes too fiercely; yet, notwithstanding, the exiles who then returned exercised no little moderation. But, as ill-luck would have it, certain men of authority summoned our comrade Socrates before the law-courts, laying a charge against him which was most unholy, and which Socrates of all men least deserved; for it was on the charge of impiety that those men summoned him and the rest condemned and slew him - the very man who on the former occasion, when they themselves had

> the misfortune to be in exile, had refused to take part in the unholy arrest of one of the friends of the men then exiled.'

There is a certain irony in hearing a free if fictitious discussion of justice among ten young men in a place which had been, supposedly, governed by another Ten, chosen by an oligarchy. Moreover, at least two of the young men had been illegally put to death by that oligarchy, The Thirty - Niceratus and Polemarchus, one Athenian and the other a metic - possibly out of 'fear and greed' (as alleged by Aristotle) and more probably for simply supporting the democrats, materially (by supplying shields?) and financially; and it bears out Polemarchus' initial view of justice as 'helping friends and harming enemies'. Even more ironically, the ostensible leader of the discussion, Socrates, was put to death by the democratic government which had overthrown the tyrannical Thirty.

It raises the interesting question about the relationship between a discussion of justice and the facts of political life. And the bearing, if any, that the fate of a thinker has upon the validity of his opinions. And vice versa. It is certainly true that if Socrates had not acted in accordance with his own principles - by not following Crito's urging to escape - it is unlikely that he would still be studied as he is. As it is, attention is drawn to the dangers of discussing 'justice' and any of its related terms in a political context.

The only other member of the group about whom we have much information is Lysias. He survived all the political upheavals and later took one of the Thirty, Eratosthenes, to court for the death of Polemarchus, his brother, and the confiscation of his property. He became a well-known writer of speeches: it is said that he offered to write a defense for Socrates at his trial, but his offer was not accepted. Although a worker with words, he is only present at the *Polity* conversation, remaining totally silent. It would appear that professional writers, as such, have nothing to contribute to a discussion of justice.

Plato introduces a speech supposedly by Lysias at the beginning of his dialogue *Phaedrus*. It is more likely a composition of Plato's in the style of Lysias, and also with content that displays his character. It argues that the non-lover should be accepted, rather than the lover, and this reveals Lysias as a predator - at least in Plato's opinion - and Socrates has to respond in two speeches to save Phaedrus and to declare the divinity of love.

Reading or hearing the *Polity* in the light of these remarks affects our understanding of Plato's meaning. It is often held that the dialogues, generally, are 'academic' in the modern, often pejorative, sense of the word, as unrelated to 'reality'. Indeed, the *Polity* itself is often thought of or is related to 'Utopia', a non-Greek word invented by Thomas More, conveying both that it does not exist (οὐ, meaning no or the negative, and τόπος, meaning place) and that it is the good place (εὐ, meaning well, with τόπος). This calls to mind the ending of Book Nine of the *Polity* where Socrates and Glaucon agree about the good man:

> 'Rather, says Socrates, he looks fixedly at the city [the polity, πολιτείαν] within himself . . . he will willingly partake of and taste those things that he believes will make him better, while those that would overturn his established habit he will flee, in private and in public.
>
> Then, says Glaucon, if it's that he cares about, he won't be willing to participate in political affairs.
>
> Yes, by the dog, replied Socrates, he will in his own city, very much so. However, perhaps he won't in his native land unless by some divine chance.
>
> I understand, he said. You mean he will in the city whose foundation we have now gone through, the one that has its place in words (ἐν λόγοις), since I don't think it exists anywhere on earth.
>
> Perhaps it is laid up in heaven as a pattern for him who wishes to see and seeing, to found such a city within himself. It makes no difference whether it exists somewhere, or ever will; he would act for this city alone, and for no other.'

Cleitophon, in a brief exchange with Polemarchus (*St.* 340), shows himself to be a partisan of Thrasymachus, and there is a dialogue named after him. Essentially, it praises Socrates for urging men to virtue, and then castigates him for not telling them what that means. He never realizes that whatever virtue is, it comes only by self-examination and not by being told, by authoritarian pronouncements.

Glaucon and Adeimantus are not satisfied with Socrates' discussion with Thrasymachus, and

think that the latter gave up too soon; modern scholarship has raised the question as to whether Thrasymachus is refuted. Now, the words 'refute' and 'refutation', even in their popular usage, belong to some form of argumentation, some form of logic, of reasoning. Looked at from this point of view, Thrasymachus is certainly refuted, but he implicitly denies that reason is supreme. From conventional considerations, and as a professional teacher of rhetoric, he cannot deny the use of speech and reasoning, but they are mere tools, instruments of whoever has power. Even if this was clear to him, it would be detrimental to his career as a rhetorician to publicly admit it; who would study speaking, words, with someone who denied their ultimate power?

This hearkens back to the opening of the *Polity*, where Polemarchus and Adeimantus playfully threaten to keep Socrates in the Peiraeus by force, and Socrates suggests the alternative of persuasion. Thrasymachus claims that power determines justice and 'virtue', while Socrates holds that 'words' and 'reason' are what determine the good. In the latter world, Thrasymachus is refuted; but in the former world he is not shown to be wrong, but he is silenced - the rhetorical counterpart of logical refutation. It is curious that Socrates, who denies the supremacy of power, has the power to silence Thrasymachus. Persuasion, it appears, is a kind of power.

The opening of the *Polity* offers the two alternatives of power and reason. It exhibits them but does not explicitly recommend either one. The existence of speech - its mere existence, which cannot ultimately be denied - can and does provide the means for holding actions - exertions of power - accountable. Speech may be silenced or may be controlled in substance for a time, but it has a life of its own, a freedom that is inherent. The λόγος is immortal.

It was remarked earlier that the *Polity* is, in a sense, circular, and if Socrates goes down to Peiraeus in the beginning, at the end he goes down, in the person of Er, to the next world - another 'beyond'. It is similar, perhaps, to the world of Cephalus, except that in his world there is no known judgment, and sacrifices are made to placate the gods and to pay for any wrongs, the specificity and nature of which is unknown. It is irrational. Those with Er know what their wrongs (and 'rights') are. Theirs is a world based on knowledge and reason, a world in which every part is an image of the whole, since all nature is akin (*Meno*, *St.* 81).

This is illustrated in Er's description of the whole cosmos which the souls see on their journey back to an earthly re-birth. In what seems an unnecessarily complex way of describing the cosmos, our planetary system, using order, width, color, and light, Plato carefully avoids actually naming the heavenly bodies. Of course, he knows the names they were given (see *Epinomis*, *St.* 986-988) but does not use them because the bodies stand for something else, in addition to their customary meanings. In fact, they stand for the members of the *Polity* discussion who speak with Socrates, and in the order in which they speak. That order is Cephalus, Polemarchus, Thrasymachus, Glaucon, and Adeimantus, representing, in their proper astronomical order, what we call Saturn, Jupiter, Mars, Mercury, and Venus. Three elements remain unaccounted for - the sphere of fixed stars, the sun, and our moon; but identification is simple, for the former stands for the many-colored democracy, against which the discussion takes place, the sun is Socrates himself, giving light to all, and especially to Plato who is the moon reflecting the Socratic sun.

The conversation in the house of Polemarchus is a reflection of the cosmos itself, a microcosm, and presumably justice, the subject of the conversation, human soul - its proper or natural polity, before the is another image. The *polity* or constitution of the disfigurements of a Glaukus (St. 611) - is yet another image. Justice is the right-ordering of the soul, that is, its ordering in harmony with the cosmos.

This returns the discussion to the opening of the *Polity*, but in a way not hitherto mentioned. From the opening few words, we learn that we are to hear the report of a conversation that took place 'yesterday' in (the) Peiraeus. At least, that is what is intimated. Immediately, any educated Athenian would have recognized that the dialogue is to be regarded musically or harmonically. The

reported work is a duplicate, a double of the original conversation, and so we are implicitly presented with a ratio of 2:1 - the original and its repeat, being two, and the original, being one. Or, alternatively, the original and its repeat, being two, and the repeat - what we have directly - being one. Of course, 2:1 defines the octave.

This suggests that the work we have, the dialogue, has an arithmetical and harmonic structure, a structure that reflects the music of the cosmos and of the soul. Truly, then, was it said that music is the numbers of the soul.

The beginnings of the arithmetical structure may be readily seen. If the dialogue is divided into 240 equal segments, by syllable count, then at unit 60, exactly one quarter through it, Glaucon is described as 'musical' (and therefore well suited to the discussion of music that follows); exactly symmetrical with it, at unit 180, he is again described by Socrates, this time, as a 'lover of music'. This is three quarters of the way through the dialogue. Half way through it, at unit 120, Socrates reminds Glaucon (and the others) that they were seeking a paradigm - not the founding of an actual city; this is immediately followed by the famous declaration that philosophers must be kings, or kings philosophers. In this mid-point unit which also defines the fifth (360:240 or 3:2), Socrates will be found referring to the numbers two and three - a more pointed reference to the musical fifth it would be hard to imagine. No more need be said.

The so-called Divided Line and the Idea of the Good are found in unit 150 - at the division of the dialogue in extreme and mean ratio, now called The Golden Section, a self-generating ratio; its symmetrical unit is unit 90, in which the soul's tripartite division is first described. Strictly, because of the irrational nature of the ratio, this division is not arithmetical but geometrical. But clearly, the Divided Line analysis is only intelligible in the light of the nature of the soul.

The harmonical structure is more complex and requires more space for a discussion than is appropriate here, but it begins with the fundamental proportion that Pythagoras, according to tradition, brought home to Greece from Babylon. This proportion was 6:8::9:12 and it can readily be seen that, reading from left to right, 6:8 is 3:4, or the ratio for the musical fourth; 6:9 is 2:3 or the ratio for the musical fifth; and 6:12 is 1:2, or the ratio for the musical octave (a term not used by the ancients, who called it diapason, διὰ πασον, through all.) The ratio 8:9 is irreducible, but is the ratio for what was taken as a descending whole tone. These numbers are all taken as measures of string length, but relatively not absolutely; since they increase moving from left to right, they indicate longer string lengths, that is, descending tones.

Reading from right to left, as 12:9::8:6, the ratio 12:9 is the fourth, 12:8 is the fifth, and 12:6 is the octave. Since the numbers diminish, the proportion when read this way represents ascending tones. The Hellenes could, obviously, measure string lengths, but had no means for counting vibrations, which are their reciprocals.

These four tones are the only invariant or fixed tones in Pythagorean tuning theory. Taking the *Polity* as a single length, they may be applied to it and structural points determined. Units 1 and 240 define the limits of the octave and, given the nature of what we call the octave, they are in some sense the same. And yet different. The octave is taken, by the Hellenes and ourselves, as in some sense natural since the two tones defining it are both the same and different. The fifth was also taken as 'natural' and so unit 160 will mark its (descending) place (since 2:3::160:240); similarly, the fourth (not so universally taken as 'natural'), will be in unit 180 (since 3:4::180:240).

Now, unit 160 makes the basic distinction between the senses and the intellect, between the visible and the intelligible; numbers are clearly intelligible and lead towards truth. This provides the basis for the ensuing curriculum, for the education of the philosopher-king. Unit 180 marks the beginning of the downfall of the 'best man' and the corruption of the best city, Kallipolis; that corruption takes place because the guardians are unable to master the 'geometric number', the so-called 'marriage number'. That number is from harmonic theory, based as it is on '4:3 married to the five' - that is, on the musical fourth 'fertilized' by the male prime number 5.

These cursory remarks have taken the string lengths as rectilinear, and treated the *Polity* as if it were a monochord. But the scale may be considered as a circle (as it is in the *Timaeus*); its strings may be bent round until units 1 and 240 coincide and the *Polity* becomes a tone circle, the numbers of which may be read both clockwise and anti-clockwise, rising in the former and falling in the latter. The same may be done with the double octave. The application of the circle of fifths would not be inappropriate. It is not that the numbers may be read as either linear or circular, they must be read as both. Dialectic always requires us to view things in as many ways as possible; there is no one, exclusively correct, way. The current fashion is to treat the *Polity* - indeed, all the dialogues, and philosophy itself - as if it were a subject-matter, amenable to technical treatment. Undoubtedly, a technical analysis increases knowledge, but only by limiting and circumscribing what is studied and ignoring what is not; more is known about a part of our cosmos, but the whole, of which it is a part, is disregarded as irrelevant or incomprehensible. The *Polity* is not a technical treatise, but it certainly examines questions about justice, about how a man should live; is it an art, a τέχνη, like doctoring, or arithmetic? Or is it different from these and from every other form of knowledge?

The existence of the *Polity* depends on a number of different factors, operating at a number of different levels; but at one level it exists because Glaucon, like every human being, faces the existential problem of his own way of life. What should it be? He is unable, unwilling, and unprepared to face this problem in the abstract - which is impossible, anyway, because a human life is not an abstraction; and so he looks for a specific living solution, using Socrates as a model or mentor, as a paradigm or preceptor. His search for his own proper way of life begins - or rather, culminates in the *Polity* - in an improper act, the deception and 'arrest' of Socrates. His quest for justice - which is genuine - is founded on injustice. Socrates is unaffected, is unmoved - he is neither deceived and gulled, nor angry and corrupted, but remains true to the *polity* of his soul.

The first eight words of the *Polity* are, certainly, an opening, but while they open the putative discussion of justice, they do not open its manifestation. A serious discussion of justice cannot take place among enemies - nor yet among those who, like the young men, are merely separated from each other. Socrates knows that before justice can be understood it must be lived - or, rather, the understanding will only come as it is lived; the being and the knowing are one and the same. His first task, then, has to be to help in the unification of the group, to make its members friends. This is achieved by unit 100, when Polemarchus and Adeimantus again 'arrest' him (St. 449) but this time with the support not only of Glaucon, but even of Thrasymachus, and, of course, with his own connivance. The young men think that they have captured Socrates - twice - but, in fact, he has captured them. Or rather, he has liberated them from the fantasy of power, and they are now open to a new beginning on the basis of the old proverb, κοινὰ τὰ φίλων, the things of friends are common, friends share.

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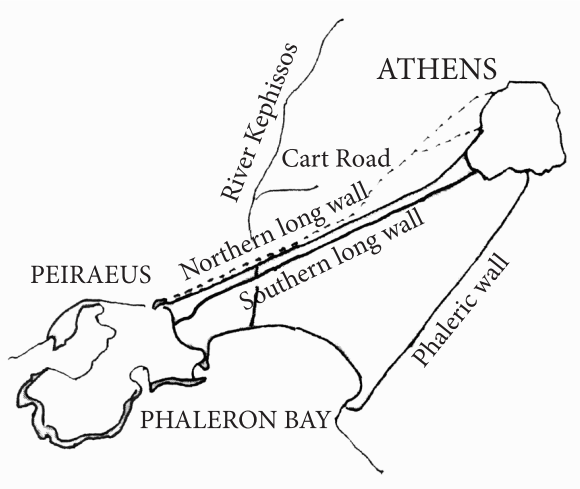
ILLUSTRATIONS



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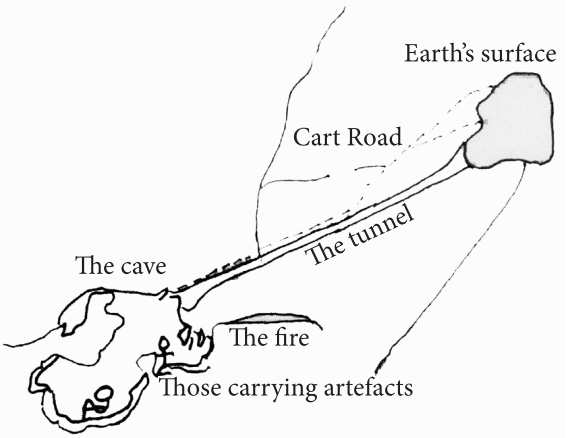

The prisoners, like ourselves, in the foetal position

The Cave and beyond

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ONTOLOGY INSIDE-OUT: *Speculation and Faith in a Musical Cosmos*

Bryan CARR

I. Once, there was a border between the earth and the rest of the cosmos. It was defined by the orbit of the moon.

There is no universally agreed upon chronological line of demarcation between the Ancients and the Moderns, but the revision of cosmography began with the work of Copernicus, and culminating with Galileo, a work which effectively abolished this lunar border, looms large in any list of 'usual suspects'. That the work of Galileo was seen by contemporaries as bringing to fruition a certain meaning of Copernicanism has recently been argued by Quentin Meillassoux, a philosopher whose work will occupy us in some detail below. This characterization is worth quoting at length:

> 'Greek astronomy had already described stellar trajectories in geometrical terms. But [in] these descriptions…what was subjected to mathematics was the unalterable form of a trajectory or a determinate surface area - in other words, a motionless expanse. Galileo, by way of contrast, conceives of movement itself in mathematical terms, and particularly the movement which appears to be most changeable of all: the falling of terrestrial bodies. In so doing he uncovered beyond the variations of position and speed, the mathematical invariant of movement - that is to say, acceleration. From that point on, the world became exhaustively mathematizable - the mathematizable no longer designates an aspect of the world…it now indicates a world capable of autonomy…a world that we can henceforth conceive of as indifferent to everything in it that corresponds to the concrete, organic connection that we forge with it - it is this glacial world that is revealed to the moderns, a world where there is no longer any up or down, centre or periphery, nor anything else that might make of it a world designed for humans…. This capacity whereby mathematized science is able to deploy a world that is separable from man…rendered possible the essential alliance between the Galilean and Copernican revolutions. In speaking of 'the Copernican revolution' what we have in mind is not so much the astronomical discovery of the decentering of the terrestrial observer within the solar system, but rather the much more fundamental decentering which presided over the mathematization of nature, viz., the decentering of thought relative to the world within the process of knowledge. The Galilean-Copernican revolution effectively consisted in the fact that both these events… were seized upon by their contemporaries as intimately connected…[it] has no other meaning than that of the paradoxical unveiling of thought's capacity to think what there is whether thought exists or not…the thought of thought's contingency for the world, and the recognition that thought has become able to think a world that can dispense with thought, a world that is essentially unaffected by whether or not anyone thinks it [1].'

This paper intends to contrast the vision of this 'glacial' in-itself with a different vision - one in which the mathematics of Galileo is situated in a broader and no less mysterious context of *musica naturalis*. In this project I am motivated by a lesson from Ernest McClain, who urged the

1 Quentin Meillassoux, *After Finitude*, pp.115-17. To this declaration of the 'glacial world' with 'no longer any up or down,' I would like to compare, without overt comment, two other citations:

Where has God gone? "he cried. "I shall tell you. We have killed him - you and I. We are his murderers. But how have we done this? How were we able to drink up the sea? Who gave us the sponge to wipe away the entire horizon? What did we do when we unchained the earth from its sun? Whither is it moving now? Whither are we moving now? Away from all suns? Are we not perpetually falling? Backward, sideward, forward, in all directions? Is there any up or down left? Are we not straying as through an infinite nothing? Do we not feel the breath of empty space? Has it not become colder? Is it not more and more night coming on all the time? Must not lanterns be lit in the morning?' (Nietzsche, The Gay Science, 'The Madman').

'We have got onto slippery ice where there is no friction and so in a certain sense the conditions are ideal, but also, just because of that, we are unable to walk. We want to walk so we need friction. Back to the rough ground!' (Wittgenstein, *Philosophical Investigations*, part I, p. 107.).

study of ancient musicology not as an academic excursion, or an antiquarian reconstruction, but in order to fathom the determining categories of the foundations of Western thought. I will attempt to spell out two different visions, a modern and an ancient one, and will point tentatively to the themes that connect them and above all to what is at stake between them.

The Copernican-Galilean Revolution development provoked a reaction, says Meillassoux. For in the wake of Newtonianism, philosophy, struggling to ground the knowability of those 'laws of nature' which always took the form of mathematical expressions of cause and effect, found itself more and more stymied; how did the mind ever get the evidence upon which it based its ratiocination? Descartes had inaugurated the modern philosophical project by ceding great swathes of territory to skepticism, in order to find the Archimedean pivot of the *cogito*. However, in practice, it had proved very difficult to gain this territory back. Berkeley pressed in one direction - towards idealism: what exists, said Berkeley, is only what is experienced, and is by *virtue* of being experienced; thus in a certain sense the *cogito* is really all one needs; or rather, one does not even need that, since perception of anything will do. Hume pressed in a different direction, towards a sort of skeptical pragmatism: a stipulation that our grounds for knowledge were always in habit, never in anything more solid and certain. Kant attempted to fend off both these (to him) unsatisfactory conclusions with a remarkable ploy: he granted that what we can know is only the (so to speak) grammatical forms of our knowledge - our minds experience things as existing in time and space, and time and space are parsed in terms of certain regularities which are consistently describable; but we cannot know what the things are 'in themselves,' only what they are like 'for us.' This, we *can* know.

In a famous passage, Kant summarized his gambit in relation to previous philosophy:

> '[I]t has been assumed that all our knowledge must conform to objects. But all attempts to extend our knowledge of objects by establishing something in regard to them a priori, by means of concepts, have, on this assumption, ended in failure. We must therefore make trial whether we may not have more success in the tasks of metaphysics, if we suppose that objects must conform to our knowledge. This would agree better with what is desired, namely, that it should be possible to have knowledge of objects a priori, determining something in regard to them prior to their being given. We should then be proceeding precisely on the lines of Copernicus' primary hypothesis. Failing of satisfactory progress in explaining the movements of the heavenly bodies on the supposition that they all revolved round the spectator, he tried whether he might not have better success if he made the spectator to revolve and the stars to remain at rest. A similar experiment can be tried in metaphysics, as regards the intuition of objects [2].'

But, says Meillassoux, the result of this experiment turned out to be disastrous. Even as the natural sciences were proclaiming result after result concerning the world apart from us, Kant recentered the project of knowledge on us. And this meant that, for post-Kantian philosophy, no scientific result can be read or understood *qua* knowledge, without translating it back into a philosophical register which undoes its literal meaning. For the scientist does avowedly intend to speak of things in the world as they are, full stop; but the philosopher has disavowed the right to speak of things in the world as they are regardless of ourselves and our knowledge; he claims only - whether or not this is said aloud - to speak of things as they are 'for us.' Far from being of the spirit of Copernicus, Kant's move amounts, says Meillassoux, to a Ptolemaic counter-revolution.

Once started, the mischief does not stop there, Meillassoux says. Philosophy, on this ground, is helpless not only to understand any scientific statements as the scientist means them, but also to defend against any manner of superstitious counter-statements. This incapacity has continued to unfold until the present day in the so-called 'return of the religious,' according to Meillassoux; rationalism proved itself incapable of critiquing any given irrationality, because it was undefended from the accusation that rationalism itself was simply one more 'story.'

This undefended position, Meillassoux claims, followed directly (if unexpectedly) from

2 Kant, *Critique of Pure Reason*, 2nd Ed., preface.

rationalism's own commitment to the quest for a first principle, an *archê*. This is because, failing to find any such necessary principle, any 'Sufficient Reason,' it nonetheless remained faithful to the *idea of* such a principle. Rationalism thus maintained the legitimacy of a form for which it could provide no content, and into which any given content could then be poured. It is thus, Meillassoux thinks, that modern rationalism lost its battle against fideism, a defeat he sees manifest clearly in both Heidegger's 'destruction' of metaphysics and in Wittgenstein's rejection of it.

This defeat is not a trivial or academic liability. In its incapacity to say No to any irrationality that comes down the endless procession of opinions, philosophy has rendered itself utterly ineffective in the struggle against fundamentalisms, but also the struggle for a just society - incapable of defending its own intuitions and articulations of values. Retreating into a desperate and reluctant relativism, philosophy has made itself a hapless accomplice to the levelling of culture by the market; and is at the mercy of every 'faith' whatsoever, because it lacks the wherewithal to critique faith as such, since it has itself dead-ended in a *de facto* fideism:

> Religious belief is considered beyond the reach of rational refutation…because it seems to these philosophers to be conceptually illegitimate to undertake such a refutation…. By destroying metaphysics…one has inadvertently justified belief's claim to be the only means of access to the absolute…. Far from seeing in fideism - as is all too often the case - a mere guise worn by antimetaphysical skepticism at its origins, before the latter went on to reveal its irreligious essence, we see skepticism as an authentic fideism, which is dominant today, but in a form that has become 'essential,' which is to say, one that has shrugged off every particular obedience to a determinate belief system. Historical fideism…is religiosity as such…the general argument for the superiority of piety over thought [3].

II. Meillassoux's counter-proposal to this 'religiosity as such' is to provide an argument for the legitimacy of mathematicised reasoning (without which he cannot argue that thought can think the Absolute). This needs to be carried out, however, without however legitimizing any sort of utter necessitarianism, because Meillassoux denies the principle of Sufficient Reason, since he thinks it was loyalty to this principle which wound up licensing fideism. To demonstrate that this condition is possible is the project of Meillassoux's book *After Finitude*. Here he argues, quite ingeniously if not always irresistibly, that the ontological Absolute is simply the radical capacity of everything to be otherwise: a Heraclitean flux he dubs *Hyperchaos*.

Meillassoux does not simply *posit* hyperchaos; he claims to deduce it, from an immanent critique of the very structure of a modern thought initiated (he says) by Kantianism. This structure, which he calls 'correlationism,' he considers *the* form of twentieth-century philosophy, whether Analytic or Continental. I will (briefly) describe this trope below, with attention to *After Finitude*. I will then (again, briefly) expound Meillassoux's argument for the legitimacy of mathematizing, developed most extensively so far (though still incompletely), in his paper 'Iteration, Reiteration, Repetition.' Meillassoux's thinking is far more detailed, subtle, and elegantly expressed, than can be done justice to in the compass of a paper of this sort. I make no pretense of having done him justice, but am concentrating on a few themes, starting with the link Meillassoux asserts between mathematization and the critique of fideism.

This link is not, so far as it goes, fundamentally different from the connection made from time immemorial. Aristotle remarks of the Pythagoreans that they turned to mathematics 'because it, alone, admits of proofs.' In a Platonic register, proof is always opposed to *doxa*, opinion. Nonetheless, Aristotle and Plato both conceded that there were limits to what knowledge could attain, and that it was (in Aristotle's words) 'the mark of an educated man not to seek more precision than the nature of the case admits[4].' According to Meillassoux, however, Kant effectively claimed that the nature of *every* case involves an inherent limit constituted by the finitude of human capacity, so that it is not a question of the thing known, but of the knower. This re-establishment of the knower in the grammar of every claim of knowledge is what Meillassoux

3 *After Finitude*, pp. 44-6.

4 Nichomachean *Ethics* I, 1094 b24.

calls Kant's Ptolemaic counter-revolution.

This anti-Kantian position does not make of Meillassoux an anti-modern, however. His orientation is, both effectively and avowedly, Cartesian. On the first page of *After Finitude*, Meillassoux already sketches the outline of his project with a call for the reinstatement of the distinction between primary and secondary qualities, terms of a theory elaborated by Locke but whose basis 'can already be found in Descartes.'

> 'In short, nothing sensible - whether it be an affective or a perceptual quality - can exist in the way it is given to me, in the thing by itself, when it is not related to me or any other living creature…. Remove the observer and the world becomes devoid of these sonorous, visual, olfactory, etc., qualities [5].'

But *not everything* vanishes. Meillassoux will defend the proposition, which he notes is in a certain important sense pre-Critical, that those qualities which remain after this sensible or perceptible dimension is subtracted, are the purely mathematizable. Mathematics, says Meillassoux, thus ushers the thinker into what he calls a 'dead world,' a world that is a-subjective: without experience, and which needs no experiencer for it to 'be.'

Most of *After Finitude* is concerned with the (ostensible) incapacity of modern philosophy to countenance this world. This is not because modern philosophy is always committed to subjectivism. Rather, it is committed to a kind of agnosticism, which Meillassoux dubs *correlationism*. For the correlationist, it is not that there is no In-Itself, but that this In-Itself *may* or *may not* exist, and is fundamentally unknowable. (Note that this is already a step beyond Kant, for whom the In-Itself certainly existed, but was not positively characterizable. Meillassoux reserves the name 'weak correlationism' for the Kantian hypothesis and 'strong correlationism' for its full-blown agnostic version.) For correlationism, 'we cannot represent the 'in-itself' without it becoming 'for us'…which means we cannot know anything that would be beyond our relation to the world[6].' This is because it is always and inevitably we who are doing the representing. Moreover, we do not know what 'we' are 'in ourselves,' what thought or subjectivity is in itself, apart from this relationship either. Hence, the question then becomes, what is this relation, or rather correlation; what are its primary terms, and what is its primary form? Is it Thought? Sensation? Language? Social mores?

Even the object's mathematizable properties do not escape correlationism's suspension, for 'they too must be conceived as dependent upon the subject's relation to the given' - at least as Meillassoux imagines his stock correlationist. This will ultimately be where Meillassoux tries to differentiate himself from correlationism, but he tries to show what is at stake, leading the way into his case for why correlationism needs to be overturned, by showing first, that correlationism occasions a disconnect between philosophy and science, and secondly, that by the same token, correlationism finds itself secretly aligned, willy-nilly, with irrationalism.

Meillassoux builds his case beginning with the inability of the correlationist to admit the 'literal truth' of any scientific assertion concerning events that antedate the existence of consciousness or life. Faced with the assertion that the earth came into existence approximately 4.5 billion years ago, or with any evidence of an event predating thought, consciousness, or life - evidence of the sort Meillassoux calls an *arche-fossil* - a correlationist cannot give an unqualified Yes or No to the question, Did it happen? Meillassoux thinks that the correlationist must answer with a '*codicil* ' added *soto voce* : Yes, that is how and when the accretion of the earth occurred - for us. This 'for us' ('us' = the 'interpretive community,' 'we who accept the measurements,' etc.) has become obligatory if usually not explicit, because correlationism is impotent to think things themselves, things *qua* mathematizable.

This does not mean that the correlationist believes that the world sprang into being a moment ago 'complete with fossils, records, and memories,' nor even that it could have done so - but, rather, that this 'did not' and 'could not have' function strictly speaking 'for us,' and faced with a seriously upheld claim that it did so, 'we' who believe in

5 *After Finitude*, p.1.
6 *After Finitude*, p 4.

the scientific pronouncements could offer no irrefutable counter-reason. Such a non-scientific position would certainly be based on faith at some crucial moment in its justificatory argument (if any), but correlationism is incapable of refuting faith, and indeed Kant famously asserted that he had established the limits of knowledge expressly in order 'to make room for faith.'

III. That Meillassoux believes this to be a peculiarly modern (or post-modern) impasse, reveals the extent to which he himself is a modern. The incapacity of philosophy to disqualify faith is a far older question. It was addressed, a propos the very issue Meillassoux raises, by Leo Strauss:

> 'It is sometimes asserted that the Biblical account of the creation of the world has been refuted by modern geology, paleontology, etc. But all scientific accounts presuppose the impossibility of miracles. Presupposing this, they prove that the age of the Earth or of life on Earth, etc., is millions of years; but what natural processes could achieve only in such periods, could be done in a moment by God miraculously [7].'

This argument was in fact famously propounded by Philip Henry Gosse, the nineteenth century naturalist who when confronted by mounting evidence that fossils were far older than the approximately 7,000 years a strict literal Biblical chronology would admit[8], suggested that such evidence was a kind of verisimilitude built in by the Creator. In much of the west, it has become the stock example of a move by which a thinker prescinds from the assumptions of the scientific community. Such an argument, frequently (though far from fairly) read as the abdication of rationality, must seem astounding in the mouth of Strauss, one of the most important, if idiosyncratic, thinkers of the twentieth century, a scholar of Lessing, Maimonides, and Plato. But Strauss is writing as a thinker in the sense (as he understood it) of ancient philosophy, which fully countenanced the strength of religion. This was a tradition, Strauss believed, that had come close to being lost, but that did not mean that religion was inherently weaker; only that it had become accidentally, i.e., sociologically, weaker. But this relative weakness is vis-à-vis science, not vis-à-vis philosophy. Strauss was adamant that modern philosophy, from the Enlightenment on, was a derivative and decadent tradition that did not know what philosophy *per se* was.

> 'It is frequently assumed that philosophy is a system. It is forgotten that if this were so, philosophy as love of wisdom, or quest for wisdom, were superfluous. Philosophy was originally not systematic in any sense…. To turn to a more simple example, according to the view of philosophy which today is generally accepted, a distinction has to be made between philosophy and science. This distinction, wholly unknown to philosophy until the later part of the eighteenth century, amounts for all practical purposes to the admission of an unscientific philosophy and an unphilosophical science [9].'

This state of things simply will not do, for Strauss. One might expect that Meillassoux will agree with him; he, too, claims to pose

> '...the demand for an elucidation of science's conditions of thinkability. Such a demand, in fact, has nothing transcendental in itself: it is proper to any philosophy which seeks to know what it is speaking of when using the term "science." My thesis is that we still do not understand what this word means, since we failed to resolve the aporia of the arche-fossil: that the mathematized sciences of nature are only thinkable under the conditions of granting an absolute scope to its statements, an absolute scope that all anti-metaphysical philosophies of the era have challenged. [10].'

Meillassoux is unlike the apostles of modern scientism in pressing for the right of philosophy to speak of a broader context than science. For him, science speaks of what happens to be the case; philosophy of the speculative context (which means, the hyperchaotic context) in which whatever laws that obtain do (locally) obtain, for as long as they do so. Nonetheless, by this same token, Meillassoux maintains the same separation of which Strauss speaks: an unphilosophical

7 Leo Strauss, 'Reason and Revelation,' in *Leo Strauss and the Theologico-Political Problem*, ed. Heinrich Meir.

8 Philip Henry Gosse, *Omphalos: an Attempt to Untie the Geological Knot.*

9 Strauss, '*Reason and Revelation*'.

10 Interview with Quentin Meillassoux, in Rick Dolphijn and Iris van der Tuin, *New Materialism: Interviews & Cartographies.*

science and an unscientific philosophy.

This account of Strauss' has been more and more fulfilled since he wrote, as philosophy has made itself more and more obeisant to science, or tried more and more to stand aloof. Science, for its part, has simply gone on its way, tending always towards the technical, as Heidegger foresaw. Finally, hubris being what it is, scientists have steeled themselves not only to claim explicitly the prerogative of philosophy to give an account of the whole - a 'theory of everything' - but to pronounce that 'philosophy is dead[11].'

Strauss famously or infamously held that the same modern era which had separated science from philosophy in this manner had forgotten that philosophers employed a mode of writing which concealed its genuine significance. Nietzsche had commented upon this:

> 'Our deepest insights must - and should - appear as follies, and under certain circumstances as crimes, when they come unauthorizedly to the ears of those who are not disposed and predestined for them. The exoteric and the esoteric, as they were formerly distinguished by philosophers - among the Indians, as among the Greeks, Persians, and Mussulmans, in short, wherever people believed in gradations of rank and not in equality and equal rights - are not so much in contradistinction to one another in respect to the exoteric class, standing without, and viewing, estimating, measuring, and judging from the outside, and not from the inside; the more essential distinction is that the class in question views things from below upwards - while the esoteric class views things from above downwards [12].

Strauss believed that this esotericism had been cultivated out of prudence, because philosophy tended toward atheism (and indeed, some readers of Strauss contend, toward nihilism), in short towards too-dangerous positions to be widely known or widely held. Nietzsche believed it was the inevitable effect of a natural distinction between three sorts of human beings - the 'herd' or great majority, the 'exceptions' or the few, and the philosophers themselves. On this account, esotericism was not so much a strategy, as a necessity cultivated as a virtue: since there would be misunderstandings between different classes, it behooved the philosopher to be intentional about these misunderstandings. But when Plato refers to his own esoteric practice, he does not speak of it in these terms. While secrecy may have had a prudential aspect, for Plato, philosophy is inherently esoteric because it aims at an experience which cannot be articulated:

> There neither is nor ever will be a treatise of mine on the subject. For it does not admit of exposition like other branches of knowledge; but after much converse about the matter itself and a life lived together, suddenly a light, as it were, is kindled in one soul by a flame that leaps to it from another, and thereafter sustains itself … if they had appeared to me to admit adequately of writing and exposition, what task in life could I have performed nobler than this, to write what is of great service to mankind and to bring the nature of things into the light for all to see? But I do not think it a good thing for men that there should be a disquisition, as it is called, on this topic - except for some few, who are able with a little teaching to find it out for themselves [13].

One cannot help but note that Plato's distinction between the few and the many is close to Nietzsche's. But Plato tells us both more and less than Nietzsche does about the 'content' of philosophical experience. It is not expressed by discourse, but discourse cultivates its conditions. Such discourse must, then, be the sort that triggers the kindling of the 'flame;' it does not directly communicate a content, but it enables the few 'to find it for themselves.'

In this context, a division between science and philosophy was unknown to philosophy, Strauss says. But then, if we turn to the ancients, to a 'philosophical science' and a 'scientific philosophy,' we are likely to be puzzled. Even if we avoid being condescending towards Aristotle's physics (for example), we cannot help but feel there is something different about the natural science of the ancients. It seems - well - unscientific. Just what this means is bound to be unclear, unless we pre-judge the case and say that it is unscientific because it does not resemble modern science - unphilosophical science; but that there is a difference between the

11 Even scientists that should know better, like Lawrence Krauss and Stephen Hawking.

12 Nietzsche, *Beyond Good and Evil*, 30.

13 Plato, *Letter* VII 341c-e, tr. Harward.

spirit of ancient 'natural philosophy' and modern science seems indisputable.

IV. Strauss claims that anciently, philosophy was the quest for the truth, and thus for the beginning of things, the *archê* , the reason for things being as they are. For Meillassoux, the first half of this - the quest for truth - is stipulated, but the second half - the quest for a principle, a beginning - is a deception. It can be shown, Meillassoux thinks, that the notion of a necessary being is contradictory, which means that the quest for a reason for things being as they are is bound to be disappointed. For Meillassoux, this can in fact be demonstrated and known: the answer to Why are things thus? is: for no reason. They simply are because, out of the transfinite set of possibilities, this one happens to be realized - for now.

This transfinite field of possibility - hyper-chaos, as Meillassoux calls it - he deduces from the *strength* of the correlationist position. As we have seen, Meillassoux has strong words for the ill results of correlationism, but he does not think it is a foolish position; indeed, he considers it to be nearly unassailable: to think anything is, indeed, to make it an object of thought, and therefore to think 'what there is without thought' is to turn this 'what there is' into something for thought. This is simply the form of the correlation as such, and as such, the correlation is, then, an inescapable fact. But this inescapability can be met, he thinks, in three ways. One is the Berkeleyan response, which amounts to absolutizing the correlation. Another, which is the response of the so-called Strong Correlationist, is simply to accept the correlation, without trying to press beyond it; one merely grants that one does not and cannot know if there is any In-Itself at all. This, Meillassoux thinks, leads to fideism of one stripe or another, notably to a kind of Wittgensteinian quietism or a Heideggerian *gelassenheit*.

But the third response is to absolutize not the correlation itself, but its *facticity*. Facticity has always had a kind of bruteness to it, and the intellect is irritated by this bruteness. The quest for reasons for things has been a sort of mitigation of this; once one sees why any given contingent fact X obtains, one feels also an accompanying relief of the intellectual irritation. But, says Meillassoux, when it comes to facticity itself, there is no deeper explanation, no rationale to be had. Facticity, the possibility that a contingent X could have been otherwise, is as far as one can get. This has far-reaching consequences: it means that in a very real sense, anything that is could be otherwise - and this absolute 'capacity to be otherwise' is what Meillassoux calls hyperchaos.

Hyperchaos is in many respects hard to tell from the flux of Nietzschean vision, or from the 'play of differences' in deconstruction, but there is a defining characteristic which sets apart Meillassoux's conception from these apparent cousins: Meillassoux aspires to set the scientific project upon as secure a foundation as can be expected from these premises. For Meillassoux, the scientist (insofar as she reasons mathematically) does indeed discover genuine natural laws, and so may legitimately reason on their basis about things in the absence of any consciousness or subjectivity.

> 'The decentering inherent in the Copernican-Galilean revolution proceeds by way of a Cartesian thesis, viz., that whatever is mathematically conceivable is absolutely possible. But it is important to note that the absolute here is not understood in terms of the capacity of mathematics to designate a referent that is assumed to be necessary… rather…all those aspects of the given that are mathematically describable can continue to exist regardless of whether or not we are there to convert the latter into something that is given-to or manifested-for. Consequently, this dia-chronic referent may be considered to be contingent while simultaneously being considered to be absolute….the Galilean-Copernican decentering wrought by science can be stated as follows: what is mathematizable cannot be reduced to a correlate of thought [14].'

The 'laws of nature' discovered by science are, then, real laws, and they refer to the in-itself, to things apart from us. But nothing *grounds* these laws. They may change at any moment, for no reason. They are, in short, not necessary. There is nothing that is necessary, for Meillassoux, except for contingency itself, which is (on my reading of him, at least) the same as facticity, the bruteness

14 *After Finitude*, p. 117.

of brute fact. The reason why there is something rather than nothing is because contingency is necessary, i.e., something has to exist to be contingent, something that could be something else - and indeed might well be something else, in a moment or in a hundred quintillion years. Thus the same argument that allows Meillassoux to claim that we do know laws of nature also entails, for him, that they could change at any moment.

N.B., this is not the same as the Humean notion that the apparently regular patterns of natural phenomena might alter; for instance, that the sun could rise tomorrow morning in the west, because there is no law which says it rises in the east. When the Meillassouxian 'speculative' philosopher is asked about the reality of natural laws, she will answer that the laws are laws indeed; but they have no (ultimate) reason to be as they are, and they could change, or be abolished, without warning. It is worth considering just what this means, however. It amounts to saying the laws are legitimate for now, or, what amounts to the same thing, for us. This addendum, almost indistinguishable from the '*codicil*' Meillassoux complains about the correlationist adding 'under his breath,' indicates that Meillassoux's vision is in a crucial sense very close to that of modern philosophy as a whole. (This should not surprise us, since despite his criticisms, Meillassoux expressly states his loyalty to Kantian critique as he understands it).

V. But what, then, would a perspective informed by ancient, 'philosophical science' look like?

As mentioned, Aristotle spoke of it being 'the mark of an educated man not to seek more precision than the nature of the case admits[15].' This is a restatement of a Platonic precept; in the *Timaeus*, it is acknowledged that often the case requires acceptance of 'likely stories' in place of absolute precision.

It is worth considering the rationale by which Timaeus, in the dialogue that bears his name, justifies the need to settle for a likely story. This is a rationale which hinges upon distinguishing the physical world, which changes, from the eternal, rationally apprehensible world. It is because the physical world undergoes change and destruction that it is apprehended by the senses and by opinion.

An account of such a world will therefore also be merely 'likely.' On the other hand, the eternal, unchanging world is knowable by our rational faculty. 'An account of what is changeless, fixed and clearly intelligible will [itself] be changeless and fixed.' This assertion is structured by an analogy: 'As being is to becoming, so is truth to belief.' Therefore, in a description of the physical world, one 'should not look for anything more than a likely story.[16]'

This countenancing of imprecision is not an unscientific shrug; it is, on the contrary, a scientific principle. The regime in which this principle has its origin is the Platonic science par excellence: music.

> 'From Philolaus in the fifth century BC, through Plato and Aristoxenus in the fourth, and down to Ptolemy in the second century AD and Aristides in the third or fourth, Greek acoustical theorists moved confidently between two modes of expression: the absolutely precise and the conveniently approximate [17].

These two principles are both at play in Timaeus' account. In the changing, phenomenal world, Timaeus proposes, since nothing 'becomes or changes' unless caused to do so, the universe must itself be 'caused,' and Timaeus puts forward a mythological 'father and maker,' a demiurge who sets the cosmos in motion. However, Timaeus deduces, the universe must have been modeled upon an unchanging form, for the universe is beautiful and orderly; it is, in a famous phrase, the 'moving image of eternity,'

> 'When he set in order the heaven, he made this image eternal but moving according to number while eternity itself rests in unity; and this image we call time [18]'

Now, the numbers which Timaeus adduces are all, demonstrably, derived from harmonic theory, and the cosmogonic narrative he elaborates is

15 *Nichomachean Ethics* I, 1094 b24.

16 *Timaeus* 28a, 29b-d.

17 Ernest G. McClain, *The Pythagorean Plato*, p. 162. There is an important sense in which my entire paper is an extended application of this crucial insight and its ramifications.

18 *Timaeus*, 37d.

a story which has its grammar quite specifically in the proportions which generate the musical intervals which are octaves, perfect fourths, and perfect fifths. Here is Jowett's translation, including his bracketed insertions:

> 'Now God did not make the soul after the body, although we are speaking of them in this order; for having brought them together he would never have allowed that the elder should be ruled by the younger; but this is a random manner of speaking we have, because somehow we ourselves too are very much under the dominion of chance.... First of all, he took away one part of the whole [1], and then he separated a second part which was double the first [2], and then he took away a third part which was half as much again as the second and three times as much as the first [3], and then he took a fourth part which was twice as much as the second [4], and a fifth part which was three times the third [9], and a sixth part which was eight times the first [8], and a seventh part which was twentyseven times the first [27]. After this he filled up the double intervals [i.e. between 1, 2, 4, 8] and the triple [i.e. between 1, 3, 9, 27] cutting off yet other portions from the mixture and placing them in the intervals [19].'

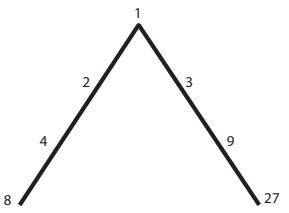


Fig 1. The numbers mentioned in the passage are all arranged here on the two sides of a triangular figure shaped like the Greek letter lambda.

There are several things to be noted about this passage. Timaeus begins by stating that the exposition does not follow the ontology or chronology: Timaeus expressly says that he is unfolding the account in an order different from how the demiurge would have proceeded; that unlike the demiurge he is putting the soul after the body (even though it is right that the soul should rule the body and thus that it be the elder); this disorder in the argument, Timaeus says, comes about from a kind of randomizing effect that chance has upon articulation. This will be important when we return to think about Meillassoux's competing hyperchaotic onto-cosmology.

The numbers mentioned are all powers of 2 and 3, and as such are those which generate octaves and perfect fifths. This musical account of the numbers' provenance is not controversial; but it is frequently hurried past as so much vague 'numbermysticism.' Phrases like these are substitutes for thinking. The cosmos here is being imagined as either an instrument or a piece of music or both, and this imagining is far from merely incidental to Plato's philosophy. An over-arching metaphor, it does not claim to be the precise truth of the matter, but *like* the truth. But the terms of this likeness need to be examined.

The octave is the interval which sounds when two strings are plucked, one of which is half as long as the other but otherwise alike in tension, thickness, and material. This 2:1 ratio holds between the successive numbers on the left side of our lambda figure. Similarly, a perfect fifth sounds when such strings are in a ratio of 3:2, and a perfect fourth, when the ratio is 4:3 - ratios which obtain between the two sides of the lambda.

One can, moreover, multiply a ratio repeatedly and so generate further tones. An octave above or below D is always D. The ear recognizes the resonance, which is in fact measurably there; one can halve the string again and again, and always get an octave. If one's beginning tone is D, a fifth above is A; and a fifth above that, E. On the other hand, a fifth below D is G. However, this 'stacking' of fifths creates an almost unsolvable problem, which arises in the following way: a fifth above E is B, and a fifth above B is G♯; if one repeats this procedure for a full thirteen times, one arrives finally at a very high D, seven octaves above one's starting-place, and seems to have come full circle. But this D turns out not to be in a 1:2 proportion with the starting D. There is a difference, which a moment's thought tells us we should have expected, for after all, repeatedly multiplying by 2/3 does not yield the same result as multiplying by 1/2. The quantitative difference between these two products is one instance of the small intervals which plague harmonic theory and which arise because of the incommensurability of products of different prime numbers. Simply put: if you pluck a length of string to get a tone, and then pluck a length of string half *that* length, you get an octave higher; half of that string gets you an octave higher than

19 *Timaeus* 34c-36a.

Ascent by Octaves			
T	8va	F.R.	S.L.
D	0	1:1	1
D	1	2:1	1:2
D	2	4:1	1:4
D	3	8:1	1:8
D	4	16:1	1:16
D	5	32:1	1:32
D	6	64:1	1:64
D	7	128:1	1:128

Ascent by perfect Fifths			
T	5th	F.R.	S.L.
D	0	1:1	1
A	1	3:2	1/1.5
E	2	9:4	1/2.25
B	3	27:8	1/3.375
F♯	4	81:16	1/5.0625
C♯	5	243:32	1/7.5937
G♯	6	729:64	1/11.3906
D♯	7	2187:128	1/17.0859
A♯	8	6561:256	1/25.6289
F	9	19683:512	1/38.4433
C	10	59049:1024	1/57.6650
G	11	177147:2048	1/86.4975
D	12	531441:4096	1/129.7463

T=Tone; 8va= Octave; F.R.=Frequency Ratio; S.L.=String Length; 5th=Fifth

that; and so on. Do this for seven octaves. Now go back to the original string, and pluck another string two-thirds if its length; you will get a tone that is a perfect fifth above. Now pluck another tone two-thirds of that, for a perfect fifth higher still. If you keep repeating this procedure, after twelve times you will have a another, very short, string, very close in length to the short string you got by halving. Very close, but not quite the same; the easiest way to figure this out is to start doubling the length of the string you got after twelve repetitions of taking 2/3. For of course, if your small strings were exactly the same length, then doubling each of them should preserve that sameness.

The table above compares repeatedly halving string length (ascent by octaves) with repeatedly taking 2/3 string length (ascent by perfect fifths). (The column shows string length as the reciprocal of the frequency; decimals are rounded to four decimal places.) After seven iterations of one and twelve of the other, the tones of the two strings are very close, but not precisely the same. The difference between them is called the Pythagorean comma. One may note that the numbers on this chart recapitulate and extend the numbers on both sides of the Platonic lambda.

If we begin with a length of string one meter long, the end of the process of halving the string will leave us, after seven such divisions, with a length of string 1/128 of a meter. On the other hand, after taking two-thirds length twelve times, we will be left with a string about 1/129.7463 of a meter long.

These lengths are pretty close: 0.78125 centimeters, and 0.77073 respectively. One might guess that these short lengths of wire will sound, when plucked, at tones very close to each other. And they will - but not exactly the same.

Of course, if we double these short lengths seven times, we get one meter when we start from 1/128 of a meter; but if we start from 1/129.7463, we get approximately 98.65403 centimeters. This is as much as to say that 'pretty close' does not mean anything at all if our standard is the absolute precision of mathematics. Indeed, it is not even very close!

VI. This discrepancy is the 'Pythagorean comma.' Probably the oldest and best-known of many incommensurabilities in music theory, it is the slight dissonance between seven octaves and twelve fifths. Correcting for this incommensurability requires a slight compromising of precision; the solution in modern Equal Temperament is to slightly untune every interval except the octave.

All of these numbers follow by simple arithmetic from the initial series of numbers on either side of the lambda shown above, which figures in the 'likely story' of Timaeus. But why did Plato choose this musical series of numbers in the first place, to serve as the skeleton for the demiurge's work? What about music made it the obvious setting for a cosmological likely story? Plato is not here to ask; but we may hazard an educated guess. In this 'likely' account Timaeus offers, approximation plays a vital role, because when the model is applied with precision, it falls apart. The story is 'likely' and approximate in effect, because the model it proposes is itself one of approximation. Plato did not hold the cosmos was a musical instrument or a musical composition; or at least, he never expressly declares this; he held that it was *like* one, in the vital respect that the effort to describe it with absolute precision was bound to untune it. Press precision too far, and one does not save the phenomena, but loses them; but this characteristic can itself be precisely understood and applied. To point out this likeness is not to point to an interesting parallelism of certain details, as if one were saying simply that numbers, or even certain specific numbers, play a significant role in both tuning a lyre and measuring the movements of the planets. It is, rather, to assert a constraint upon any articulation of analogy concerning the cosmos, because music itself is a matter of play between precision and approximation. In music, one 'settles' for inexactitude, but in the name of, as it were, a 'higher' precision, the precision of experience and not measure.

Crucially, this 'likeness,' this imprecise precision, pertains not just to the material world but to reality as a whole. Plato does say that 'likeness' is what is possible for an account of the sensible world - not of the eternal, noumenal world, which in fact can be known. And yet - as is well established but all too often unremarked - each Platonic dialogue commencing with a search for a precise definition of a thing or a concept - friendship, piety, justice, love - concludes in aporia and the admission that the interlocutors still do not know. It is as if the 'eternal' realm of the forms decidedly eludes our intellectual grasp, while the phenomenal, 'apparent' world offers itself to our senses to be grasped, seen, heard, tasted. It turns out that intellectual apprehension, which we would have guessed was blessed with precision, is almost rather cursed with it. To transmute this curse into a blessing requires a philosophical shift of perspective. The Platonic tradition points to this experience in more than one place, e.g. the Seventh Letter or the *Epinomis*, but this is not only beyond the scope of this paper; it apparently is beyond the scope of what can be put into words. At this point, the quest for definition and precision opens onto a different experience of contemplation. Nonetheless, it is important to recall that this method of 'likelihood' implicated not only the senses but the discursive intellect as well.

The 'likely' cosmology of the *Timaeus* passed into tradition, albeit not without contest. Aristotle argued against it; the Stoics and the Epicureans also. Aristotle's objections, in *On the Heavens*, centered upon Plato's having described the cosmos as having come into existence; both Plato and Aristotle agreed that the cosmos was sempiternal; this entailed, for Aristotle, that it could have had no beginning in time. The argument, which takes up the end of the first book of *On the Heavens,* culminates thus:

> 'It cannot truly be said of a thing now that it exists last year, nor could it be said last year that it exists now. It is therefore impossible for what once did not exist later to be eternal. For in its later state it will possess the capacity of not existing, only not of not existing at a time when it exists-since then it exists in actuality-but of not existing last year or in the past. Now suppose it to be in actuality what it is capable of being. It will then be true to say now that it does not exist last year. But this is impossible. No capacity relates to being in the past, but always to being in the present or future. It is the same with the notion of an eternity of existence followed later by non-existence. In

> the later state the capacity will be present for that which is not there in actuality. Actualize, then, the capacity. It will be true to say now that this exists last year or in the past generally. Considerations also not general like these but proper to the subject show it to be impossible that what was formerly eternal should later be destroyed or that what formerly was not should later be eternal. Whatever is destructible or generated is always alterable [20].'

In a remarkable study[21], Jules Vuillemin has argued that this passage ought to be seen as the source for Diodorus Cronus' 'Master Argument' which derives fatalism from the fact that the past is unalterable[22]. He also remarks:

> 'This text is fundamental for insuring the separation of the eternal, though sensible, substances which are not subject to generation and decay from the properly material substances which are subject to generation and decay. It is here that the separation of Heaven and Earth and astronomical theory find their raison d'être [23].'

We are thus met with a nexus of questions that bear upon the issues of fate and freedom, the boundary of the universe between the region of chance and that of necessity, and the possibility of the revision of the past.

An earlier passage in *De Caelo* also deserves attention:

> 'Some of those who hold that the world, though indestructible, was ungenerated, try to support their case with a parallel which is illusory. They say that in their statement about its generation they are doing what geometers do when they construct their figures, not as implying that the universe really had a beginning, but for didactic reasons facilitating understanding by exhibiting the object, like the figure, in the course of formation [24].'

The argument Aristotle is undertaking to refute clearly has to do with the tension between exposition and being; Aristotle in fact rejects the analogy because it seems to him that the exposition cannot do away with a narrative, and so chronological, element. Apparently, then, some philosophers had invoked a kind of disjunction between exposition, which must unfold in time, and the eternal matter being expounded. This disjunction seems very close to the 'rule of chance' which Timaeus proffers as the reason for the 'random way of speaking' which expounds the development of the soul of the universe after that of the body; a rule which disrupts the orderly chronological or logical unfolding of an argument.

Simplicius in his commentary infers that Aristotle is referring to 'Xenocrates and the Platonists' in this passage, and his comment contains an interesting gloss on Aristotle's argument, to wit, that

> 'no one would say that it was merely hypothetically that intercourse preceded conception and birth, or the stones and wood the house [25].'

This (apparently foresworn) argument is remarkably like the Gossean hypothesis mentioned above, about the fossil record: to wit, that the past could simply have been virtual. When Gosse argued that Adam's navel had been bestowed by God for the sake of appearances, he was indeed advancing the argument that the intercourse, gestation, birth and childhood which would have preceded Adam's adulthood were all 'hypothetical,' just as were the years of growth attested to by the tree rings in the trunk of the Tree of Knowledge of Good and Evil, or the generations of imaginary animals whose 'bones' were being discovered in Gosse's day. Despite the analogy, Simplicius defends the general procedure, arguing that Plato is describing not a chronological event but an eternal and divine cause of the universe.

Elsewhere, in Simplicius' commentary on *De Anima*, we find a further reflection:

> 'Because of its declension outside, as a whole [the soul] simultaneously both remains and proceeds, and it has neither completely without the other. Whence, its immortality is at that time filled with mortality in its whole self, and it does not remain immortal only. Its ungeneratedness somehow happens to come to be [26].'

20 Aristotle, *De Caelo I*, 283 b.

21 Jules Vuillem, *Necessity or Contingency; the Master Argument*, 1996.

22 See below, note 51 (on fatalism & Diodoros Cronos).

23 Vuillem, *Necessity or Contingency*, p. 15.

24 Aristotle, *De Caelo*, 280a.

25 Simplicius, On Aristotle's *On The Heavens* 305, 1.

26 Commentary on Aristotle's *De Anima* 90.20-24. The attribution to Simplicius is a matter of debate in modern scholarship. On the term 'declension,' νεῦσις, see below, note 61.

The plain interpretation of this remarkable claim would seem to be that by its removal from the upper world to the lower, the principles in the soul are so disordered that even its quality of having-no-beginning would be an ontological, and not merely discursive, disruption of principle; at least, it concedes that exposition renders inevitable a mismatch between What Is and What is Said. This mismatch is an extended application of the 'tuning' of articulate description in the face of intractable mystery - a function of 'musical' cosmology. Without trying to completely unpack this very paradoxical argument, we may simply place it as part of the Neoplatonic project of reconciliation attempted between (at least) Plato and Aristotle. Nearly everyone in the Neoplatonist tradition accepted some form of the musical cosmos, but only a few whose work survives wrote extensively upon music. One who did was Aristides Quintilianus.

VII. Aristides is not the most famous of the Neoplatonic philosophers. It is not clear where to situate him in the tradition; we do not know who his teachers were; his dates are uncertain, and although he lived in the Roman Empire and wrote in Greek, his place of origin is unknown[27]. His only surviving work, the three books *On Music*, was only published in English translation towards the end of the twentieth century[28]. This work is one of the very few complete musicological treatises to have reached us from antiquity, and it provides a wealth of technical information not only on the musical practices of the Greco-Roman world but of the way music and metaphysics informed each other, to the point of blurring together.

This 'blurring' is important to underscore; Aristides' treatise sets out to provide a complete account of music, but its compass is far wider than what would be included in a strictly musicological work today. It shifts back and forth between a detailed exposition of the specifics of music theory which Aristides had inherited - details pertaining to tuning, rhythmics, scales and modes, and musical psychology - and broader analogies between music and the cosmos as a whole. It bears emphasis that these are not, for Aristides, separate aspects; and the word 'analogy' here must be carefully understood.

When Aristides shows how some aspect of the cosmos, or the cosmos itself, is musical, he does not think of himself as drawing a parallel between two more or less unlike regions and pointing out an important similarity. Indeed, Aristides seems to think of music as the very principle by which relation as such is constituted. At the beginning of his work, he cites the otherwise unknown Pythagorean philosopher, Panaceus, who, he says,

> 'confirms my view. He says that it is the business of music not only to organize the parts of sound to one another, but also to assemble and harmoniously join together everything as has a nature [29].'

For Aristides, as for the entire Pythagorean tradition, human music, made by voice or upon instruments, is a subcategory of music as a whole. Most music is imperceptible by human ears; indeed, it is not even sound, but it is music nonetheless. The illustrations of this range from the familiar ratios of planetary motions, to many more abstruse and (to modern minds) implausible associations; for instance, between, on the one hand, different notes, modes, or rhythms, and on the other hand, the seasons, the tides, animal bodies, medicine, ethics, and politics.

Towards the end of his treatise, when he is preparing to wrap up his demonstration of the privileged place of music in the preparatory curriculum for philosophy, Aristides produces a compressed litany of phenomena from history, medicine, astronomy, ethics, botany and biology, pointing out, sometimes in detail and sometimes

27 Aristides' career can be assigned objective termini from textual evidence: he cannot be earlier than Cicero, since he refers to him; on the other hand, his work is extensively by Martianus Capella in the fifth century AD. This gives a span of approximately between 50 BC and 400 AD. However, since his vocabulary and argumentation seems so fraught with resonances with Porphyry and Plotinus, it has often seemed reasonable to presume that he shares some milieu with these thinkers, though guesswork about the nature of the relationship is perilous.

28 Thomas Mathiessen, *Aristides Quintilianus: On Music*, in three books, 1982; Andrew Barker, *Greek Musical Writings* Vol. II: *Harmonic and Acoustic Theory*, 1989. I will make use of both of these translations, occasionally modifying one in light of the other.

29 Book I, 1.

incidentally, how each of these things manifests numerical proportion. This enumeration culminates in a sweeping assertion:

> τὸ δὴ ταῦτα μὲν οὕτως ἐναργῶς δι΄ ἀριθμῶν καὶ μεσοτήτων συνεστάναι, μουσικὴν δὲ μὴ ἂν ὑπονοεῖν παντελῶς ἀμαθοῦς καὶ ἀμούσου τὴν φύσιν ἐστίν.
>
> 'To have organized these things so palpably through numbers and means, but not music, is to suspect nature of being wholly ignorant and unrefined [30].'

That is: if we acknowledge that number and mean are principles of structure for all these disparate phenomena, we ought also to recognize music as an organizing principle as well, if and insofar as we regard nature as seemly, fitting, and intelligently ordered.

I can think of no passage which more succinctly summarizes the difference between the ancients and the moderns, than this sentence from Aristides Quintilianus.

In the modern age, we do indeed imagine things to be 'organized palpably through number and mean, but not through music;' indeed, what it would mean for things like nebulae or mitochondria or plate tectonics to be 'organized' musically is hard to even imagine. Concomitantly, we do suspect (and far more than suspect) nature of being both ignorant and unrefined. It is almost part of the *definition* of nature that it is ignorant and unrefined; for the opposites of these are not natural, but cultural. Dawkins' anti-Paleyan 'Blind watchmaker' is in fact a fairly weak trope for this. Nature is, on Dawkins' assumptions, more (or perhaps less) than merely blind; it is planless, and indeed fundamentally incapable of either vision or plan, to make a watch or anything else[31].

Aristides' analogies culminate with a parallelism between certain melodic modes (on the one hand) and (on the other) the Stoic-inflected distinction between the sublunary world, where chance (and by the same token, freedom) has a foothold, and the higher heavens where necessity reigns supreme. These exemplify, for Aristides, two sorts of time, and especially two sorts of future: one future that is in some wise 'up to us,' and another that is inflexible and inevitable. These Aristides calls (in Mathiesen's rendering) what-may-be, *μέλλον*, and *what-will-be*, *γενησόμενον*, and respectively these concern, he says, what is contingent in part (*τὰ ἐπὶ μέρους συμβαίνοντα*) and what is contingent in general (*τὰ καθόλου συμβαίνοντα*[32]). (Barker: 'things which occur in particular cases' vs 'things which occur universally'.) And this distinction corresponds to a specific line of demarcation in the universe: the lunar orbit.

> The wise say that the quality of the future is twofold: the one type, indeed, is necessary and not to be turned aside, which is called what-will-be; the other type is alterable and in no wise appointed, which they say is what-may-be. The necessary presents and comprehends the things above the moon; the uncertain and possible, the things here. Things contingent in general are not to be turned aside and are necessary; things contingent in part are easily and continually changed [33].

Aristides' distinction between two sorts of future draws upon ancient tradition, according to which Fate is such that it constrains even the gods. But there are also always cultural procedures possible for attempting to outmaneuver fate, implying a certain limited leeway. Herodotus relates numerous stories of such attempts to avoid or alter destiny, which of course (as the paradigmatic episode of Oedipus had already shown) do not always work. This distinction is known across the ancient world: within a very small compass, human beings have a certain capacity to shape their own destiny. Beyond that, our fates are determined.

VIII. The two kinds of future have musical paradigms. There is melody which moves in a straight line, up or down; this is made of notes which are contingent in general, each note "following"

30 Book III, 8. (Note 30 continued at end of article).

31 Richard Dawkins, *The Blind Watchmaker: Why the Evidence of Evolution Reveals a Universe Without Design.*

32 This precise formulation (unlike τὰ καθόλου συμβαίνοντα) does not occur in the text but is parenthetically provided by Mathiessen in his commentary, to clarify the parallel with 'contingent in general.' The word *sumbaionta* is more difficult to render than its commonality would suggest. It can often mean 'facts,' or '(logical) consequences,' 'events,' 'what happens' or 'what follows.' There is in any case a sense of being dependent upon, and belonging with, something preceding either logically or chronologically.

33 in Book III, 26.

(from) the note before, and so is a representation of what-will-be. There is also melody made of notes that move disjunctly, in which the notes of the melody skip notes of the scale and occur out of sequential order. Such melody is a paradigm of what-may-be.

Following Aristotle and the geocentric model of the universe in general, Aristides presents this degree of freedom as corresponding to this world "below the moon." Beyond or above that, however, is an inflexible destiny which corresponds to the laws which the heavenly bodies apparently follow without wavering. Aristides cites evidence from Homer and from Herodotus to demonstrate the relevance of the distinction: there is a destiny that is inexorable and non-negotiable, but there is also a degree of room for modifying the decrees of fate. His most succinct example, a prophecy which Herodotus attributes to Musaeus, is also his most important for our purposes:

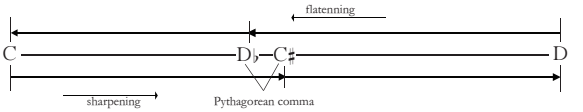
> I have evidence for the truth of this from an ancient oracle delivered from the Pythian tripod, concerning the expedition of the Medes against the Greeks at Plataeae…: "There in death shall lie, beyond what Lachesis and fate doomed, / Many a bow-bearing Mede, when the due day of calamity comes [34]."

Aristides interprets this prophecy as apparently containing a contradiction: "Beyond what Lachesis and fate doomed" means, he says, "in excess of" the decree of destiny; but "when the due day… comes" means when the day arrives, as appointed *by* destiny.

> The dissonance of the phrases will at once be apparent. But….can the prophet of the universe really have uttered in so short a space of time statements that so blatantly contradict one another? Far from it. It is rather, I believe, that he is covertly pointing to the dual nature of destiny [35].

In speaking of the *dissonance* of the two phrases, Aristides not only uses a musical term for a conceptual tension, he also points to a moment when, as he sees it, Herodotus (or the Pythian oracle) intentionally uses two propositions in implicit dissonance with each other to prompt a further insight upon reflection on the part of readers. We will later observe Aristides himself cueing the reader in a similar fashion.

It is important to note that this distinction between "futures" is drawn with regards to the lunar orbit. Pliny mentions[36] that Pythagorean tradition defined the interval from Earth to the moon as a whole tone, defined as the amount by which a perfect fifth exceeds a perfect fourth; the relevant ratio is 9:8. Within this ratio one can find the Pythagorean comma in the smallest compass, as the difference or overlap between two adjacent semitones. For instance, although D♭ and C♯ are equivalent in Equal temperament, in Pythagorean tuning they are different: sharpening C actually raises the pitch above the point reached when D is flattened, and vice-versa. The amount by which either flattening the higher, or sharpening the lower, of two notes separated by a whole tone is, again, the Pythagorean comma.



Aristides has remarqued about the division of the tone that his predecessors,

> 'since there is no number between 8 and 9, doubled them to make 16 and 18, and found between them 17. They said that by this number the tone is partitioned inro senuitones. But it is discovered that these are not divided equally, but rather into a greater and lesser semitone [37].'

Later in his book, remarking upon this same procedure, he expands:

> 16 and 18 alone of all the plane numbers have areas equal to their own perimeters, showing the symmetry of what surrounds and what is surrounded—the body and the soul; and 17, which is the middle term of the aforesaid pair, and shows the natural mean to both of them, explains the first natural emanation of the moon towards terrestrial things [38].

Thus Herodotus' (or Musaeus') apparent contradiction, within two lines ('so short a space

34 Ibid. The reference is to Herodotus, *Histories*, IX 43.
35 Book III, 26. Tr. Barker.
36 Pliny, *Natural History*, II, 22
37 Book III, 1.
38 Book III, 12.

of time'), is treatable as a figure for the tension that obtains within the interval between the earth and the moon. But this tension, between semitones, in turn recapitulates the tension in the cosmos as a whole, between two sorts of contingency, below the moon and beyond it.

Now it is plain that Aristides is deploying mathematics in a way that would raise eyebrows in any mathematics department today (and the examples I have given are very mild in comparison to some), a way that many would say does not qualify as mathematical at all. All of this would perhaps lead us to believe that whatever is going on in Aristides' thought, it is very far indeed from the rigor of Galilean mathematized science.

And yet, when we come to Aristides' phrase "contingent in general," *τὰ καθόλου συμβαίνοντα*, and his characterization of this as *necessary*, we cannot help but note that Meillassoux characterizes necessity in exactly the same way. Necessity is, for Meillassoux, precisely contingency in general; what us necessary, in the last instance, is simply *that something contingent be*. The Meillassouxian account of hyperchaos turns out, remarkably enough, to be readable through the lens of the Ptolemaic cosmology, and vice-versa; but the hinge of this is the analogy between "two kinds of future" on the one hand, and Aristides' musical modes.

What matters here is not first of all whether this musical comparison is plausible in today's terms. We are rather concerned with the very distinction between two kinds of future, the fact that they follow from two sorts, two sorts of "happening," of contingency (what happens in general, and what happens in particular cases[39]), and especially the fact that they correspond to the regions on either side of the lunar orbit, for it is precisely this distinction between the super- and sub-lunar which the Galileo's conceptual innovations abolished. Recall how Meillassoux presents the Galilean revolution:

> Galileo…conceives of movement itself in mathematical terms, and particularly the movement which appears to be most changeable of all: the falling of terrestrial bodies [40].

Note here that the movement of the celestial bodies is explained in terms of the same dynamics as hold sway below the moon. From this point on, there is no distinction between the *kinds* of change in the cosmos; the orbit of a planets is simply another case of a falling body. To be sure, telescopic evidence of sunspots and lunar craters force thought to countenance the notion of flaws in the heavens; but philosophically, the revolution is a change in the way mathematics is used to decode the 'Book of Nature.' As Meillassoux notes, the implication of this was the construal of the universe as a "glacial world." Thus the most far-reaching effect of erasing the cosmic border between sub- and superlunary realms is not to introduce changeability into the empyrean of the planets and fixed stars, but to show the absolute regularity of terrestrial change; not to press mutability upwards, but to draw necessity down.

IX. However, though Meillassoux maintains that this mathematicization of sublunary reality is absolutely legitimate, it remains, for him, local. Hyperhcaos is yet a further, super-superlunary realm, in which changeability *as such* is the only rule. This extreme materialism proposes that Nature is, finally, exponentially more random and radically intention-less than Dawkins ever dreamed; moreover, it presses home a mathematization of the real every bit as drastic as Aristides', but which it expressly contrasts with music. This contrast is not highlighted by Meillassoux; it passes almost unremarked. Nevertheless, it indicates just how much is transformed in the revision of mathematics in these terms.

This turn of Meillassoux's argument[41] is extremely dense, and will require some attention; but I am eliding many details for brevity's sake.

39 This distinction also corresponds to the way Aristotle distinguishes between poetry and history, which he says respectively treat 'the sort of thing that happens' and 'what actually happened.' As *Poetics* IX has it: 'poetry is more philosophical and more elevated than history, since poetry states more universal things, whereas history states particular things.' (9.1451b5-7). This is an indication that for the ancients, the privileged vantage Meillassoux accords to mathematics belongs to a different register.

40 Meillassoux, *After Finitude*, p115.

41 Meillassoux, 'Iteration, Reiteration, Repetition:

Meillassoux focuses upon the oft-remarked claim that the mathematician is concerned with the rule-governed manipulation of signs, but signs which are in themselves meaningless:

> An essential part of the enigma of mathematics—in what does mathematics consist? What does it speak of?—turns upon the elucidation of the following question: how can we think a meaningless sign? And what exactly do we do when we produce such a notion mentally [42]?

Crucial to this investigation is Meillassoux's contention that a sign remains a sign even if it does not refer to anything. There are many steps in this argument, but the decisive moment comes with a distinction he illustrates with a parable, "the contented paleographer." He imagines an archaeologist excavating a dig in the site of a little-known civilization which is presumed not to have had any writing. The researcher uncovers a tablet with two lines of marks, e.g.:

§ § § § § § § § §
++++++++

The archeologist assumes this pattern is a series of similar visual motifs decorating the tablet's edge. But then,

> 'suddenly, her heart leaps: for she realizes that this frieze might in fact be two lines of signs—the equivalent of a child's schoolbook, in which one learns how to write a character. She now grasps the motifs as occurrences reproducible at will:'

§ § § § § § § § §, *etc.*
++++++++, *etc.*[43]

Meillassoux then asks, what is the difference between these two ways of seeing the series? In the course of his answer, drawing upon Bergson's analysis of time, Meillassoux's notion of the meaningless sign will be expressly distinguished from the musical note, in a way that foregrounds (though this is clearly not Meillassoux's concern) the conceptual inversion which has taken place between the ancients and the moderns.

Bergson illustrates his notion of *durée* with the privileged example of melody[44]. In a melody, a note does not have an independent significance for us; notes are heard not separately, but successively, and their import for us varies depending on their position, their precursors, and what follows them in anticipation or in fact. The note C with which a melody begins is heard differently than the C which occurs in the middle of a phrase in passing or upon which the descending cadence lands. 'Each time, the same note becomes different, and differently different from its predecessors,' says Meillassoux. And thus aesthetic judgment becomes relevant.

"Similar sounds acquire a differential significance from the sole fact of being repeated," he glosses. And this difference arises even if the notes are all the same, as in the chiming of a clock.

> There is a sensible differential effect that is not identifiable with a dissimilarity. It is the pure passage of time—at least, conscious, sensible time—that produces this difference [45].

For Meillassoux, this Bergsonian effect of a different significance despite material identity is applicable to all sorts of phenomena, including the visual motif understood as an artistic effect. When the series

§ § § § § § § § §
++++++++

is understood as a set of decorations on a clay tablet, each instance § of the pattern can be looked at individually—as per its execution, its placement in the whole series, or as to whether its occurrence makes the series too long, or breaks its symmetry, and so on. Meillassoux will say that if such judgments are possible, the series is a *repetition*.

But when the series is a recurrence of signs, then in essence the marks do not differ from each other, and their recurrence is not subject to these evaluations. Every § in the sequence is the same,

a Speculative Analysis of the Meaningless Sign.' https://cdn.shopify.com/s/files/1/0069/6232/files/Meillassoux_Workshop_Berlin.pdf

42 Ibid.

43 Ibid.

44 We are not concerned here with whether Bergson is correct in restricting this differential effect to temporal experience, or whether, as Meillassoux insists, he has identified a phenomenon which pertains to space as well. Our experience of music is of course temporal; but Aristides would not concede that the human experience of music is dispositive for what all music is. What is crucial for our purposes is the position of music as privileged instance or paradigm.

45 Meillassoux, op. cit.

and is the same whether it stands alone or in a series of n such marks, even if n is infinite. This is the case not only regardless of any accidental material differences between the marks (one being differently written, larger or smaller, in another color, etc.) but regardless of the differential effects which arise from their ordinal setting. Such a series, Meillassoux calls an *iteration*.

> 'In a way that escapes all conventional explanation, I end up being able to recognize as perfectly identical in type, occurrences of marks that I can for this reason alone think as iterable at will. Because, thought as rigorously identical to their type, lines of such signs escape the differentiating-finitizing effect of repetition, and instead open me up to the universe of writing, beyond that of design or music [46].'

Meillassoux's grounding of mathematics, hinging upon the possibility of the meaningless sign, the sign qua sign and utterly irrespective of any use it is put to, thus is expressly and dramatically shown to be an inversion of the ancient conception of music; and this apparently entirely accidentally (for Meillassoux certainly does not have any polemic in mind here with Pythagoreanism or Neoplatonism). For Aristides Quintilianus, music is inherently meaningful, with a veritable pluripotence of significance that is not a matter of a "link" or "correlation" with a meaning, but organic and inseparably inherent in it. This experience of music has a legacy that lasts to the present day. It was forcefully and succinctly put by Mendelssohn:

> People often complain that music is too ambiguous….with me it is exactly the reverse….the thoughts which are expressed to me by music that I love are not too indefinite to be put into words, but on the contrary, too definite [47].

The reversal which Meillassoux effects, then, asserts the priority of the meaningless over the meaningful. Since the meaningful is always meaningful *for someone*, it should not surprise us that this reversal is of a piece with the ostensible discovery of the "glacial world" by the "Copernican-Galilean revolution," the world that exists (says Meillassoux) regardless of whether it is experienced by anyone.

X. For the ancients, the moon was the boundary of one kind of changeability. This is strongly connected to doctrines concerning matter. Matter per se underlies all change, but this matter is not a being; it is rather something very close (in certain respects) to Meillassoux's facticity: any given thing's capacity to be changed into something else. We never encounter matter qua matter; we encounter a hill that may be fortified by being dug and shaped; forests that furnish wood we can make into ships or furniture, or may be burned. Natural things can act, and they may be acted upon. Some can even be acted upon to such an extent that they become another thing altogether. This is possible in the sublunar realm, where the four classical elements interact and change: fire making water boil and become air, for instance. By contrast, heavenly things—above the lunar sphere—can be caused to move from place to place; but they cannot be changed in in their being.

The regularities which characterized the heavens for the ancients are associated over and over again with an absolute destiny, a rigorous and unbreakable decree. From Africa to China, the notion of this fate or destiny recurs, for obvious reasons: the heavenly bodies do in fact apparently repeat their courses. Out of this arose an intricate body of astrological doctrine that underlay (though it was far from exhausting) the practices of forecasting the future in the ancient world. The gods themselves are known to have been considered subject to this cosmic order, which constrained even their great power. Thus, Zeus is bound by the decrees of the Moirae; Indian deities subject themselves to ascetic discipline in a context which involves enormous cycles of time; in the Sassanid era, the god Zurvan is said to have sacrificed "for a thousand years" to obtain offspring, in the form of Ahura Mazda and Ahriman; and in certain Sumerian myths, supremacy belongs to the god who holds possession of the Dup Shimati or Tablet of Destinies. This smattering of examples derives from enormously diverse cultural,

46 Ibid.

47 Felix Mendelssohn, Letter to Marc-Andre Souchay, 15 October 1842.

geographical and historical settings; I only adumbrate this list, whose instances could be multiplied and systematized, to indicate how widespread is this intuition that some lawlike context exists in which even the gods are obliged to act.

Near the beginning of the twentieth century, F.M. Cornford traced[48] reflection on this pre-existing regime of cosmic order and explained the arising of philosophy in this context. According to Cornford's thesis, which is still remarkably viable after over a hundred years of intervening scholarship, the PreSocratics' abstract speculation arose within the context of Greek religion, and was in fundamental ways not different in kind. "Unless we have some grasp of [its] history," Cornford argued, "we are not likely to understand the speculation, which, however scientific its spirit may be, constantly operates with these religious ideas." For Cornford, even under the impact of philosophy, this religious cosmology remained in effect in its broad outline across the ancient Near East and Mediterranean world. It did, however, occasion more radical questioning, which Cornford does not explore; and this from two separate directions.

There was, first of all, a skepticism or incipient nihilism, which insinuated that the apparent stability of the heavens was an illusion; the emergence of everything out of the void was happenstance and could be undone. This position, in a number of variants which did not concur with each other in every respect, is associated for instance with the names Epicurus or Pyrrho. It is a properly philosophical questioning, a critique operating solely with the resources of rationalism.

But there is a different alternative which is articulated in Biblical and post-Biblical literature. As with philosophy, so too here religion comes under a profound critique; but in this case it is a *religious* critique. And here, the cosmic order, to which religion had implicitly and philosophy explicitly subjected the gods, is, on the contrary, rendered utterly dependent upon the will of God. For the radical skeptic or the radical atomist, the order of the cosmos was either completely illusory, or else completely adamant, so that there was *no* "sublunar" realm at all in which anything like the experience of freedom remained meaningful.

In the Biblical world, by contrast, the cosmic order itself is a *creature*. (That this account resembled the "likely story" of the *Timaeus*, despite certain important differences, was not unnoticed by early Christians; Augustine for example remarked that of all the philosophical schools, the Platonists came closest to the teachings of the Gospel.) As such, the cosmos in its entirety—"above" and "below" the moon, regardless of whatever other differences this border may be thought to signify—is wholly under the sway of the divine will.

There is, however, an extremely important qualification, which gradually grows in significance. While in this vision, God's power is unquestioned, it is His loving kindness, mercy, and faithfulness which come to characterize Him. In short, the decisive point is not that God is powerful, but that the power is God's. Power becomes secondary to love. The phrase "His mercy endures forever," from Psalm 136, is indexical here, and the Christian declaration of the Gospel of Jesus Christ, which read Jesus' life, death and resurrection as betokening this mercy as undercutting the severity of "Law", is perhaps the most obvious example.

The contrast between the strictness of destiny and the mercy of forgiveness is movingly enshrined in the prayer from Rosh Hashanah and Yom Kippur services:

> 'You shall apportion the destinies of all Your creatures and inscribe their verdict. On Rosh Hashanah will be inscribed and on Yom Kippur will be sealed—how many will pass from the earth and how many will be created; who will live and who will die; who will die at his predestined time and who before his time… who will enjoy tranquility and who will suffer, who will be impoverished and who will be enriched, who will be degraded and who will be exalted. But Repentance, Prayer, and Charity avert the severe Decree [49]!'

Everything in this excerpt before the final sentence indicates the discretionary power of God: everything that occurs is envisioned as being settled by the final arbitration of God's will, including the revision of previous decision—some will die "at"

48 In Cornford, *From Religion to Philosophy*.

49 *Unetaneh Tokef*.

the predestined time, and some "before" it. God's freedom is unequivocally asserted to preside over fate itself. But the final sentence, which concludes the entire prayer, asserts something astonishing: the very decree of God is potentially mutable by the purely human devotion of "Repentance, Prayer, and Charity." The far-reaching significance of this reversal—the declaration that the "severe decree" is itself alterable—is considerable and paradoxical. In terms of the mythical cosmos systemized by Ptolemy, it amounts to a radical inversion of the significance of the "lunar sphere," the border between the world of inexorable decree and the world of freedom. In the pagan cosmos, this sphere is the effective limit to what is "up to us;" a limit that is inexorable, immutable, and utterly impersonal—a limit there is no petitioning. Here, on the other hand, there is envisioned instead a cosmic order in which the rigid pre-determined decree of fate may yet itself be modified by the personal dimension. The possibility of human devotion averting the "severe decree" is grounded in the freedom, and more, in the goodness, of God; in the cosmos finding its *archê* and *telos* in the personhood of God. What the Biblical paradox asserts is that the "elbow-room" of the sublunar realm is in some manner *ontologically primary*, that it comes first, *before* the "severe decree" of necessity.

This problem, for the ancients, concerned the question of *fatalism*. Could belief in human freedom be maintained in the face of cosmic regularities?

In the fourteenth century Nicholas Oresme essentially accepted the terms of this debate even as he contending strenuously against fatalism and the astrological vocabulary and apparatus which were fatalism's chief expression at the time. He did this by arguing that the cosmic cycles would not ever recur; that it was overwhelmingly more likely that all the orbits of heavenly bodies were incommensurable, and so the probability of any recurrence of a heavenly configuration dwindled to zero. This being the case, however, meant that astrology was deprived of its theoretical basis: if no configuration of planets ever repeated, astrological predictions had no empirical ground[50].

When Oresme deploys the incommensurability of orbits, he is essentially arguing that there is an irrationality to be found in the very citadel of cosmic order. He did not in fact concede that human freedom and cosmic law are incompatible; but he argues against the absoluteness of law as a way of making a *prima facie* case that the claims of fatalism are vanishingly unlikely to hold. Oresme's critique of astrology is measured and nuanced (he did not insist that every aspect of either the tracing of celestial movements or terrestrial prediction based on them was worthless) but the point to note, for our purposes, is the link he deployed between mathematical incommensurability and freedom.

Like many moments in Medieval philosophy, Oresme's argument is a philosophical one, motivated by a religious impulse. The grounds of his intuition are informed by the Biblical paradox: freedom is ontologically prior. (This however is not to be understood anachronistically in the mode of German Idealism, as for instance a Schellingian ur-grund which harbors good and evil. For the Bible, the existence of the world is grounded in God's freedom, and all is declared Good.) This putting argument into the service of "faith" can be criticized as an illegitimate application of rationalism, or a refusal to press rationalism all the way. But it can also be seen as a recognition of the place where faith legitimately takes its rightful role. And it is notable that incommensurability serves to articulate the intuition at the basis of this faith.

XI. The term *faith* translates a number of different words, crucially Greek *πίστις* and Hebrew אֱמֶנָה, which connote the relationship between God and humanity, or—to speak slightly artificially—the faculty by which they are related. The significance of this term does not reduce to a bizarre positing of entities in the teeth of counter-evidence, as per the modern deformation of thinking. It is an ancient word, irreducibly personal, and refers both to humanity's stance towards God and God's reciprocal stance towards humanity; a close relation in the biblical lexicon is the term

50 For Oresme's work, see e.g. *A Source Book in Medieval Science*, ed. Edward Grant.

"covenant." It is necessary to underscore this in order to emphasize that the position which Meillassoux criticizes as "fideism" has precursors far antedating the Kantian "revolution." Meillassoux of course understands that there was "faith" before there was (in his sense) fideism; but the significance of the Biblical notion of faith seems lost upon him, or read through a modern lens as simply one possible content among many which could fill up the fideistic function, which is, for him, the positing of reasons *per se*. For Meillassoux, this is simply religiosity as such. For the Bible, however, the fundamental category is not the reason, crucial though that may be, but the person, who is known, and indeed exists, only in relation. This is, again, the far-reaching paradox of the Biblical world: the ontological primacy of the person.

To call the 'person' ontologically prior is not simply to say that the person 'comes first,' either in time or in principle, but also to aver that this priority is radical. It comes before the very terms by which firstness would be asserted: before *ontos* -- before Being; before firstness, before priority. To say the person comes first ontologically means one cannot speak of ontology until afterward. This is why the 'Biblical paradox' is a *paradox*.

This Biblical paradox did indeed, in one sense, turn inside-out the Ptolemaic cosmos of concentric spheres. If, nonetheless, those ancient philosophers for whom the Biblical paradox was compelling did not feel obliged to abandon that cosmology, this is not because they were incapable of understanding the stakes; much less because the Biblical paradox had somehow inoculated them against critique, as contemporary prejudice would so often have it. Neither were they merely keeping a prudent silence, as until they were emboldened by the Reformation. Rather, they were possessed of a kind of grammar which enables this cosmology to survive as a meaningful worldview, precisely because it was non-absolute. That grammar is the deeper understanding of the cosmos-as-music. This grammar has already assumed that there is a degree of "leniency" built into the strictness of destiny. Since the same heavenly bodies whose courses are determined by the heavenly spheres also sing out the music of the spheres, a music unheard by human ear but apprehended by intellect, the apprehension of the cosmos is itself conditioned by a degree of looseness. It is the intellect, above all, which apprehends the incommensurabilities of which the Pythagorean comma is only the most famous. The philosopher who apprehends the cosmos as music, knows that its "melody" or "harmony" is always a function of certain incommensurabilities being glossed over. Far from destroying the pattern, this essential inexactitude is in fact the pattern's condition of possibility. The experience—whether by ear or by intellect—of apprehension of music *qua* music, as opposed to quantity *qua* quantity, is one of a limited but crucial freedom. To experience the cosmos as music is to feel this freedom as deeper and more fundamental than the "severity" of absolute precision.

This ease works both ways; for despite the infamous case of Galileo's trial, the stance of Biblical faith towards God has never been unduly troubled by revisions in cosmology, precisely because for this faith, the cosmos was already implicitly inverted. Moreover, all the details about how exactly to formulate a cosmic description, together comprise that precision which is always counterbalanced by an acceptable looseness or a pragmatic making-do. The notion of cosmos-as-music already released the philosopher from any deep anxiety about absolute precision in cosmology. Certainly the cosmological project cannot do without precision. But precision is not, for the musician, an end in itself; rather, in tandem, but serves in tandem with approximation, it serves the end of setting forth a likely story.

If, however, we reduce music to an aesthetic quirk of animal or even merely human habits, and think of nature solely in terms of "number and mean, but not music," then we have only the severe decree of necessity; and if we recoil from it or seek to ground it in something else, we have not freedom, but only hyperchaos[51].

Hyperchaos, as an object of (ostensible) certitude, however, is preferable to any likely story for Meillassoux, who declines to be satisfied with less than the precision of the mathematically

51 See this note at the end of the article.

absolute. Meillassoux's argument against Kant, recall, was that Kant had established a kind of inescapable uncertainty in our knowledge of the world, based upon our lack of access to the in-itself. On the other hand, an ancient philosopher would have held that the case into which inquiry is conducted is what establishes the limits on our knowledge. This would not mean, however, that some matters permit certainty and others do not. To be sure, Mathematics "admits of proofs," but for Plato, mathematics is not the highest knowledge; there is the dialectical *periagoge*, or about-face. Of this conversion, Meillassoux's famous teacher Alain Badiou has remarked:

> [M]athematics is the only point of rupture with δοξα that is given as existing, or constituted. …Everything else that exists remains prisoner to opinion, but not mathematics. So the effective, historical, independent existence of mathematics provides a paradigm for the possibility of breaking with opinion. Of course, there is dialectical conversion, which for Plato is a superior form of breaking with δοξα. But no one can say whether dialectical conversion, which is the essence of the philosophical disposition, exists. It is held up as a proposal or project, rather than as something actually existing. Dialectics is a programme, or initiation, while mathematics is an existing, available procedure [52].

The question of whether the *periagoge* is an actually possible maneuver for thought, a genuinely realizable regimen of ascesis, will take one right to the very edge of the question of fideism. For if "no one can say whether [it] exists," we are clearly edging towards an overlay between philosophy's progamme of articulable reasons, and a *hypothetical* register, the question of (in a Kantian key) "what we may hope." To put it otherwise: the "likely story," operating in the realm beyond certainty and demonstration, turns out to have not just a stop-gap function; it has, rather, a position as a properly *philosophical* narrative.

Nonetheless, this hypothetical stance, this As-If, is different from the Biblical use of the word "faith." It is indeed the "form" of the religious in one sense, but the modern innovation, or catastrophe, was to separate this form from the personal mode of experience which was primary and which made it sensical. The apprehension of this form as form, in an ancient philosophical register, would have been very close to the Buddhist experience of *sunyata*, form-as-emptiness. In broad strokes, one could call it wholism as such. What Meillassoux proposes, instead, is reductionism as such—a reductionism that reduces to the bruteness-itself of brute reality. It is no surprise that this reductionism has a close but misleading relationship with modern scientism.

The Galilean revolution subsumed the sublunar world under the mechanisms of the heavens: a ball rolling down a plank is subject to the same inevitability as a planet moving across the sky, and for the same reasons. Ultimately extended, this rigidity subsumes all experience whatsoever; neuro-nihilism now asserts that every phenomenological moment is in theory describable by this scientistic determinism (with however much quantum uncertainty one cares to insert[53]). Meillassoux fully accepts the scientistic reduction of any given contingent reality. And he expressly sees this position as defining an alternative to the Idealist stance which regards subjectivity as inherent to existence:

> The real (present) world is for me only accessible through a Galilean science. This absolutely does not refute the idea that every reality in our world is in fact subjective, but it reinforces the contrary hypothesis (since) I can decree the subjectivity of the inorganic real only by hypothesis, but I can establish by way of a proof the capacity of mathematics to describe this same inorganic real [54].

52 Badiou, *Theoretical Writings*, tr. Brassier & Toscano, p. 29.

53 In this respect, a polemicist like Sam Harris (who is at one with Meillassoux in asserting the deadliness—the 'poison', as he calls it—of the notion of revelation, which he claims is simply identical with superstition) is nonetheless closer than Meillassoux is to the ancients. (Harris' readiness to countenance quasi-Buddhist meditation techniques while rejecting the reality of the *personal*, expressly denying the reality of thoughts and of freedom, is an instance of the philosophical project coming up to, and recoiling from, the Biblical paradox.) See Sam Harris, *The End of Faith; Free Will;* and *Waking Up: a guide to spirituality without religion*. Meillassoux differs from Harris in accepting the reality of thought, but he concedes the local applicability of reductionism.

54 Meillassoux, '*Iteration, Reiteration, Repetition…*' p. 19.

The argument here is Occamist, appealing to parsimony. Subjectivity *à la* Idealism could be inherent to existence, but this cannot be demonstrated by proof; it would, rather, be a supplement to an already-sufficient account. For Meillassoux, idealism can only be hypothetical, but materialism can be "proven" to be *sufficient*. What this indicates is that idealism is an index of possibility for other scenarios. This is why idealism and correlationism are close, and materialism and correlationism are far apart. It is also why there is a close isomorphism between either correlationism or idealism and the religions which philosophy has traditionally critiqued, whereas materialism has historically religion with hostility and these other philosophical stances with wary suspicion[55].

The admissibility of idealism under the heading of what "cannot be disproved," is clearly another aspect of philosophy's inevitable intersection with the limits of the articulable, and the threshold of faith. This has undergone a mutation in modernity, however: Meillassoux's notion of fideism (and indeed Kant's of faith) remain hypothetical; both are concerned with what may be permitted to belief, not in the face of counter-evidence, but without being *ruled out*. It exists under the regime of the As-If. The Biblical *faith*, on the other hand, is an experienced *relation*: an existing-for-.

Nonetheless, Meillassoux is an inheritor of the Biblical inversion. For like the Biblical paradox, his notion of hyperchaos *also* turns the Ptolemaic universe inside-out, so that when one presses as far as one can to the ultimate "metacontext," one finds not utterly rigid law (the "stern decree") but rather a kind of ultimate freedom, a freedom to be anything. That this hyperchaos is not anyone's freedom, but rather pure indetermination, follows from Meillassoux's rejection of revelation. Meillassoux turns out to be—quite surprisingly, given how existentialism seems to be practically the apotheosis of correlationism—not so far from Sartre, for whom human freedom—the aspiration to be God—foundered upon God's status as a contradiction. Hyperchaos is what is left when one rejects God's existence, but affirms the freedom which hitherto pertained to God.

XII. This pure flux does admit of a kind of mathematical description. Gregory Chaitin has provided a rigorous and detailed articulation of how the number line turns out to be almost entirely composed of randomness, in a certain well-defined sense[56]. And this randomness, in turn, has a musical analogue: the sonic continuum is punctuated by certain very specific points which are the distinguishable tones. As Viktor Zuckerkandl underscored, it is remarkable that a discrete series of tones like the musical scale should be experienced as a continuum:

> What do we hear—a progress advancing in uninterrupted continuity, or an alteration of skips and halts?... There can be no doubt about the answer: we could not hear the melody as motion if we did not hear it as continuous…. [But] look at what takes place… the tone f sharp sounds; so long as it continues, it does not move from its place, does not change its pitch; it is directly succeeded by the tone g, which, as long as it sounds, shows as little alteration as the preceding tone; in the same manner the tone A follows, and so on. There is no question of any transitions, whether gradual or sudden; tone stands beside tone without connection…where is the continuously progressing line, the symbol of continuity of motion? Stasis-gap-stasis-gap; our graph is the perfect image of discontinuity. One is at a loss to understand how this can ever be understood as a continuous process [57].

This puzzle, of the continuity of melody in time, is essentially that which led Bergson to lay out his account of time as *durée* in terms of melody; terms in opposition to which, as we saw, Meillassoux expressly defines his position. The puzzle is all the more confounding because the specification of these discontinuous tones which comprise the continuous melody is constantly beset by incommensurability, even as the tones are

55 In a more full analysis, this point would need to be supplemented by an exploration of the true, deep materialism of Biblical religion, which is unambiguous about declaring the created world 'very good.' (This implicit decision for a certain materialism against idealism is an extension of what I have called the Bible's 'religious critique of religion.')

56 See Gregory Chaitin, *The Limits of Mathematics and The Unknowable*.

57 Viktor Zuckerkandl, *Sound and Symbol*, pp 118-119. Zuckerkandl goes on to consider, and reject as superficial, the analogy of melody with the cinematic film, on the grounds that neither ear nor mind supplies illusory glissando-like 'transitions' between the notes.

defined by ratios. Aristides put the matter thus:

> Perhaps one would assume that our argument is inconsistent, since on the one hand we make our examination of the things pertaining to music by numbers, and on the other hand, we assert that the intervals are not perfectly capable of accepting these same numbers [58].

Aristides then has recourse to a rationale which again relies upon a distinction between the sub- and superlunary realms. Here as elsewhere , he expressly appeals to an unwritten tradition[59], and we are left with the dangerous task of guessing how much he is leaving unsaid.

> If we are to tell the reason for this, we must call into play a divine and esoteric argument. …Things here are composed by mimesis with the more valuable things, and … in that place a perfect and unhindered actuality is born, while here it is defective, maimed, and troublesome….Surely music itself also has a beginning from the whole universe, just like other things—to speak in a not unconvincing manner—and by its mixture with bodily matter falls away from its precision and excellence in numbers, since at least in the regions above us, it is strict and incorruptible [60].

Aristides' vocabulary whenever he speaks of such "ancient" arguments is laced with Neoplatonist tropes:

> I shall refer to an argument that is ancient, certainly, but since it comes from wise men, it is not untrustworthy. Even if it is unconvincing in other ways, at least as far as what appears to be is concerned, it undoubtedly proves true. … For the soul—while it is seated in the purer region of the universe, being unmixed with bodies—is unadulterated and immaculate and goes around together with the sovereign of the present universe. Whenever from a declension to the things here it takes on certain appearances from the things around the earthly region, then little by little the soul is forgetful of the beautiful things of that place and sinks down [61].

In particular, the term "declension," *νεῦσις*, is a technical term from Plutarch, Plotinus, and others. Priscian, speaking of this declension by which the soul loses its strong bond with the intelligible world, says that

> the connection between the two worlds is somehow slackened, and is not precise, as was its unity in the unseparated Intellect [62].

This appeal to a two-storey world is bound to seem disappointing if we take it literally, for it leaves unaccounted-for the existence of the corrupting "bodily matter" in which music is entangled "here below." That this would have been obvious to Aristides and any discerning readers is hard to deny. While I am loathe to offer too quickly a standard esotericist defense ("he doesn't mean what he seems to say here"), it is worth pointing out that it is not difficult to understand why merely appealing to a far-away realm does not solve the problem of incommensurable measures (a fact which Oresme exploited), and that this would hardly have been unapparent to Aristides. The unsatisfactory answer is not meant to remain unchallenged, but to spur the thinker on to a further series of questions and answers, culminating in a vision of affirmation. One should recall here the way Aristides cites Herodotus' citation of Musaeus' prophecy: Can he, Aristides asks,

> 'really have uttered in so short a space of time statements that so blatantly contradict one another? Far from it. It is rather, I believe, that he is covertly pointing [63]…'

This is not to say that Aristides doubted the validity of his geocentric universe, with its significant change in climate above the sphere of the moon. But the contradictions in the theory ought to point us towards a resolution beyond the letter of the text. That such a resolution may well exceed the bounds of what is expressible at all is not just a cliché of Neoplatonism; it is built into the musical matrix which affords the whole architecture of this ontology. Music is *the* self-transcending model.

A real resolution of the problem does not lie in positing a region in which the precision which fails

58 Book III, 7.
59 Cf Book II, 7, and Book II, 17.
60 Book III, 7.
61 Book II, 17.
62 Priscian, Commentary on Theophrastus' *On Sense Perception*, 26.19. (Note 62 Continued at end of article.)
63 Book III, 26. Tr. Barker.

to obtain "here" can be found. It can only lie in the acceptance of the limits to this precision. That this acceptance, however, might feel *more* precise even as it leaves behind the standards of articulability is consistent with the Platonic dialogues' repeatedly concluding in aporia. It is also aligned with what Plato says of ineffable experience of illumination:

> To the man who pursues his studies in the proper way, all geometric constructions, all systems of numbers, all duly constituted melodic progressions, the single ordered scheme of all celestial revolutions, should disclose themselves, and disclose themselves they will, if, as I say, a man pursues his studies aright with his mind fixed on their single end. As such a man reflects he will receive the revelation of a single bond of natural interconnection between these problems [64].

This "revelation" is nowhere spelled out in the Platonic or pseudo-Platonic corpus, and could not have been, since it is beyond what can be articulated. The most specific characterization offered is the notable fact that it is not characterized as an answer to the problems, but as a connection between them all. However, as beyond articulation, beyond the discourse of rationale, it also re-opens the question of faith.

XIII. Earlier I characterized Meillassoux's position as decidedly modern, in part because of his Cartesianism; I measured him, with some severity, against certain characterizations of modern philosophy offered by Strauss. This evaluation needs to be modified, however, with respect to the motives which underlie Meillassoux's work. The question *quid sit deus*, ('what is God?', or, 'what would God be?') Strauss frequently stressed, is "coeval with philosophy[65]." This is in marked contrast with much modern thought for which the issue, or non-issue, of God is too obviously "settled" to bear serious consideration. For Meillassoux, on the other hand, the history of thought is a history of combat between religion and philosophy, and

> the term 'God' does not designate one of the camps, that of religion, but names the battlefield where the two camps confront one another…. 'God' is the name given to the stakes of the struggle between immanence and transcendence [66].

Meillassoux's article "Spectral Dilemma[67]" presented the outline of his curious theology ; it has since been more spelled out in public in excerpts from his *L'Inexistence divine* (still unpublished in whole) which appeared in the appendix to Graham Harman's monograph *Quentin Meillassoux: philosophy in the making*. The gist of the argument is as follows: Those facts of the world with which we are most incapable of making peace are deaths; in particular, deaths that are "unjust"—deaths which came too early, which were of grievous brutality, whose victims were innocent. In the face of these deaths, we are met with an intolerable feeling of incapacity, and no available stance, whether of "belief" or "unbelief," resolves it for us. Traditional theism, says Meillassoux, either proffers rationale and theodicy, or else takes refuge in mystery-mongering about the dark counsels of God; by contrast, atheism is by turns sad, or bitter, or cynical, or resigned, but in no case does it offer a genuine answer; only a mood or an attitude. Meillassoux refuses both of these horns of the dilemma, and he claims to do so on the basis not of faith but of speculative knowledge. Hyperchaos means that everything is contingent, that everything could be otherwise. And because the notion of God is not self-contradictory, we may yet hope that while God does not now exist, God might yet exist in the future, and fulfil with the inauguration of a realm of justice a real recompense for all the intolerable deaths. Nothing guarantees that this God and/or justice will exist; but they are legitimate objects of hope; and (just as importantly), such a God would also be innocent of the atrocious suffering that preceded this advent.

64 *Epinomis* 991e.The issue of the disputed authorship of this passage (attributed by Diogenes Laertius to Plato's student Philip of Opus) does not concern us directly.

65 See, e.g., Strauss, *The City and Man*, p 241.

66 Meillassoux, 'Conclusion' to *L'inexistence divine*, in Harman, *Quentin Meillassoux: Philosophy in the Making*, p. 229.

67 Meillassoux, 'Spectral Dilemma,' in *Collapse* issue IV, 2008.

L'Inexistence divine concludes with a four-way set of alternative stances which can be taken with regards to God. First is the classical theist attitude: to believe in God because he exists. Second, the classical atheist stance: to disbelieve in God because he does not exist. Meillassoux rejects the first option because it is intolerable to believe in a God who is responsible for the catastrophic ills of human history, let alone those of the natural world; he rejects the second because it leads merely to cynicism or despair. A third, paradoxical, position, is *disbelief* in God because he does exist: a stance of rebellion, whose archetype if the Miltonic Lucifer. But Meillassoux articulates a fourth attitude, which he says with some plausibility has been hitherto unremarked: one *may believe* in God precisely *because* he does *not* exist.

Doubtless, one might be forgiven for meeting this argument with a baffled incredulity. Meillassoux offers no reason to expect the emergence of such a God, and indeed cannot consistently do so, since he has foresworn recourse to reasons for hyperchaos taking one swerve over another. By this same token, however, Meillassoux rejects considerations of probability or improbability as relevant criteria for philosophy. This virtual god would appear, if ever, *precisely* for no reason; it is therefore (he may rejoin) not valid to object that no reason is presented. In answer to an imagined interlocutor mocking the notion of a fortuitous appearance of God and a subsequent resurrection of the dead, Meillassoux cites Pascal's remark that such a resurrection would be no more incredible than our having been born in the first place.

Graham Harman has perceived the important shift at work in this argument:

> Rather than concerning ourselves with what is likely to happen in the world as we know it, we focus instead on the most important things that could happen [68].

This presupposes a grammar and criteriology of "importance," and Meillassoux does indeed provide such. For him, there have been three unanticipatable events: the advents of matter, of life, and of thought. When life arose, subjective experience erupted on the cosmic scene for the first time. Likewise, when thought arose, the possibility of knowledge came with it, newly born. A fourth such authorless miracle *ex nihilo* would be the birth of the divine and the instauration of a world which is, and knows itself as, the recompense for the ills of what went before, a perfect realization of beauty and justice. Meillassoux details, in far more depth than I can recapitulate, a grammar of ethics, desire, and immanence, which spells out his conception of how this unanticipatable advent can and should motivate action in the present world.

Meillassoux spells out an articulate and reasoned development of his thinking in relation to the history of philosophy, its rationale and themes, and responds to many possible objections. Nonetheless, we may remain troubled even after seeing fairly clearly what his own logic dictates. In particular, we are left with a bafflement over just how Meillassoux is using the word 'God'. It is clear that he means by this the advent of a *being*, an entity, endowed with capacities and powers—in particular, the power to effect the resurrection. But this is, for certain orientations of thought, far more problematic than the notion of such a being coming into existence; it is essentially a variation of onto-theology.

Paul Tillich has spelled out the case succinctly. Belief in such a god, a god which is an entity, he said,

> must be transcended because it is wrong. It is bad theology…[this] god…is a being beside others and as such a part of the whole of reality. He certainly is considered its most important part, but as a part and therefore as subjected to the structure of the whole. He is supposed to be beyond the ontological elements and categories which constitute reality. But every statement subjects him to them. He is seen as a self which has a world, as an ego which is related to a thou, as a cause which is separated from its effect, as having a definite space and an endless time. He is being, not being-itself. As such he is bound to the subject-object structure of reality, he is an object for us as subjects. At the same time we are objects for him as a subject. And this is decisive for the necessity of transcending theological theism. …This is the God Nietzsche said had to be killed because nobody can tolerate being made into a mere

68 Graham Harman, 'Meillassoux's virtual future,' *Continent* vol 1 issue 2, 2011.

> object of absolute knowledge and absolute control. This is the deepest root of atheism. It is an atheism which justified as the reaction against theological theism and its disturbing implications. It is also the deepest root of the Existentialist despair and the widespread anxiety of meaninglessness in our period [69].

Or, as Jean-Luc Marion has asked[70], what if the question is not whether God attains to being, but, does Being attain to God?

A second problem, not unrelated to the first, is that as a creature of hyperchaos, i.e., of the capacity of everything to be other than it is, Meillassoux's God is a function of power, rather than the other way around: a Scottist or even Occamist inversion. While Meillassoux insists that "to believe in the existence of God is not to believe in God but to believe in existence[71]," it is also the case that to believe in the hyperchaotic God is not to believe in God but to believe in hyperchaos; in effect, to believe in power—the power (capacity) to be otherwise. (Again, this power is not the power *of* anyone in Meillassoux; it is itself a brute datum.) "Believe" may be a contestable term here; Meillassoux claims to have deduced hyperchaos by a rigorous procedure, and so there is no question of belief, but a claim of knowledge. But the more serious charge is that hyperchaos is what is absolute here; the world of justice, if it ever came about, may exist as the most important or significant event ever, but it remains as foundationless as every other development; there is no sense in which it is secure, because, like everything, it is "capable of being otherwise." It is just as solid, and just as ephemeral, as every other particular contingency.

From the perspective of Biblical paradox, Meillassoux has faithfully returned to the scene of the crime. Meillassoux brings to the longing for justice a properly philosophical *eros*; but his fourth world cannot offer satisfaction of this desire, since this desire is precisely that such satisfaction would be *sure*, beyond the proclivities of hyperchaos.

Meillassoux denies that the different productions of hyperchaos can meaningfully be described in terms of probability, because he frames hyperchaos in terms of Cantorian transfinities. Given a list of n possibilities, the probability of any one of them being randomly selected, is 1/n; but, he argues, this does not obtain once one has recourse to the transfinite. Strictly speaking, however, this is not quite exact. Given a list of "all" the infinite decimal strings, one can of course produce an infinite augmentation of them via the famous Cantorian diagonal procedure. But if I am faced with a single such strand, what are the chances that it appears or does not on my infinite list? What the Cantorian proof tells us is that we do not know, and cannot know, whether it "already" appears, not even if we have an infinite amount of time at our disposal to check it against every entry. This does not mean that it does not appear, that adding it would not constitute a redundancy. In other words, this Cantorian move is an *epistemological* foundering of knowledge. But this would mean that Meillassoux's turn to the virtual actually comes perilously close to reinstating correlationism, for it was correlationism which argued that there was a meaningful sense to *suspending the question* when it came to certain ontological claims. In fact, however, it is a (weirdly) more realist correlationism than Meillassoux's covert kind, because it simply *says we do not know*, rather than that the question is meaningless. (In the terms Meillassoux has used before, it reinstates a "weak" correlationism rather than a "strong" one.)

If this is true, then it is not pointless to ask what the "chances" are of such a scenario arising as Meillassoux imagines. And the chances are, indeed, transfinitely small.

The Roman Catholic writer J.R.R.Tolkien frequently wrote of the necessity to have "hope without assurance," or "hope without guarantee[72]." This is in some measure bound up with the nature of faith, which is, in the famous definition from the Epistle to the Hebrews, "the substance of things hoped for, the evidence of things unseen[73]." What Meillassoux on the other hand offers is a kind of "guarantee without hope:" This can be said, despite his insistence that the virtual god is a legitimate object of hope, because he expressly

69 Paul Tillich, *The Courage to Be*, p. 184-5.
70 Jean-Luc Marion, *God Without Being*.
71 In Harman, p. 235.
72 E.g., J.R.R. Tolkien, *The Silmarilion*, p. 265.
73 Epistle to the Hebrews 11:1.

refuses any reason for hope at all, and yet maintains that one can *know* that this virtuality is somehow real.

XIV. As we saw, Meillassoux hinges his project upon the possibility of a mathematical account of the in-itself; and he explicitly distinguishes this from a musical account. This careful distinguishing may seem merely incidental; I believe, on the contrary, that it is essential. Meillassoux's position does not simply happen to reject the musical cosmos as one part of a pre-scientific worldview with which we have all long since had done. He must reject it, because he must posit that it is mathematics and mathematics alone which is the satisfactory language—a language made of meaningless signs—for the in-itself.

I have not here marshalled any definitive defense of this musical ontology, in either its classical philosophic, or its Biblical, forms, though I have tried to suggest why the Biblical paradox and the musical paradigm could nurture each other. Nor have I even attempted to refute any of Meillassoux's positions. All I have tried to do here is to suggest some of the philosophical stakes in the confrontation between the moderns and the ancients, with Meillassoux and Aristides as convenient foils for each other. Nonetheless, I insist, these are real stakes, and not merely "as if" stakes in a false battle. The musical ontology is not bound to an "outmoded" geocentric map of the solar system or imaginary crystal spheres in the heavens. It is an ontology that asserts that there is a fundamental *almost-ness* to any representation of Being, that this gap between What Is and What Is Said is inherent in What Is. This is why ancient philosophy, to which this cosmology was native, was felt to open upon the question of faith; whereas modern philosophy, which insisted that a flush fit between representation and reality is possible in principle, has had either to feel itself at war with faith, or reduce it to the purely hypothetical mode. For the ancients, philosophy was a way of life and a set of spiritual exercises, which cultivated a philosophical ascent of the soul, an ascetic philosophical experience. What makes Meillassoux's thinking so bracing is (in part) his awareness that such spiritual stakes are real. His project is challenging and dizzying in its audacity and scope, as great as the ancients—refreshingly so, it must be said; it is only that he attempts, as a modern, what the ancients did, and here, his premises are outstripped by his ambition.

In one crucial sense, I maintain, Meillassoux is right: philosophy as it culminates in correlationism (and it is indeed a culmination, for Meillassoux) does yield to "religiosity as such," i.e., to fideism. What Meillassoux misses is that the Biblical paradox is, like philosophy, a critique of "religiosity." Only, whereas philosophy opens upon the pure "form" of faith—without content—Meillassoux would rather give us pure content—brute content, as it were: contingency as such. The Bible's critique, however, does not pursue "reasons," as philosophy does, but the Person. Because of this, it was able to navigate the upheavals of cosmology. Ancient philosophy does indeed lead up to the question of revelation; and music is the grammar of this *preparatio*. Modern philosophy accepts the formal critique of religion by the Bible, but not the experiential one; it thinks it can stipulate it and move on. Thus "faith" in modernity becomes a formal "as-if", and is either uncritiquable but empty, or (if there is any content to it) superstitious. Meillassoux's novel move is to reject all this, in favor of "content" without form—pure contingency. The question, then, is whether philosophy can possibly be satisfied with such a conclusion and remain philosophy. If not, there remains the ancient path of ascesis.

Aristides' work comes towards the conclusion of a very long process, in which ancient philosophy gradually settled and codified its responses to the waning of a mythical worldview. Myth waned under the rise of reason[74], and philosophy, which aspired to be reason raised to perfection, nonetheless attempted to deploy reason in such a way as to maintain access to the experience of the world which had been given by myth. In the Pre-Socratics and especially in Plato, we see the rising of this mode of thinking; in the late-classical neoplatonists (and other schools), we see its rear-

74 And particularly as the technology of writing spread; that this was clearly seen by Plato is evident from the myth of Thamus and Theuth, as recounted in the *Phaedrus*.

guard as it moves into a kind of scholasticism, while in the background, as Plutarch relates, the oracles fall silent. Aristides' work shows some of the best assets, and some of the gravest liabilities, of such thinking. On the one hand, it is over-confident and over-specific in its tables of correspondences, and one cannot help but be struck by the knowing way Aristides lays out his various examples of numbers found in the astronomy, biology, grammar, and ethics, as though it was clear that the same principle was manifest in each instance[75]. One almost feels in reading Aristides, at least as regards this over-specificity, that he has pressed precision too far and lost an appropriate approximation. One must be cautious in this assessment, however, for it is not clear whether, or to what degree, Aristides believes these correspondences to be literally true, and on the other hand how far he holds them to constitute a "likely story." The philosophically compelling moments in his work are not these lists of exact correspondences but the spirit which inspires them, which takes for granted that something like this is so, because Nature is neither ignorant nor unrefined, even though there are aspects of it, and of the soul, that are irrational.

Aristides' notion of the relationship between philosophy and music corresponds to this distinction. As he says, learning may be distinguished into two types:

> One type preserves the rational part in its natural freedom, through the impartings of judgment both making it sober and undefiled. The other treats and tames by habituation the irrational part….the sovereign and teacher of the rational part is philosophy; music rules the irrational [76].

Thus the transition between music and philosophy itself would also be a kind of lunar orbit, as one moved from "things below" to the perfectly harmonious. But again, this harmony is not actually the numerically precise accounting of every last proportion; it is the preservation of beauty by way of an appropriate elision of such sub-perfect perfection. Finally,

> Philosophy provides a real escape from the processes of becoming; if something unwelcome occurs, it endures it calmly and bravely, and counts nothing evil or shameful save wickedness and enslavement to vice…. It is as its greatest consort and attendant we must name music; and by comparing as it were the lesser and the greater mysteries, we must assign to each its proper value and honor. … music transmits the beginnings of every kind of learning, philosophy the extremes [77].

Beginning with Descartes, modern philosophy abandoned, because it no longer understood, the foundation of philosophy in music; it opted for mathematics. Meillassoux has the merit of understanding this new foundation and seriously attempting to ensure that it is solid. As we have seen, he even expressly distinguishes it from music. This paper has attempted to make clear what the stakes are in such a decision.

Despite, perhaps, initially seeming merely a curiosity, the contrast between these two thinkers, one of whom is among the last of the Ancients, the other a rising contemporary, is thus neither a five-finger exercise in relevance-mongering, nor an academic essay in idleness. Nor is it meant simply as an illustration of a "Before" and "After" for the supposed watershed of "secularism," "disenchantment," or whatever moment one takes to decisively cleave the ancients from the moderns. The comparison is set in motion by the deployment, in both cases, of particular key-terms, in ways that weirdly and crucially coincide before, just as crucially, diverging. I must leave it an open question to what degree the juxtaposition of my two writers is a matter of necessity—whether it obeys some secret law—or whether it is purely caprice. Indeed, since the very key-terms here are those surrounding contingency, the question of the

75 It is, especially, his great specificity which is problematic here. In this respect, Aristides' work is not unlike that of a very different rear-guard, the authors of the *Brahmanas* in India, whose confident explanation of each detail of a sacrifice as meaning precisely such-and-such seems likewise a symptom of an over-reaching instinct to codify what is nonetheless a legitimate sense of correspondence.

76 Book II, 3.

77 Book III, 27, Tr. Barker. Mathiessen, however, has:

> *Though we count both of them as lesser in comparison to the greater mysteries, we must afford to both philosophy and music their proper worth and honor,*

which raises the possibility that Aristides was aware that philosophy itself opened upon a mode of experience beyond it.

"meaning" of these coincidences must be, itself, a matter of "tuning"—of Meillassoux's contingency ringing in a particular harmony with Aristides'. To ask how precise such coinciding is, then, is not an objection to the project, but an application of it.

Appendix:
Continuation of notes 30, 51, and 62

Note 30, continued: The translation here, by Mathiessen, is not without its problems. Note, first, that the phrase 'but not music,' as placed, is ambiguous. It could mean:

Either,

(1) 'To have organized these things, but not music, so palpably through numbers and means…'

Or,

(2) 'To have organized these things so palpably through numbers and means, but not through music…'

It must be acknowledged that the grammar of the Greek does suggest meaning (1), which is in fact close to how the passage is rendered by Andrew Barker:

'To suppose that these things are so clearly constituted on a foundation of numbers and means, and yet that music is not, would be the mark of someone outstandingly stupid and uncultivated in his nature.'

Barker's translation differs from Matthiessen's in another respect: the 'nature' that is found to be stupid and uncultivated, ignorant and unrefined, is that of the observer who is making the mistake, rather than Nature as they mistakenly conceive it. Barker's translation, however, supplies a possessive pronoun ('his') which the original lacks; strictly speaking, one should translate something like '…as far as nature is concerned' or 'with regards to nature.'

It is to be noted that in such renderings, the term 'nature' itself is ambiguous. This ambiguity affects the passage as a whole. While it is true that meaning (1) above seems the more immediately obvious rendering, the question is far less clear in the context of Aristides' whole treatise. In light of Aristides' citation of Panaceus (see previous note), it seems reasonable to conclude that Aristides means to think of music as pertaining to everything without exception. Compare Barker's translation of the same passage (Book I, 1): '…to bring together in harmonious relation all natural things.')

I therefore treat as stipulated that Mathiessen's rendering of III.8 is representative of Aristides' general position, even if philologically disputable. (I am indebted to David Z. Crookes for his insight into these issues of translation. He is not, of course, responsible for my final interpretation.)

Note 51: The question of fatalism and freedom is not in the forefront of Meillassoux's concerns in his polemic with correlationism or his attempts to found scientific reasoning. This is again an indication of the distance between him and the ancients. (It also serves as another index of his difference from Bergson, for the Bergsonian notion of *durée* serves as a fundamental grammar for freedom. This is seen even in the very title of Bergson's first book *Time and Free Will*, in which his account of *durée* was initially laid out. Meillassoux's own most extensive engagement with Bergson to date is in his paper 'Subtraction and Contraction: Deleuze, Immanence, Matter and Memory' in *Collapse* vol III (Nov. 2007)).

The 'master argument' of Diodorus Cronus purported to demonstrate that a contradiction followed from three commonly-granted premises: (1) Every true proposition about the past is necessary, i.e., it is now impossible for it to be false; (2) The impossible does not logically follow from the possible; (3) What neither is presently true, nor will be so, is yet possible. Although all three taken together are incompatible, it is possible to maintain any two of them and discard the third. According to Epictetus, Diodorus went on to grant the first two premises and formulate an argument against the last, thus concluding that 'Whatever is not true now or will not be so in the future is impossible.' This obviously identified possibility with inevitability; it was, in short, an assertion of fatalism.

Meillassoux on the other hand affirms premise (3)—he explicitly insists that it is possible for many possibilities never to be realized. And he clearly affirms premise (1), since his argument about ancestrality relies upon it. On the other hand, he may certainly be read as denying premise (2), since he denies that anything ever has sufficient reason for occurring; though he does acknowledge that there is such a thing as the impossible—it is simply the unthinkable as such.

Note 62, continued: The verb 'slacken,' χαλάσθαι, here and in the excerpts below, recurs frequently in Neoplatonist doctrine. It means to 'loosen' (as from bonds), to relax a tether, or (significantly for our purposes) to lower a pitch with a tuning peg.

Likewise the term νεῦσις, 'declension,' has a metaphysical or metapsychological significance in Neoplatonism (as, too, in Gnosticism), but its earlier provenance is mathematical; it is still used today with the same meaning it had among ancient Greek geometers: it denotes the construction of geometric figures with a compass and a *marked* straightedge, which may rotate and slide. With neusis construction, many classically insolvable problems (insolvable, that is, with unmarked straightedge and compass) may be answered, including, notably, the doubling of the cube and the trisection of the angle.

Compare Ps.-Simplicius, Commentary on Aristotle's *De Anima* 241.8-12:

> not the soul's activity alone but also its essence and the highest part itself— of our soul, I mean— is somehow dissipated and slackened and as it were sinks down in the inclination towards what is secondary. It does not entirely abandon itself…but it no longer preserves its own purity, with the result that it keeps itself simultaneously the same and not the same.

This should be read alongside Simplicius on Xenocrates in the Commentary on *De Anima* at 408b.32:

> Xenocrates, who himself calls the forms numbers and knew that every form was indivisible, while what undergoes motion is divisible and altogether posterior to the forms, exhibits the mean position of the soul by means of both the extremes, calling it a moved number, since it is not simply form but has descended as a whole into division, not totally divided but not remaining as form, having reached a mean position by in a way slackening and loosening the indivisible unification. For this reason he did not say that it simply suffered motion but that it was self-moved, in order to make clear the distinctive character of the mean position, being a slackening, as the term signifies, but not so as for it to be torn from the self.

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THE HALLELUJAH EFFECT:
Archaeology of a Harmony in Three Parts

Babette BABICH

Prelude

I could not have written The *Hallelujah Effect*[1] without my epistemological interest in Friedrich Nietzsche's studies of rhythm and quantitative metrics on the language and sound of ancient Greek or the *Spirit of Music*, just as Nietzsche spoke of it in the extended title of his first book on *The Birth of Tragedy*. Nor could I have written *The Hallelujah Effect* without being interested in modern technology and the working phenomenology of radio and related media on our consciousness, individual and collective, or without being interested in the 'space' as Theodor Adorno put it, of music as such. In addition I had to be interested in the complex reasons that Nietzsche had for writing as much as he did write about Beethoven and voice (rather than and as would be more obvious or in conformity with expectations, about Wagner and voice). And, perhaps above all, none of this complex background would have led to the writing of The *Hallulujah Effect* without the serious and dynamic engagement of Ernest McClain, someone to whom some of the above themes would be at best peripherally connected but who nevertheless responded and quite literally played with all of my efforts, every step of the way.

No book gets written without something of that last and what was and what remains intriguing is just how broad McClain's influence on other thinkers, in other ranges and intersections of thought remains, from ancient philology to music and musicology in recondite or so-called 'esoteric' spheres[2], to physics, if one includes the work of the late Patrick Aidan Heelan (1926-2015).

What follows is a kind of a retrospective prolegomenon is a three part account of the three sections of *The Halleljuah Effect*. Initially composed in May 2011, beginning on Mayday and in and through direct email correspondence with the musicologist, the late Ernest G. McClain, which is why this 'archaeological' account is dedicated to him. The 2013 book as such grew out of that correspondence and developed the video lecture recorded at Fordham University, first published online on May 19, 2011, and the text of a discussion which appeared in the online music journal PerfectSound in Autumn of the same year[3].

On the Technological Reproduction of Sound: Music, Pop Opinion, and Antiquity.

What is the technology of sound? What is the technology of the technical reproducibility of sound? Writing after Martin Heidegger's questioning of *The Origin of the Work of Art*, and at the conclusion of a list of a variety of technological means of reproducibility in the various spheres of art, Walter Benjamin reports smoothly (which is just how we scholars like it[4]) that 'the technical production of sound was

1 The trouble, of course, with anything that has as many facets, even three is too many, is that the reader in general tends not to be able to make much sense of it. That is yet another reason for my gratitude to the late Ernest McClain for being a first reader even as he protested his own limits: this is a model for all scholarship. I have subsequently published the section on radio on its own as Babich, 'Adorno's Radio Phenomenology' and I am preparing a discussion of 'Nietzsche and Beethoven' for a separate publication.

2 See McClain's *The Myth of Invariance*, with its introduction by Sigmund Levarie and see too Jocelyn Godwin's '*The revival of speculative music*' as well as and including a broadly ranging bibliography, Graham Pont, '*Philosophy and Science of Music in Ancient Greece*.'

3 See the list of references below.

4 Benjamin's essay on the artwork easily outmatches Heidegger's essay on work of art for influence and as Hans Ulrich Gumbrecht and Michael Marrinan observe, Benjamin's essay on technological reproducibility 'is probably the most frequently cited and most intensely debated essay in the history of the academic humanities in the twentieth century' in Mapping Benjamin, pp. xxx-xxiv.

tackled at the end of the last century[5].' Theodor Adorno adds complexity (which is not how we like it), raising the phenomenological question of the techno-mechanical transmission of music in his *The Current of Music* and with yet more complexity, going back to the 1870's and indeed to 770 BC with the earliest system for the technical reproduction of sound (i.e., the Greek invention of 'truly phonetic writing' as Ivan Illich argues[6]), there is the spirit of music at the heart of Nietzsche's *The Birth of Tragedy*. It is in order to raise the question of the technical reproducibility of sound, that we begin in the present day, with YouTube and other media, turning then to radio and the current of music in the era of the second World War, and finally exploring the implications of reading, as Nietzsche does (and Illich only accords with Nietzsche's reading of), ancient Greek as a technology for reproducing sound: the spirit of music.

'*The Hallelujah Effect*' was presented in non-digital form (i.e., 'live') for the first time at the February 2012 meeting of the Society for Phenomenology and Media at National University in La Jolla[7]. In addition to offering a discussion of two songs by Leonard Cohen, namely Cohen's *Hallelujah*, covered by seemingly everyone (specifically analyzing a YouTube video of k.d. lang at the 2005 Juno awards) and Cohen's Suzanne (analyzing Nina Simone's cover in a 1968 YouTube video[8]), the spoken presentation also foregrounded the phenomenology of advertizing (that would be what cognitive psychologists call 'priming' and marketers and others call 'branding[9]") reviewed by way of the Frankfurt school's analysis of the culture industry particularly Adorno's studies of radio and what he called the 'current' of music, finally touching upon (just because the theme is more esoteric) Nietzsche and Beethoven[10].

To summarize once more, the first thematic consideration offered a reflection on the effect of the effect (as it were) in *The Hallelujah Effect*. The second thematic undertook a review of the phenomenology of musical sound, highlighting Adorno's radio physiognomy[11] while the third and culminating theme sought to highlight, as has heretofore not been explored as such, the 'live' spirit of music in Friedrich Nietzsche's study of antiquity, including his studies of quantitative or perhaps better said quantifying rhythm as he also studied the relation of ancient Greek musical drama and word.

That lecture as such would be developed into a published book in June of 2013 as *The Hallelujah Effect: Philosophical Reflections on Music, Performance Practice and Technology*[12]. There, the effect of the 'Hallelujah effect' corresponds to the specific mediation of music in the age of digital, broadcast media, i.e., music reproduced technologically in recordings of all kinds and via

5 Benjamin, '*The Work of Art in the Age of Mechanical Reproduction*,' p. 219.

6 In Ivan Illich's his paean to the technology of phonetic writing, *In the Vineyard of the Text*, the first 'truly phonetic writing was a one-time invention, made in Greece around 770 BC.' This 'one-time invention' was characterized as Illich explains with beautiful concision, by the use of 'signs for both consonants (which are obstacles to breath) and for vowels (which indicate the color given to the column of air that is spirited out of the lungs).' Illich, *In the Vineyard of the Text*, p. 103.

7 The San Diego lecture presentation, 'The Hallelujah Effect - k.d.lang, Nina Simone, Leonard Cohen: Reflections on Radio, YouTube, and Adorno' grew out of the video lecture Babich, 'The Birth of kd lang's *Hallelujah* out of the 'Spirit of Music.' See for a short account written on the basis of the current discussion, 'On The Hallelujah Effect: Priming Consumers, Recording Music, and The Spirit of Tragedy,' *Proceedings of the Society for Phenomenology and Media*. (San Diego: National University Press, 2015). Pp. 1-12.

8 I discuss Nina Simone further in my 'Hallelulah and Atonement,' which appears in a new anthology edited by Jason Holt, *Leonard Cohen and Philosophy*.

9 For a discussion of priming see the work of John Bargh, 'What Have We Been Priming All These Years?' and see for branding, Ap Dijksterhuis, et al., '*The Power of the Subliminal*' in addition to Tor Nørretranders' *The User Illusion*. There are numerous marketing studies, but see, for example, Susan Gunelius's recent publication in Forbes, Susan. 'The Psychology and Philosophy of Branding, Marketing, Needs, and Actions' as well as the ad marketer, Douglas van Praet's *Unconscious Branding: How Neuroscience Can Empower (and Inspire) Marketing*.

10 *The Hallelujah Effect* (see note 5) is also and to be sure all about *Hallelujah*, not only the 'holy' and the 'broken' Hallelujah, as Leonard Cohen invokes King David's own Hallel psalms but also the musically constitutive phenomenon that is Handel's *Hallelujah* Chorus in addition to (this testifies, as an homage to the late Ernest McClain, the musicologist who corresponded with me as I first worked my way through the project), an American hobo song, *Hallelujah, I'm a bum*.

11 Adorno, *Current of Music*.

12 See here the first part of Babich, *The Hallelujah Effect*.

a variety of media: on radio, on television, online, especially YouTube, shared on Facebook, Twitter, personal and news blogs, etc.[13] Reflection on the effect in question calls for a reflection on the object medium of media itself, especially on the 'web' and the corresponding means or medium of access: on one's computer, mobile phone, tablet, etc.[14] A little later, I would write a short chapter on some of Leonard Cohen's songs, as it became clear that if Cohen's *Hallelujah* inspired the book, and if I talked about the song in length, there was, there will always be, more to say on a poet and his voice[15].

I. Prelude to the Effect: Sonic Branding, Media, and Online Porn.

The Hallelujah Effect studies the 'effect' of Leonard Cohen's *Hallelujah* beyond its effect on radio or television as this is simultaneously echoed on YouTube, which last is itself a media version of the eternal return of the same in potentia (i.e., depending, and this is meant as a joke, on the number + 1 of relevant hits), an echoing effect related to the resonant frequency that is a pop music 'hit' or a viral video and so on. Some commentators echoing the scholarly analysis of mimesis speak of 'memes' but the term (like the remix recoil of what the electronic remix artist Steve Goodman calls 'memetic' music) emphasizes not only repetition but also evanescence, poised to suppose such things no more than passing fads, like a mental hula hoop, here today, gone tomorrow. Beyoncé, Miley Cyrus, *Game of Thrones*, any latest thing.

I argue that the 'effect' of Cohen's song in and through its many covers works the way advertising works, that is by way of what psychologists (and marketing consultants) call 'priming' and marketers name 'branding,' whereby the covert is the key. Priming or branding works because we do not notice it. Thus our thoughts are entrained, whereby, as John Bargh argues (in a convergent argument with Nietzsche's critique of causality), we suppose per contra that our will is free[16].

Hence *The Hallelujah effect* is about the working of 'entrainment,' 'effected' as it is and most efficiently so by sound: one's brain, so to speak, rather more literally than one thinks, aligns itself with certain wavelengths[17], heard and unheard (the sonic including both subsonic, as Goodman points out, and the supersonic), which last 'unheard' can also be same as saying what is in effect 'heard' as Sharon Weinberger writes 'only by you[18].' The phenomenon has military applications and these in turn bleed over into popular culture, as Goodman has explored this from a number of perspectives studying both the military uses of sound as well as the military influences on club and pop music[19]. As

13 And the focus on Cohen's *Hallelujah* explores the 'effect' of a song that remains so persistently popular that a parish priest in Ireland varied the words to sing it from the altar at a wedding catapulting himself into the now proverbial fifteen minutes of YouTube fame. Father Ray Kelly, has been able to enjoy an effect of his own, as Ronan McGreevy reports: 'Singing Priest Fr. Ray Kelly Offered Deal with Music Label.' I thank my friend Tony Beavers, philosopher of technology and media, for first bringing this to my attention as did my sister, Frania Shelley-Grielen, who both alerted to me to this via Facebook.

14 We should pay attention to the word 'media' and as such just as the late Friedrich Kittler does in his *Gramophone, Film, Typewriter*. To be sure, academic parsings of what 'media studies' studies vary widely. Thus in addition to my own focus on Günther Anders (who remains unfortunately still too little discussed and that means that he remains even less understood), as well as Theodor. Adorno, who is at the heart of this book. See Guy Debord and the Situationist and Anarchist approach to both politics and art (today there is some sense of this influence in the work of Jacques Rançière, as well as Jean Baudrillard, Jacques Ellul and Paul Virilio, in addition to and in spite of his datedness and sometimes his insularity, Ivan Illich as well as the for some reason difficult to integrate Michel de Certeau who continues, not always to advantage for an understanding of his work, to be read together with, as he himself read with profit, Foucault and Bourdieu. And there are many, many others one can name in addition to the well-known Marshall McLuhan, Neil Postman, and, a bit more insightfully, Jerry Mander, *Four Arguments for the Elimination of Television and so on*.

15 See Babich, 'Hallelujah and Atonement.'

16 See Bargh & B. D. Earp's 'The Will is Caused, Not Free.' Nietzsche argues a similar point contra free will, but to the extent that Nietzsche's argument is offered in the epistemological context of his critique of causality, the parallel is obviously limited.

17 See Persinger, 'On the Possibility of Directly Accessing Every Human Brain by Electromagnetic Induction of Fundamental Algorithms' as well as the various contributions to Radványi, ed., *Psychological Operations and Political Warfare in Long-term Strategic Planning*.

18 Sharon Weinberger, 'A Voice Only You Can Hear: DARPA's Sonic Projector.'

19 Steve Goodman, remixer and electronic music artist as well as a philosopher of what he names memetic music, writes that '[s]onic warfare is therefore as much about the logistics of imperception (unsound) as it is perception. The bandwidth of human audibility is a fold on the vibratory

all of us are well aware, beyond club music culture and apart from the military, everyday television shows employ distinctive opening sound sequences and certain commercial jingles are associated with certain brands.

This is your brain on drugs.

Like drugs, the joke in question can be varied: *this is your brain on line: on YouTube, on Facebook, on Twitter*.

Consider the sing-song AOL ping reminder, itself the eponymous title of the 1998 Hollywood film *You've Got Mail*. The movie itself offered a popular cultural riff on the erotic subversion that drove the AOL experience. But the study of the connection between the larger phenomenon (and the even larger profit margin) of internet pornography, underscoring the internet as source for our absorptions has yet to be exhausted.

The very point of media is mediation, that is connection as the journalist Vance Packard and the communications scholar Marshall McLuhan both observed themselves drawing upon a phenomenon already adumbrated (and to be sure inaugurated) by Edward Bernays[20]. Both the sexual and the social drive or 'effect' priming, programming, branding. This is how advertisement works and, to argue contra a popular internet meme, the internet is not so much full of cats as it is full of, suffused with, percolated through and through by ads, ads interrupted by further ads, ads everywhere, all the way down, and all the way up. Where ads once crawled across the page (drawing the user to track them and thus fixate upon them to click them away), today studies of eye movements are used to determine placement such that we are often unaware of the bill-boardification, as it were, of the webpage as indeed our email inboxes, spam filter and all).

Today we turn to the internet for nearly everything, including online dating, from hookups to marriage, a turn amplified by the praxical detail provided by mobile phones and tablets (the internet is hardly limited to anything as deskbound as a computer). This accessibility is especially important for surfing for erotic distraction or porn or just in the airbrushed fashion of the 2013 version of *You've Got Mail*, the film starring Joaquin Phoenix's relationship not with another movie star, but with his computer's operating system. *Her*, a film highlighting the allure - and above all the convenience - of the disembodied presence of the voice heard but unseen and dramatizing the autistic thrill of zipless phone sex starring Phoenix co-starring (in effect) with a California style breathy, sexy, playful, nonmale threatening, i.e., Siri-style female voice (Scarlett Johannson). Add the even more comprehensive phenomenon of sexting/texting to which one must also add the rules for the same (rules unspecified but which must nevertheless be learned and observed). The romantic dynamic of *Her* depends on machine stimuli, now no longer as obvious as AOL's vinyl sing song ping as in *You've Got Mail*, and consequently all the more effective. Thus *Her* features a sonic tapestry that signals the future world, not only the whispering vocal programmers (a wish fulfillment of the Internet of Programmed Things, these objects dream of prowess voice recognition programs and apps never actually have in actual use - Dragonfly Software fantasies) a soundset of background tones and reminders, a softer chirping reminiscent of the evolution of digital chime effects as we have learned to distinguish the same from our congress with cellphones and Windows (mostly). The stimulus effect in question is what interests me in *The Hallelujah Effect* and it the stimulus effect that interests marketing researchers just as it also

continuum of matter. With reference to military research into acoustic weaponry, this molecular backdrop will be mapped as a vibratory field into which the audible is implicated' (Goodman, *Sonic Warfare: Sound, Affect, and the Ecology of Fear*, 9). Goodman offers this definition of the title of his book: '*Sonic warfare* then, is the use of force, *both seductive and violent, abstract and physical*, via a range of acoustic machines (biotechnical, social, cultural, artistic, conceptual), to modulate the physical, affective, and libidinal dynamics of populations, of bodies, of crowds.' (10). See also Clive Stafford Smith's 2008 article in *The Guardian*: 'Welcome to "the disco": For US interrogators seeking to disorientate and break Iraqi prisoners it's 'torture lite' - rock music played at excruciating volumes' as well as Weinberger's 2007 contribution to *Wired News*, already cited: 'A Voice Only You Can Hear: DARPA's Sonic Projector.' See, too, the various contributions to Radványi, ed., *Psychological Operations and Political Warfare in Long-term Strategic Planning*.

20 See Bernays, *Crystallizing Public Opinion* as well as Ellul, *Propaganda*.

interests the military[21].

One waits for the stimulus and today, with or without AOL, one's phone signals a tweet 'connection,' a Facebook comment, an appointment, a text. One looks for - one checks, as one says - one's messages, one's email, Facebook account, blog posts, etc. Anticipation *and* satisfaction are the same. Indeed, one of the reasons for the great success of Twitter is that we at no point want to find ourselves without a text to read. We do not want to miss a connection, a mention, a message and to that end, although studies of twitter do not usually highlight this: we arrange to 'feed' ourselves tweets as texts, just as we might subscribe to blog posts so as to clutter our own inboxes on our own initiative - really, it can seem when surfing the net, like the metonymic analogue of channel surfing that anything will do.

The checking activity reinforces the feed activity because the mini-event, the little tone or vibration is all the reward we get and, as advertisers have learned to their profit, as Twitter also knows: this is all we need.

In *The Halleljah Effect* I quote Slavoj Žižek's reminiscences of the French minitel I myself also encountered as a student in Paris[22].

Žižek deconstructs his reminiscences by way of his well-known tactics of (over)sharing intimacies and trademark Hegelian-cum-Lacanian termonology:

> '…the idea of minitel is 'sex is an Other.' You type in your password but you do not communicate with a paid prostitute, you communicate with hundreds of people doing the same thing you are doing. So you pick up one of the messages and you do it: you send in your own message to him or to her - you don't know to whom, that is the charm. You only have the family name: it may be a man or a woman. You send your message to someone you don't know, you exchange dirty messages: 'I will do this to you, you will do that to me.' The point is that people became obsessed by this. Lacan says - he even uses vulgar terms - that if I'm speaking now about fucking it's the same as if I'm fucking. This is literally realized now in France; sex can be purely the matter of a signifier of exchanging dirt [23].'

The little Minitel boxes have gone the way of all eclipsed modern technology, maintained beyond its time for the sake of capital investment (it worked), and then finally discarded as the detritus that is the human legacy to the world - accretions of badly degrading plastic carcasses, wire guts - but the dream of the minitel continues unchanged in texting and online dating, especially online porn (and phone sex as depicted as automated simulacrum - what is the difference between anonymous or chat room phone sex and the simulation of AI or computer sex in *Her*, the Hollywood film mentioned above, and which contrast of convenience the film itself duly illustrates).

I quote Žižek quoting Lacan 'that if I'm speaking now about fucking it's the same as if I'm fucking[24]' invoking such vulgarity less to demonstrate the supposed open-mindedness of academic word choice[25] than for the sake of illustration. Phenomenologically speaking, talking of fucking, but especially sexting, and in particular the example of online porn may be seen to illustrating the Hallelujah effect - less as a disjunction between either the holy or the broken Hallelujah, as in Cohen's lyrical invocation, than a jaded and exaggerated one.

Talking about fucking, phone-sex, sexting, chatrooms, minitel, etc, rendezvous via phone apps or other online instant dating services (even Craigslist in a pinch), is the exchange. The extension of communication in time is part of the problem of our dependency on checking in. If the stakes are erotic, the effect is addictive. Students and lovers

21 See the overview and contributions to the US government publication, edited by Frank L. Goldstein and Benjamin F. Findley, Psychological Operations: Principles and Case Studies, particularly Col Dennis P. Walko's Psychological Operations in Panama During Operations Just Cause and Promote Liberty, p. 249-277.

22 See Jaron Lanier's *You are Not a Gadget* for his reminiscence of the 'Well' experience that he himself cofounded, and thus for a kind of virtual media archaeology in the mode of nostalgia. See too Lee Siegel's *Against the Machine* as well as, if in general, Patrice Flichy, The Internet Imaginaire, especially pp. 101ff.

23 Žižek, *Flash Art*. For a political discussion of the French Minitel qua telephone based electronic information technology and in terms of its distinction from the web or internet, see Moulaison, 'The Minitel and France's Legacy of Democratic Information Access.'

24 Žižek, *Flash Art*.

25 And, to be sure, there is a growing philosophy of porn dedicated to demonstrating such open-mindedness.

(and politicians) have to check their cell phones, someone 'good' might have signaled a possible connection, something (unclear what) might be in the offing, but to keep the game alive it will not do to answer too slowly (and, so the adumbration of time also holds, as these are all potentially endangered or vulnerable liaisons of possibility/impossibility, too quickly). The result is a constant on-edge, enduring sense of breathless connectivity explored in *Her* but also more heavy handedly in the surprisingly uncreative 2011 documentary film, *Connected* [26].

The problem is the disconnect of online connections. Hence there are a number of recent articles and books urging that one disconnect, urgings duly offered online, presupposing that one be connected enough to click on a link and predisposed to disconnect (the tip-off is always that this is recommended from time to time, for, as Jean Baudrillard pointed out some time ago, if we do not count Günther Anders, as we should, this is an already determined schema).

Reality is integrated, *branché*, connected. And in order to be part of what counts, as Baudrillard and your career advisors alike underscore, one needs to be connected as well: academics cannot function without the internet, not if they want to be admitted to universities, succeed at them or teach at them, not if they want to publish their work or indeed and to an ever increasing degree, read the work of other scholars.

Here the baseline is the having of an email account or a Facebook account (or the increasingly appealing thought of not having one: shall we call that the Garbo Effect?) or Twitter or whatever, but and just the ubiquitous business of the internet. The one specifically internet business (as if tailor made for it) and which has continued to profit despite economic downturns of one kind or another is, of course, the porn business. But my concern in the context of *The Hallelujah Effect*, and here I extend the reflections of the book itself, is not the profitability of this particularly ancient industry.

The problem with porn, where pornography is the graphic depiction of erotic themes (we all know this and this has been known since antiquity) and porn is the online version of the same, is the absence of the erotic. There is an obvious parallel to Heidegger's observation that the essence of technology is 'nothing technological.' And in its current instauration, (online) porn is addictive and the problem with addiction, once we get past the moralizing, is that it is flat, a flattening that leads to more of the same: attenuating and numbing. There are images on images, web-sites on web-sites, and the enticement is to find and click on them all. To this degree, porn thus has nothing to do with desire as Lacan recognized and which Lacanian insight Žižek happily channels along with Lacan's vulgarity, while adding his own.

As addictive as it is, the porn effect, like the Hallelujah effect also changes minds and sensibilities, just as advertising or 'branding' does. As studies of its cognitive effects suggest, especially as experienced online, porn tends (not accidentally) to draw the user away from the user's 'own' desires (however constructed these may be in the garden variety concatenation of power plays and subject structures) to desires not the user's own. Thus some social scientists and psychiatrists have made this point by analyzing the porn industries' deployment (and often innovation) of internet engineering tactics, tactics that have everything to do with the nature of the 'search' as such (and its reward structure in the brain) and hence everything to do with the nuts and bolts of internet search engines.

The same internet marketing strategies highlighted in Eli Pariser's *The Filter Bubble* work on us in the porn industry. Yet the language of a bubble can be misleading, suggesting rather more benign tactics than those actually in use. As the recent debacle on Facebook's social engineering experiment illustrates, it is rather less that things are 'hidden' from us or 'filtered' per se than it is that users are deliberately and quite specifically funneled or directed towards specific sites, like the actively directed 'feed' employed by Facebook's illustrate the Hallelujah effect - less as a disjunction

26 This is the limitation that is perhaps obvious for such recent (all-too) quick films such as Tiffany Schlain's 2011 *Connected: An Autobiogarphy of Love, Death, and Technology*. But quite without such limitations, see Sherry Turkle's *Alone Together*.

social networking engineers[27]. Similarly specific, and similarly surreptitiously,

> 'The online porn industry uses many strategies to promote use of their sites, including Pop-up windows (trap users in an endless loop of porn), Home page hijacking (planting a Java script command on computers to change the user's default home page to a porn site), Stealth sites (a variety of techniques, including buying up expired domain names, exploiting common misspellings, or using wellknown names of companies or artists) and Hidden key words that are picked up by search engines (Porn operators bury key words, including brand names of popular toys or names of pop artists, in the code of their Web sites to lead children and teens to their sites) [28].'

Beyond this one can also refer to the studies beloved of cognitive science enthusiasts - and this standard (and standardizing) deployment of behaviorism may be tracked all the way down to the advertising executives on Madison Avenue and all the way up to the campaign managers at the White House. Still more importantly from a phenomenological-existential point of view, one can see the results of this standardizing priming in one's own (and in others') actions and responses.

Thus we have all the illustration we need in anyone (whether that anyone be observed in one's own person or that of another) with access to a cellphone or an iPad or other tablet or laptop. As a user one tends not to notice one's absorption in the branding dynamic but signaled via *non-responsiveness*, these effects are easiest to note in others. Indeed, my students, most of them in their late teens and early twenties, can interrupt their own multitasking online and texting activity (in class) to condemn the behavior of their nearest cohort of still-younger youth: family members or friends. One part of the reason we take note of such attention paid to texting behavior is that the phenomenon of texting - sending and receiving - inspires anxiety by its nature: unlike a phone message which requires that the recipient not only actively check his or her inbox but just as actively or deliberately choose to listen (or not) to any message sent, the next message comes across already in its entirety (whole that is to say but only if successfully 'sent' - and there is an entirely separate phenomenology that attends upon the anxiety of composing a message or series of messages only to see a spate of 'message not sent' messages on one's mobile phone, frozen texts, like preserved dead letters), already there to be read. Corresponding to this phenomenon of known reception, as it were, there are rules for texting and responding, like the often unobserved rules for email reception (and often delayed response), adds to this pressure. For young people in love, communicating by cellphone (though this includes the old as well and age differences are often exaggerated by media), the rhetoric of desire is adumbrated via time delays and over- (and under-) responsiveness to texts sent.

Two Cousins (Günter Anders and Walter Benjamin), Freud's Nephew, Edward Bernays, and the Psychology of Priming via Radio and Film:

Text and online connections mediate (and at times trump) the so-called 'real world,' the so-called 'reality' of which the phenomenological critical theorist Günther Anders had analysed at length in his 1956 *Die Antiquiertheit des Menschen* (The Obsolescence of the Human). Writing, as Adorno cites him, Günther (Stern)[29]Anders reviewed the

27 Adam D. I. Kramera, Jamie E. Guillory, and Jeffrey T. Hancock, 'Experimental Evidence of Massive-Scale Emotional Contagion through Social Networks.'

28 Manjeet Singh Bhatia, 'Internet Sex Addiction – A New Distinct Disorder,' p. 3. See too the recent article from Mark D. Griffiths who has been writing on this for more than a decade, Griffiths, 'Internet Sex Addiction.' Thus the journalist Naomi Wolf observes that the online images and sounds of porn have come to stand in place of reality, pointing out that 'Today real, naked women are just bad porn.' See also Wolf's more recent article, 'How Porn is Destroying Modern Sex Lives.' Agreeing in part with Andrea Dworkin's radical insights into the dangers of pornography, Wolf observes that the result is not however and as Dworkin had imagined an increase in real-life rapacity (although one can argue that ordinary sexuality has become perfused with ordinary violence). Rather, so Wolf argues, men have become less 'libidinous,' this is the jading effect of porn (and we have study after study on this, from rats to college university volunteers before ethics committees began to put a brake on this) which Wolf compares to a kind of programmed Pavlovianism: 'if you open your focus to an endless stream of ever-more-transgressive images of cybersex slaves, that is what it will take to turn you on.' Ibid.

29 Günther Stern or Anders is better known via his relation with famous other names (as Walter Benjamin's cousin and Hannah Arendt's [first] husband). A student of Husserl and Heidegger among others, Stern/Anders crossed paths with Adorno in his academic trajectory (although he

spectral qualities of radio experience as this is increasingly 'mediated' for us today not merely via radio but cellphone and tablet everywhere we go.

The 'spooky' language used in Anders' so-titled essay 'Spuk und Radio' articulates phenomenology of the ghostly construct of broadcast media and Anders himself wrote as a student of Husserl and the same Martin Heidegger who would himself later write about epoch of his day as 'world image' and even later claim at the conclusion to his 1949 Bremen lectures that our capacity to hear and to see diminishes or succumbs 'through radio and film under the rule of technology[30].' It was Anders who wrote about radio in the early thirties just when Heidegger offered the lectures that became his reflections on 'The Origin of the Work of Art,' both in advance of the reflections of Walter Benjamin on the 'Work of Art in the Age of Technological Reproducibility' exploring both photographic and film media.

The same monetized image is all corporate media needs to control the world (and our thinking along with it). But if the synesthesia connecting sound and image may be dissonant for those who like to keep their c.v.'s neat and clean, writing about sound, writing about image, writing about film, media inevitably involves the whole array as Anders detailed in 1956 book, presciently well in advance of McLuhan or anyone else analyzing the consumer's self-dedication to his or her own production as an unpaid homeworker, creating him or herself as the mass human being[31].

It is in this biographical array that one may support that it is arguably no accident that in the same 1927 in which Heidegger published *Being and Time*, Hollywood brought Al Jolson to the screen in *The Jazz Singer* which is regarded as the first feature length 'talking' picture. Thus Anders as a refugee from Nazi Germany would travel to California's Pacific Palisades and Hollywood, led as he was by insolvency (he was not as cossetted as Adorno or indeed the great majority of other academic expats, which surely affected Anders's sensibilities or overall moodiness), to write one of the few texts from his 1956 book to have been translated into English, 'The World as Phantom and Matrix[32].'

To use the language of cognitive psychology: we are 'primed,' although the point of priming, that is in order for priming to be priming is precisely that we not notice it[33], and indeed dismiss it as irrelevant if we do - for nothing touches our assurance of our own free will and consequently our conviction of our own utterly autonomous selfdetermination[34].

In behavioral (or cognitive) studies in psychology, 'priming' is a technical term but it also features in marketing or advertising research and scholarship. It applies to what Adorno characterizes as the 'ubiquity' standard as the very same standardizing impetus for conditioning of all kinds, especially qua 'covert priming' which,

himself would disclaim the assertion sometimes made that this collision was what blocked his habilitation). On his writing on music and in addition to Adorno's reception of his work, see Reinhard Ellensohn, Der andere Anders.

30 Heidegger, '*The Turning*,' p. 48.

31 Exceptions to this priority would be Horkheimer and Adorno with their 1944 *Dialectic of Enlightenment*, in addition to Adorno's already mentioned publications on music and radio.

32 Anders, 'The World as Phantom and as Matrix.'

33 See for a retrospective, Bargh's 'What Have We Been Priming All These Years?' as well as Dijksterhuis, et al., 'The Power of the Subliminal' in addition to the more classic philosophical authors discussed below.

34 It is to make a related but different point that Steve Goodman's *Sonic Warfare* points to the relative absence of scholarly material on classified, speaking from a military or political perspective, affairs. As Goodman writes exploring the important role of fear in the poltico-social dynamics of modern capitalism: 'Much speculation can also be found in conspiracy theory, which is only natural when research related to the defense industry is concerned. These sonic fictions and urban myths can form a starting point for a more careful philosophical investigation. For, in addition to the paranoid sensationalism that enlivens these often spurious accounts, they remind us that the sonic (and unsonic) body is always poised precariously in a processual disequilibrium with the acoustic environment, and that even minute perturbations of this environment can set in motion resonant events and generate and provoke unforeseen cultural mutation.' (12). But to speak of 'fictions and urban myths,' and 'sensationalism' is already to play into the same point Goodman is making, which is a great convenience for public relations or what Edward Bernays' (who indeed originally set both the tone and developed the methodology for the 'public opinion control' or what we call 'spin' and which he) called the crystallization of public opinion. See here and this is also relevant to the Facebook social engineering experiment which only adds empirical confirmation to Bernays' theoretical study *Crystallizing Public Opinion* which was itself first published in 1923.

as already noted, is the only kind of priming that can matter as such[35]. To this day we speak of 'prime' time and talk about radio and television 'programming' without reflecting on either the origins or the literality of such terms[36].

Arguably dating back to Plato's *Republic*, the theorizing of specific strategies for the manipulation of a target audience may be dated in its contemporary mass media manifestation to the early decades of the 20th century. To this extent, priming works[37]. Part of the mechanism has to do with the difference between conscious and unconscious motivation and controlling popular opinion is a concern for both government and the advertising industry. Thus Edward Bernays, author of *Crystallizing Public Opinion*[38], drew upon the Freudian theory of the unconscious (Bernays happened to have been Freud's nephew)[39].

In marketing, motivational researchers have gone on to develop Bernays' 'crystallization' in advertising practice[40]. As one 'primes' the pump, one primes the consumer. Priming is used as a term in cognitive psychology although, in what may be a bleed over from marketing research, the term branding is becoming popular. Some social science scholars will insist that there is either no priming effect at all or else that there is but it is general or minimal[41]. Perhaps the idea here is to reassure confidence in consumer choice and free will, i.e., *rational choice*, itself the name of a leading research trend in the social sciences. Thus a number of studies (*nota bene*, in disciplines not contributing to marketing research) argue for the inefficacy of priming in a fashion akin to US academic confidence in the non-existence of propaganda in United States (we don't have any of that, only other 'evil' nations like Nazis or the Russians do) or those who maintain despite the clear corporate control of government on both legislative and judiciary levels that the popular voice is adequately expressed by the expedient of voting at four year intervals or, failing that, by signing online petitions.

Yet advertising 'priming' works, contra our persuaded sense of our own and secure 'free will[42],' down indeed to the brand itself - given to be sure that the subject or target is in the mood for what is so primed. This efficacy is not unlike the similarly physiological efficacy of Viagra. "Drink Coke" does indeed lead to consumers buying this particular soft drink. The point of the advertising campaign for drinking Coke produces consumers primed to consider Coke as an option throughout a lifetime, in observed as in observed instances, and of course
well beyond the duration of any given social science study on human subjects.

II The Sound of Music and Radio.

Along the way, the first part of the book inevitably centers upon the question of desire for men and for women (by and for men and women and so including gender differences). This itself is probably the most troublesome aspect of the book and it needs its own discussion and that in turn also requires its own background[43].

35 Such 'covert' priming corresponds to what Vance Packard triumphantly reported as making news in 1956, featured as it was on the front page of the *London Sunday Times*. As Packard reports, 'certain United States advertisers were experimenting with "sub-threshold effects" in seeking to insinuate messages to people past their conscious guard.' Packard, *The Hidden Persuaders*, p. 62.

36 Adorno to be sure would explore just this 'current' in his thusly titled *Current of Music*. Adorno, *Current of Music*.

37 Bargh, 'The Most Powerful Manipulative Messages Are Hiding in Plain Sight.'

38 Bernays, *Crystalizing Public Opinion*.

39 The connection between Bernays and Freud (and as Anna Freud) is the topic in the context of the exploration of political manipulation and control in Adam Curtis's 2002 BBC television documentary, *The Century of the Self*.

40 See Dichter *The Strategy of Desire*. See further Stefan Schwarzkopf and Rainer Gries, eds., *Ernest Dichter and Motivation Research* as well as Franz Kreuzer et al., eds., *A Tiger in the Tank*. We live our lives watching television modulated by that same practice and this is part of the reason Anders could analyze the political transformation of consciousness by all-too-commercial and very effective means. As to tribute to this efficacy, the 2011 obituary printed on the last page *The Economist* (this is exceeding pride of place in this journal) highlighted Dichter's transformation of marketing by means of sex as 'Retail Therapy.'

41 See Bower, 'The Hot and Cold of Priming.' The focus on free will (and determinism) is key to this debate and to some of the controversies surrounding it.

42 Bargh, et al., 'The Natural Unconscious,' including patently enough 174 references. Reprinted in part under the senior author's name as 'Priming Effects Replicate Just Fine, Thanks.'

43 See the original video version on this essay for this emphasis but see too Babich, 'Great Men, Little Black Dresses, & the Virtues of Keeping One's Feet on the Ground' as well as

With regard to the seduction of digital media per se, it is useful to review not only Adorno's *Current of Music*[44] and Anders' phenomenology of listening[45] but also the 'recording consciousness' offered by H. Stith Bennett in his sociological phenomenology of musical practice, *On Becoming a Rock Musician*[46]. Bennett's study is fairly unengaged, apart from the nearly ubiquitous allusions to two of his words: 'recording consciousness.' Bennett's study of the rock musician (rather precisely on the model of what Bruno Latour calls the 'actant'[47]) offers a phenomenological approach to sociology, an approach that makes, as perhaps only Latour would be able to recognize, Bennett's style of doing sociology both rigorously scientific and stunningly creative. For reasons that have only to do with what I elsewhere analyze by way of a reflection on the continental analytic divide in philosophy and as this crosses over into the social sciences, scholars are increasingly losing touch with such applied specifically phenomenological methodologies. Indeed if one fails to consider Bennett's approach from an 'actant's' perspective (on the recent model of Latour's 'modes of existence' of the thing[48], and indeed hermeneutically via an ethnography of so many rock musical actants as Bennett follows them in his archaeology of becoming a rock musician) one overlooks what Adorno called the 'natural sound' of music.

And to say such a thing already upsets all musicology specialists, especially the graduate students among them reviewing books they have trouble reading, for which trouble they blame not the limitations of their own background or their own interests but they do fault the author. As a reviewer's technique, this always works (it gets the review finished and submitted) but as Nietzsche says in his *Ecce Homo* reflecting not only on his own readers but on reading in general, 'Ultimately no one can extract from things, books included, more than he already knows.' Knowing all the truths they need to know about music and culture, including political culture, such readers wonder why one has to bring Adorno in (ah, no, not: again)? Being 'afraid' of Adorno, as middle-aged and elderly ladies in New Jersey fear the sharks, it occurs to members of both groups that discretion recommends that one keep clear of the sea. A complicated style, like a complex thematic puts one off and these features characterize Adorno (but hardly all of his readers many of whom simplify the object of their expertise).

What is at stake in the technological 'current' of music in the age of technological reproducibility reflects the mediation of media mentioned at the outset above. Hence if the initial or first part of the book is on 'priming' or what is colloquially called 'entrainment' by those who who write about music and cognitive science, the second part not only looks at Adorno's critical approach to what he called the

the related and more 'branded' (and briefer) version, 'Women & Status in Philosophy.'

44 See Adorno, *Current of Music*.

45 See here Günter Anders' early essay on listening, published as Stern, 'Zur Phänomenologie des Zuhörens.'

46 Perhaps one reasons for this is that an excerpt from his work has been anthologized in more than one place and as a result tends to be taken for the whole. I refer to Bennett's 'The Realities of Practice.'

47 Here I am using this in Bruno Latour's sense. See for a relevant discussion of the influence of the American sociologist Harold Garfinkel on the course of his own work, Bruno Latour, 'Biographie d'une enquête.' Also online as 'Biography of an Investigation' Latour seeks by way of an autobiography of the fortunes of his own reception to highlight the 'modes of existence' of the thing, in Latour's case that would of course be a hybrid thing: technology *and* technique and (although Latour to be sure does not mention him) that is also what Michel de Certeau called a 'tactic' (Latour's disarming Anglophilia leads him to give pride of place to Harold Garfinkels' 'microtechnique,' more technical as it was than de Certeau's reflections).

48 If Stuart Elden can advert to the mixed qualities of Henri Lefebvre's style in his work on Lefebvre as being at least part of the reason for the limits of Lefebvre's reception (limitations Elden's own work has done a great deal to remedy), the same could be said for de Certeau if the mixed quality is de Certeau's case is simply a diffuse note, rather as if filmed by Jean Cocteau. On Lefebvre (not de Certeau), see Elden, *Understanding Henri Lefebvre*: 'As anyone who has read him will attest, his was not the most fluent of styles.' (Ibid., p. 5) Elden goes on, almost in the spirit of Kittler's *Gramophone, Film, Typewriter*, to point out the erotic dynamic of Lefebvre's dictation-style for the writing of his books, 'Lefebvre sometimes used a typist to transcribe monologues on topics, which makes sense of the way in which his work is repetitive, digressive and meandering. … Many of these typists were 'women he desired or loved', for whom he would improvise his ideas…' Ibid. Just as Giorgio Agamben has argued that *Being and Time* was composed under the 'sign of love' (see for references and discussion, the first chapter of Babich, *Words in Blood, Like Flowers*, Elden reports Lefebvre's conviction that the improvisation that was the result produced some of his 'best' work.

culture industry but also the working 'current' as he spoke of it or mediation of the same. If, and by the nature of things, those who know about music know little about cognitive science and vice versa, those who know about music and neuroscience know even less about Adorno despite the fact that his research was itself part of the wartime-instigated Princeton Radio Project directed by Paul Lazarsfeld. My approach in *The Hallelujah Effect* undertook to explore a given phenomenon, namely the working of Cohen's *Hallelujah*, via a return to the things themselves, in this case literally so, which is to say the medium of the song as heard, exploring the experience of transmitted music as this was once experienced not by me, but by those like Adorno (and like Günther Anders and to be sure also like Heidegger and Arnheim) who first experienced both radio and recorded music, as it were, for the 'first time,' by which I mean that they had an experience of music both prior to and after Benjamin demarcation of the 'Age of Mechanical Reproduction.'

This last set would include Nietzsche, but only at its cusp or edge. This, among other things, is why it is complicated to raise the question Nietzsche raised about an even more difficult to imagine situation, namely that of ancient Greece and the birth (out of the 'spirit' of music), of the tragic art form. I take this up in the third section below and it is important that Nietzsche who loved music as he does (without it, he says, 'life would be a mistake') does not simply cut to the chase and give music direct genealogical honors but interposes the spirit - that is to say, as we shall see, spirit as inspirited intonement, tone as such: the sound of the letter. This is the life of music but it is also its breath, *spiritus*.

If Nietzsche's writing on the 'spirit of music' tends to go unnoticed, Adorno's claims regarding the space of music and sound has tended to be refused, to an extent, perhaps, one no longer hears what he is talking about and one certainly does not see his point. This usually goes along with defending jazz against Adorno as if he had an attack worth worrying about. But what Adorno was talking about had to do with the sound-stripe, to get all Princeton Radio Project technical about it[49]. In other words, he is talking about the decays, the losses, the 'depradations', in Robert Hullot-Kentor's terminology, undergone when music 'is subjected to[50]' radio production, not in Adorno's era of bad technology and bad media but exacerbated by what scholars like Greg Milner (very technically) name 'perfect sound[51],' just 'when broadcast artifice endeavors to appear as pristine nature, when sonic copy lays claim to origin, when music on the air acts as the reproduction of an original[52].' And to be sure, in the case of popular music we listen to 'recorded sound' that is already 'predominantly electronically sampled to begin with[53].'

Hullot-Kentor goes on to observe that Adorno's investigations depended upon on the specific material or basis of radio itself:

> '[v]irtually all radio music presented performances of live vocal and instrumental music …[r]adio, in other words, consistently claimed to reproduce live music as natural sound, every bit as live in the home and with the intention at least of a transparency of transmission, as if the radio itself played no part [54].'

Historians of recorded sound explore the specific conditions for generating this perfect sound as does Adorno: fitting the orchestra to the constraints of recording technology and even designing studios around that same technology (the engineering of musical sound only continues this project today). But the ideal of what Adorno calls 'natural sound' hardly holds any longer (in

49 In addition to specific studies of Lazarsfeld and Adorno, on which much more is needed, see Rose Rosengard Subotnik on the complications of style and sound with respect to Adorno in her exceptionally rigorous study, Developing Variations and see too her *Deconstructive Variations*.

50 Robert Hullot-Kentor, 'Second Salvage,' p. 112. The text corresponds to the editor's introduction to Adorno's largely composed in English (with George Simpson, as McClain lets us know) *Current of Music*.

51 I refer here to Greg Milner's *Perfecting Sound Forever* in addition, more generally, to Chanan, *Repeated Takes*, see further Arved Ashby's *Absolute Music*, as well as with specific reference to Adorno, Alf Björnberg, 'Learning to Listen to Perfect Sound.' Björnberg himself draws upon Roth's 2004 *Capturing Sound* to which Milner's study is likewise indebted as is Peter Wicke's 'The Art of Phonography.'

52 Hullot-Kentor, 'Second Salvage,' p. 112.

53 Ibid., p. 112.

54 Hullot-Kentor, 'Second Salvage,' p. 113.

part because we don't and perhaps cannot know what that would mean in Adorno's sense) and Anders' description of what he called the 'ghostly' tone of radio invoking as he does the auratic aspect of sensed invisibility as this today could hardly be more dominant in our wireless world (even if the sense of invisibility is reduced from loudspeakers to earphones that have grown or evolved from minimal earbuds to fashion headware, featuring radio-broadcasters headsets, themselves connected via Bluetooth to a personal ipod or other device - this is the point of individualization that Anders would insist upon, manifest today in the more rather than less *autistic* quality of online social networking. Today, and usually without adverting to the fact of it, what we hear on the radio (when we bother to listen to radio at all, rather than programming our own programming, as it were, programming ourselves, with our own recorded 'playlists'), are nearly always pre-recordings broadcast on the radio, and in this sense the musical work of art is technologically reproduced or mediated in several takes or at several removes.

Using the example of k.d. lang's and Leonard Cohen's and other popular musicians live concerts or 'tours' (the term already begins to give the game away), I point out by the middle of the book that 'recorded' sound is no longer just for records or radio or ipods or what have you but increasingly characterizes so-called 'live' concerts as well. The Broadway stage experience would not be what it is without amplification (meaning microphones and loudspeakers) and although opera fans vigorously deny the possibility (as vigorously as most opera houses), some use of broadcasting technology can and has been defended even by the Metropolitan Opera itself by appealing to the same acoustic constraints acting on the space of sound that Adorno invokes. As Kai Herada points out:

> 'Both the Metropolitan Opera and the New York City Opera have similar systems - the Met system includes Meyer CQ-1s, 650Ps, UPA-1s, and MSL-4 cabinets; additionally, a number of loudspeakers are semi-permanently mounted in the domes above the audience, perfect for otherworldly effects [55].'

Herada's article is titled 'Opera's Dirty Little Secret' and indeed it is. Thus opera fans are inclined to insist that there is no use of acoustic 'enhancement,' this would be what makes opera opera and all opera fans turn out to share Adorno's views in this regard, willing or not. Yet how opera fans can insist on this is a bit odd given that almost every opera fan also is well acquainted with the much loved live radio broadcasts scheduled by New York City's Metropolitan Opera, not to mention CDs made at particular performances. But, *mirabile dictu*, there are no broadcasts without microphones. The New York City Opera, defunct in the interim, was perhaps the most unapologetic in its use of acoustic 'enhancements' but, and as Herada notes, such 'acoustic enhancement systems' are in fact already used 'in many opera houses, including the Deutsche Staatsoper in Berlin, and the Hummingbird Centre in Toronto, which both use a LARES system; many theatres, such as the Ahmanson Theatre in Los Angeles, the Royal National Theatre in London, and the Vivian Beaumont Theatre in New York City, use the SIAP system[56].' Indeed, the recent Broadway hit, After Midnight, only coincidentally also starring k.d. lang for a very short stint, was staged at the Beaumont on Broadway, and depended on microphones for the After Midnight effect, an effect including all the Harlem jazz scholars pretend to have been the sole object of Adorno's ire. And, as Herada explains SIAP stands for 'System for Improved Acoustic Performance' and it

> 'operates by placing microphones in close proximity to the direct sound source on the stage or in the orchestra pit [to] process a mix of the signals with delay, phase, and frequency-response changes. The resulting signal is fed to a large number of loudspeakers placed in extremities of the performance venue [57].'

Effectively undetectable, large as life and twice as natural, such sound systems acknowledged to be in place for acoustic 'enhancement' (this would be the correlate to transhumanism in an operatic context) are also inevident enough that WQXR's Fred Plotkin insists that opera houses be required, a la the Monsanto debate, to declare the use of such

55 Kai Harada, 'Opera's Dirty Little Secret.'

56 Ibid.

57 Ibid.

technology (which opera houses are reputed to do and which opera houses roundly deny so doing) on pain of being proclaimed 'guilty of defrauding their public[58].'

A Phenomenology of Listening in Adorno's Current of Music: Radio Broadcasts, Live and Recorded, Microphones and Loudspeaker "Fidelity".

Like many others, the author herself did not first "hear" Leonard Cohen's as sung by any of those who made his *Hallelujah* famous (that is the effect), most especially the inimitable John Cale who, so I contend, really makes the song, setting the bar for as well as influencing all subsequent covers (including as observe in the book, Cohen himself). No, I first heard Leonard Cohen sing his *Hallelujah*. Leonard Cohen, 'himself.' I put the 'himself' in quotes because of course I didn't hear Cohen in any sort of 'real-life' sense: I was never, and this is the key point to Adorno's thesis on the space of sound, where he was (never mind the use of microphones and loudspeakers and the lot of 'public defrauding' devices to which Plotkin objects). A recent apology for the use of new technology for recording musical performance, rock, pop, and classical, points however to the very same relevance of the space itself, the very space of the place itself to which I argue that k.d. lang listens when she sings. Thus John Meyer defends a development of a new technology (paradoxically named with one of the titles of Adorno's essays) called the Constellation Acoustic System that is even more invisible than the SIAP system of acoustic enhancements already in place for use (but denied as being used) in various opera houses throughout the world. As Meyer informs Anthony Quint, 'After 50 years of recording, we've learned that musicians actually interact with their spaces … It's not a source and reverberation event - it's one event talked about in two different ways[59].'

Meyer and his recording forbears might have well have taken a tip from Adorno because what they are recording (and note that opera houses need never admit to ever using such technology and be right on the money in so doing) is the room itself. The result is a virtual concert hall, 3-D printing in sound:

> 'Constellation is a custom-designed installation of a large number of miniature microphones and selfpowered loudspeakers connected with advanced digital processing software. It should be emphasized that this is not sound reinforcement. There is no pickup of direct sound; Constellation deals only with reverberation and early reflections. A powerful computer is processing 20,000 echoes per second to, in Meyer's words, 'make them go where they're supposed to go[60].'

58 Fred Plotkin, 'In Opera, There is No Such Thing as a Magic Mike.' Plotkin reprises an exchange from a previous post in 2011, noting his opposition and the opposition to his opposition, writing 'This reader felt there is a place for mikes in the opera house: 'Microphones, and actually all sound reproduction, are not what it was a decade ago. Digital signal processing has made huge strides in pickup and reproduction quality... Singers do need some practice with mikes. They can pick up every inhale, every throat clearing, every click of a heel on a floor and every costume ruffle. All workable issues. But the electronics can also pickup more vocal nuances, that last tear-drop of a note, that would be lost in the large volume of air where there is also the competition with the sound of the orchestra.' This commenter would not agree: 'I am firmly in the camp of 'No' regarding microphones in opera performances. I have heard miked performances in opera houses, and they are invariably very discernible, distorting and distracting. Microphones cheat opera-goers of one of the great aural experiences - the glory of a human voice, producing sound that is awe-inspiring, shimmering in the air like a jewel and soaring through (usually) vast spaces. As one whose earliest opera experiences were semi-open-air (the Cincinnati Zoo - yes, Zoo!) with no amplification, no amplified speaker (even with modern advances in sound) could capture accurately the glory of the sounds produced by great singers and a great orchestra, and no amplification was needed even in that open-air venue. It has been my experience that even recordings fail to accurately capture certain indefinable qualities of voices that make them great. I believe that the younger generation will become captive opera audiences simply because un-amplified opera is so different from the concerts they normally attend; it is so amazing to hear that unique sound, unavailable even in the best HD broadcast or on DVD! I hope opera houses will not cheat future generations out of this incredible experience by miking performances. And I agree that the ones that do should make full disclosure of the use of 'sound enhancement', or be guilty of defrauding the public!'

59 Cited in Andrew Quint. 'Big Voice in a Small Room.' Yet, the head of Meyer Sound Laboratories, Meyer also offers support to both Plotkin's and the observations of Hullot-Kentor and myriad audiophiles: 'On the subject of combining multiple run-throughs at the editing stage, Meyer adds: 'You can tell when they're spliced no matter how well you do it. Because you're taking two different events. The rooms change every ten minutes; humidity and temperature are never completely static. When we take a measurement of a room, store it in the computer, wait ten minutes, and store it again, it's a different room. Not very much different, but enough to change how we hear reverb.'

60 Quint. 'Big Voice in a Small Room.' As Quint informatively details: 'It's complex all right. (sic) For example,

This point, and one can find other examples too, complicates while also confirming Adorno's claims in *The Current of Music* and to a lesser extent in his *Introduction to the Sociology of Music*. And to this same extent dependent upon pro sound techniques for its recordinding production, today's music as remix artist Kode9 (Steve Goodman) as already cited and many others also have observed[61], depends on loudspeakers. Indeed, Goodman's most recent release Martial Hauntology, is characterized as 'an audio research box set which "investigates the properties of newly emergent super-directional speakers when coupled with infrasonic devices[62]."'

We tend to take technical reproduction or mediation for granted, which is to say that we pay no particular mind to the where or the how of it. Thus when one says that one has heard a singer or a particular song one does not mean that one heard the song 'live.' To that extent my clarification at the beginning of this section that I heard Cohen himself but wasn't actually in his presence is a superfluous one. All we mean is that that we have heard some recording, somehow.

Reproduction is the important word here. One need not have direct access to a given physical recording 'itself' to hear a recording and today's digitally mediated 'recordings' are virtual, reproducible, transformable, translatable (though and to be sure, never without loss, just as the expert John Meyer quoted above could also attest however much his new innovations may be expected to go some way towards mitigating that loss).

For Adorno and for Anders too, as rigorous object phenomenologists, it is all about the radio as such, the object or thing that is a radio as a tuning and as an attunement. Songs broadcast on the radio have a quality all their own. Nor do we have trouble recalling the particular acoustic quality of radio: we know the sound radio makes and we can notice the quality of radio transmission as such and very specifically, as Adorno observes, when tuning a radio set. But if new technology can (and it is important to note that it does not always) make this a less common experience - even satellite radio drifts, and the internet has a rather famous instability from time to time - we nonetheless notice the 'sound' or the fact of radio transmission when the weather changes or else when we are moving about or driving, due to the call letters constantly announced as well as the strong/weak character of broadcast sound. Like our cellphones to this day this fading quality of wireless broadcasting is one of the things that has not changed since the beginning of publically accessible radio broadcasting in 1922[63]. What we may call this quality of inherently unstable or bad reception is also behind the rise in texting - in an obvious instantiation of technology subverted by its users and subsequently transformed and ultimately evolving in tandem with that original subversion. When I was a student, phone phreakers were fond of playing with acoustic tones to fool phones into thinking that coins had been deposited when none were[64]. Like some music compositions that famously replicate the clacking sounds of a typewriter as well as the bell interface that told the typist to return the carriage at the end of the line, the rotary dial is part common culture: *Dial M for Murder* has a certain invocation of a certain time - the rotary dial had its own restrictions, like all technology and one of them was time, and

the Constellation implementation at Zellerbach Hall at the University of California involves 44 microphones and 105 (!) speakers of various sizes. The result is far more acoustic flexibility than can ever be achieved with physical modifications to a hall - curtains, shells, moveable walls, chambers, etc. - and at much lower expense.'

61 As Goodman is a performance artist, so I prefer to take his point at his word, even though the theorist Michael Chanan says the same, as does one of Adorno's editors, Robert Hullot-Kentnor and a number of others.

62 Glenn Jackson, 'Kode9 and Toby Heys Launch AUDiNT, Release "Martial Hauntology" Box Set.'

63 Such public broadcasting should be distinguished from Nikola Tesla's invention of the technology needed for radio broadcast or Guglielmo Marconi's first transmission in 1895. Indeed, it is fairly difficult to talk about the 'first' radio broadcast as this is a highly *national* (and not accidentally politically charged) concept but also because it is essential to define what one means by such a first event: broadcast radio or wireless transmission requires the technology, this is the reason I have acknowledge Tesla's contributions in the text above as, in practice it requires both a broadcast source and a receiver tuned at the right time. A lot of things have to come together to ensure that this works as it should and to this day radio broadcasts may not dispense with studio engineers.

64 The song dates from 1984 and in the mid to late eighties I was using both ordinary letters and intra-university and by the end of the decade and early nineties I was using dial up email connections for correspondence.

this is how earlier phone phreakers wreaked their mischief (this would have been before my time), utterly lost, a very tiny point that makes all the noir difference there is, in *Push M for Murder*. What cell phone users soon discovered is that unlike landlines, cellphones work very badly at being phones but very well at being portable secretaries, able to take a message at length, any length, though the shorter (this is also a matter of flip out screen sizes and such like) the better. The key was not the keypad, though pushbutton technology was the sine qua non for this subversive transformation, but the phone's capacity to redial and its memory of a number or, as in this case both a number and a message to be transmitted as soon as the signal returned. Older people continued to wander around, shouting into their phones, asking 'Can you hear me?' while younger people simply sent text after text oblivious to the lack of service but also using a technique that could often connect in the small bursts of connectivity where a duplex, first me, then you, back and forth of a phone conversation would fade into frustration, repeated and failing, again and again[65]. Texting transcends the limits of phone connectivity, likewise, and here the machine handshake is key, phone to phone. Unlike a written letter, in this instauration of Lacan's assertion that a letter always arrives at its destination, if the cellphone text goes through, the sender knows that truth (and unlike email where there is a certain server leeway still) the receiver has likely received the text. This knowledge in turn produces the expectation of a response and the anxiety of waiting for the same, playing the game of which is a delicate and all-too-human story. If there is a supplement to be added on this point, that has more to do with the section of The *Hallelujah Effect* on desire.

What Adorno calls the 'radio voice' (even where many of us no longer listen to the radio per se) frames the way we hear music today, even so-called 'live' music. Adorno's filigree-feel for the physicality of things, which Adorno recounts in overt coordination with his citation of Anders'/ Stern's musicological research corresponds to the complex issue Adorno seeks to raise by means of what he calls radio 'physiognomy[66].' In addition, Adorno deploys a '*descriptive* or '*'phenomenological*" method[67]' to study the radio experience (the listeners themselves, as it were). As Adorno writes, his concern was with 'the characteristics of the radio phenomenon as such, devoid of any particular content or material[68]' and in this statement we recognize the Husserlian *epoché*. Hence Adorno considers 'the way any voice or any instrumental sound is presented over the radio,' adding (in a clarification reminiscent of Husserl's 'to the things themselves')

> 'it will be very difficult to abstract this expression of the 'radio itself' from the expression of what is actually broadcast, and we shall see later that these two layers of expression influence each other [69].'

In this way, Adorno summarizes his project phenomenologically, specifying that what he calls:

> '[p]hysiognomics intends only to define more correctly the inherent features of radio phenomena and to elaborate within these features certain relations … we may confess here that the axiom which governs all these attempts is our conviction that the unity of the radio phenomenon, in itself, as far as it really

65 Even Maurizio Ferraris in his outstanding reflection on the mobility of cell phone as object (and including a foreword by Umberto Eco), *Where Are You?*: An Ontology of the Cell Phone. Sarah De Sanctis. Trans. Umberto Eco New York: Fordham University Press, 2014, does not offer an analysis of the subversive dimension - and directionality - of texting. This does not mean, and this is part of the perspective to be drawn from this study of *The Hallelujah Effect*, that texting does not have an effect on the user: it surely does. And to the extent that young people are particularly enthralled with texting one is hard pressed to make a distinction between texting and sexting. See on this albeit from a clinician's or therapeutic rather than a philosophical perspective, Clay Calvert, 'Youth Produced Sexual Images, Sexting, and the Cell Phone' in Fabian Saleh and Albert Grudzinskas eds., *Adolescent Sexual Behavior in the Digital Age*. Oxford: Oxford University Press, 2014, pp. 89-116. More generically there are number of 'rules' guidebooks to texting (and dating) as well as to the intrusion of cell phones in marketing (when a summons from the device is used as an authoritative intervention to justify an abrupt end a listening to a sales pitch).

66 See Adorno, *Current of Music*.

67 *'The question of why we follow this descriptive or 'phenomenological method can easily be answered. We are dwelling on the phenomenon ["of music pouring out of the loudspeaker"] because it is actually the phenomenon which determines the reaction of the listeners, and it is our ultimate aim to study the listeners.' Adorno, Current of Music*, p. 107. Italics in original.

68 Ibid., p. 77.

69 Ibid., pp. 77-78.

> has the structure of a unit, is simply the unity of society which determines all the individual and apparently accidental features[70].'

Taking the example of the symphony as one might hear it on the radio, Adorno contrasts it with the symphony as experienced in a concert hall, that in space and time. As Adorno notes, the reference to the radio symphony corresponds to the transmission of a live broadcast, which was itself specifically structured to the particularities of radio, just in the way that radio orchestras were structured to the mechanical constraints of the recording studio as such.

Adorno highlights the space of sound, as 'the sonic' is always as Goodman notes: 'a phenomenon of contact and displays, through an array of autonomic responses, a whole spectrum of affective powers[71].' Today's shock DJs expand their 'music' into the felt domain as Nietzsche speaks of the 'science of aesthetics' in the first line of his *The Birth of Tragedy*. The shock DJs new 'new' music is designed for more than listening, produced by way of a 'prepared' or techno-synaesthetic effect. Such preparation requires technology, microphones such as the kind that transform the architecture of a symphony hall or concert space, as John Meyer places by the hundred fold, as well as the well analysed loudspeakers of the Psyops protocols developed during the second world war and to which I have already referred above. But the problem in a sense is that the new sonic world is too much with us. Thus, as Simon Reynolds points out:

> 'The concept of 'noise' has made a big comeback in recent years . . . the irritating end of it is all those artist[s] aiming for ye old 'shock effect,' their pure noise laden with content of tediously 'transgressive' nature (all the old cliched faves of vileness and violation: serial murder, neo-Nazis, yawn . . .). The blindingly obvious fact is that no one shockable is within earshot; there's no disruption or challenge in these scenes, because they're screeching to the converted[72].'

It is one thing to talk in improvisational terms with Jacques Attali (of course); it is another to take an insider's look at the 'urban audiosocial' or club music experience.

If 'sonic culture' or indeed, Lefebvre-style, if allusions to the 'cartography of sonic force,' are increasingly preferred today, Adorno's terms hearken to the faces of things. Adorno's physiognomic discussion includes as I have already suggested space itself, and in this physicophenomenological sense, the space of musical sound includes (as we noted the recording specialist John Meyer's surprise at this) the room itself qua sounding board inasmuch as all rooms and not just concert halls or recording studios happen to be sounding boards and it is also about the radio for Adorno, or about the choice of an array of loudspeakers for the audiophile, etc. A distinction between Adorno's 'radio loudspeaker' and today's earphone experience makes a great difference for the sound engineering that must be considered although it is usually taken for granted in the very technical business that is the 'perfection' of recorded sound today. These issues relate not only to the terms of production as such, whether in recording practice (as the esoteric centrality of mixes and soundchecks also makes clear), but also to music heard 'on the radio' and even, albeit counter-intuitively, to live performance[73] [74].

Goodman's analysis of the 'sonic' in his Sonic Warfare as well as his work as Kode9, makes no more than a passing appearance in *The Hallelujah Effect*, along with important others like Siegmund Levarie, a mathematical and musicological theoretical schematist writing on 'sound' and harmonics with the composer and pianist Ernst Levy[75]. Indeed

70 Ibid., p. 118-119.

71 Goodman, *Sonic Warfare*, p. 9. For Goodman, who composes/performs as Kode9, 'Sonic warfare is therefore as much about the logistics of imperception (unsound) as it is perception. The bandwidth of human audibility is a fold on the vibratory continuum of matter. With reference as noted above to military research into acoustic weaponry, this molecular backdrop will be mapped as a vibratory field into which the audible is implicated.' (Ibid.)

72 Simon Reynolds, *Bring the Noise*, p. xii.

73 This same notion of 'perfect' sound inspires the title of the online music journal, *PerfectSoundForever*. See again, Roth's *Capturing Sound* as well as Björnberg, 'Learning to Listen to Perfect Sound' etc.

74 Again, this is one of the central and very practical themes of Bennett's *On Becoming a Rock Musician*.

75 See Levarie and Levy, *Tone* in addition to their *Musical Morphology* as well as Levy's book (as edited, but in this case, this is a close collaboration, by Levarie), *A Theory*

the very fact that there are such echoes at all already exemplifies the dynamic of interaction (an achievement realized in the spirit of Latour's 'Biography of an Investigation[76]'), responding to a network of scholarship and thinking and drafted in the dynamic time of email exchanges with the musicologist, the late Ernest McClain[77].

And there would be enough if *The Hallelujah Effect* stopped at the radio 'effect' of the effect. But the project began from the start as a phenomenological (or what Nietzsche very classically named an 'aesthetic' or sensed) reflection on Nietzsche and Beethoven, hence not as a philosophy monograph wandering incidentally, accidentally into the field of music and musicology but precisely on the intersection of philosophy with music in Nietzsche's thinking. To situate this intersection required the addition of a set of reflections on the broader *musiké techné* of antiquity, and not only as exemplified - as this harmonic theory would be exemplified for Levarie and Levy as well as for McClain - in terms of what he as a musicologist analyzed as the '*Pythagorean Plato*[78]' (all three of them by way of the explicitly harmonic analyses of Albert Freiherr von Thimus on Pythagoras[79]). There the associations converge on the esoteric, and how could they not? In this and related cases, the unheard refers to the soul in Plato as in Plotinus as well as Boethius. At the same time, as C.F. Audry Williams argued in his study of Aristoxenus more than a century ago, the only access to thinkers on harmony in this tradition will be by way of analogy with the modes of medieval or church music. As Georgiades and Dahlhaus (and Nietzsche) all argued, medieval church music as this itself can be seen, in Williams words, to have

> '...made possible the building up of the art of polyphony. This art arrived at its culmination in connection with the modes during the sixteenth century; and now music was to enter on its new phase, in which counterpoint and the young art of harmony were to be used with the major scale (derived, like the ecclesiastical modes, from the Greek diatonic system), and to be brought under the influence of the rhythm of the dance; while the whole of music was to be permeated with the culture that had been acquired through a study of Greek literature [80].'

Williams proceeds to illustrate his discussion of Aristoxenus by using examples from Bach, Handel, and Gluck as well as Beethoven. In *The Hallelujah Effect*, I offer a discussion of some other authors who trace such parallels, including Jacques de Liege in the work of the nearly utterly neglected phenomenologist of music and listening, the late F. Joseph Smith[81] and his studies of esoteric harmonics in the careful context of historical musicology[82] as well as to the Greeks themselves for whom such esoteric harmonies go without

of Harmony. See too Levarie's *Fundamentals of Harmony* as well as Josef Mertin and Siegmund Levarie, *Early Music*.

76 It matters that Latour's study is rather less than actually 'interactive' - and despite the allure of thinking that social media permits a democratic engagement, the appeal is as much illusion as it ever was and as Anders argues. But as Terence Blake who offered a review of some length only to test the limits of such interaction, the promise is more apparent than real. See Blake's review of Latour's *Modes of Existence*, 'On the Existence of Bruno Latour's Modes: From Pluralist Ontology to Ontological Pluralism.'

77 The exchange was useful in unanticipated ways. McClain, born in 1918, remembered knowing George Simpson as the Queens College adjunct geographer who was drafted to help Adorno with writing English for his work on the Princeton Radio Project during his time in New York.

78 See McClain's *The Pythagorean Plato*.

79 On which see for example Werner Schulze, 'Number and Proportion in Plato's Political Theory, Plato's Political Philosophy and Contemporary Democratic Theory.'

80 See Williams, *The Aristoxenian Theory of Musical Rhythm*, xiii. Williams offers a summary of the 19th century philogical tradition (that is also Nietzsche's own background), especially Rudolph Westphal (whom Nietzsche also cites) and particularly drawing upon the second volume of F. A. Gevaert's 1881 *La Musique de L'Antiquité*, abstracting a summary of Greek sources in Williams, *The Aristoxenian Theory of Musical Rhythm*, pp. 3-6.

81 See F. Joseph Smith's own contribution 'Variation in Music and Thought.' See for a discussion (in Italian) of Smith's contribution to music and phenomenology, Nicole Pedone, 'Musicologia e Fenomenologia in F. Joseph Smith.'

82 This is also a coordination of musicology and numerological studies as these are inevitably Pythagorean but without which there is no access to the sacred medieval musical tradition, including Boethian number theory, as Smith shows. An expert on the Jacques de Liége, see his essay 'Greek Letter Notation in the *Speculum Musicae* as well as Smith's key study of Liége, F. Joseph Smith, Iacobi *Leodiensis Speculum Musicae*. On the broader historical tradition, which may also be connected with Nietzsche's studies of quantitative rhythm, see Dorit Esther Tanay, *Noting Music, Marking Culture*. See too, much more broadly afield in theological or metaphysical readings of music and time, Peter Manchester's *The Syntax of Time*, pp. 29-30. See too for a more extended discussion, the section on Augustine's *On Music* in Manchester's *Temporality and Trinity*.

saying. But in particular, I turn to Nietzsche and despite the wide range of scholars who have written on Nietzsche and antiquity, and the even broader range of scholars who discuss Nietzsche and music there are few discussions, apart from my own studies, that seek to bring the two together.

If the original inspiration for *The Hallelujah Effect* was the role of Greek music (and word), the conjunction of music and word is found in Nietzsche's related investigations into and the sounding, the pitching, the tonality of ancient Greek (there Nietzsche's rhythmic study differs from the Westphal's and William's discussion of the same Aristoxenian account of rhythm, which is for Nietzsche already an Alexandrian and so late conventionality) and in consequence of the Greek music drama, the musical art work that was tragedy. But it is Beethoven's music (and word) that serves as an illustration for Nietzsche in his *The Birth of Tragedy out of the Spirit of Music*.

III. On Nietzsche's Greeks and Nietzsche's Beethoven.

The third part of The Hallelujah Effect features Friedrich Nietzsche's theory of quantifying or quantitative [*quantitierenden*] rhythm in his studies of Ancient Greek lyric together with his account of ancient Greek musical drama as a *Gesammtkunstwerk* even beyond Wagner's perfect imaginings. Important too and in addition to the more mainstream and received discussions of mathematics in antiquity (the standard account of Brumbaugh et al.), and one can expand this retrospective prolegomenon to include add the archaeological layerings of Friedrich Kittler's mathematico-musical reflections[83] but which already include the not always noted but essential researches done by Ernest McClain to whom this essay is dedicated in addition to Jay Kennedy's more recently popular rediscovery of McClain's and other work (done for many years also by many others) on stichometry in Plato's dialogues (i.e., on the very music of the text in a sense related to the 'music' about which Nietzsche himself writes in reference to tragedy[84]). In addition, although more complicated still, there is Nietzsche's long standing writing on causality, deliberately looks backwards and forwards (anticipating Freud on the workings of the unconscious). Hankinson's study of cause includes several chapters matching Nietzsche's approaches to the question of causality, including Stoic causality and skepticism, without to be sure mentioning Nietzsche[85]. To this should be addedvancient reflections on metallurgy and medicav physiology (I have touched on related themes in my explorations of the techniques of ancient bronze with reference to Pliny[86], in the context of a review of the mechanical practical technology available in ancient Greece[87]). And

83 See Friedrich Kittler, 'Number and Numeral' along with to be sure both parts of his first study of music and mathematics in antiquity: *Musik und Mathematik. Band 1: Hellas, Teil 2: Eros and Band 1: Hellas, Teil 1: Aphrodite*. I discuss Kittler and Nietzsche in terms of what Kittler and other media theoreticians spoke in Foucault's spirit as 'archaeology' in Babich, 'Archaeologies of the Aexandrian.'

84 Kennedy's claim to priority here works largely because he eschews references and by failing to credit others ingeniously finds that he has made a discovery. The tactic depends for its success upon what Nietzsche called a 'failure of philology' - i.e., no one knows any better. Thus see Eva Brann (who puts her claim into the very title of her study), *The Music of the Republic: Essays on Socrates' Conversations and Plato's Writings* as well as John Bremer's On *Plato's Polity* and several essays laying out his earlier work, particularly: 'Plato, Pythagoras, and Stichometry' in addition to Ernst McClain's *The Pythagorean Plato* where McClain himself gave the credit to scholars of ancient musicology, and so on, particularly including Richard Dumbrill's theoretical (including demonstrative or experimental explorations of the kind I have referred to above with respect to Adorno and Anders as phenomenological), *The Archaeomusicology of the Ancient Near East*.

85 I have been writing on Nietzsche and causality in connection with science, and thus from Nietzsche's post-Kantian perspective for many years. See especially my forthcoming essay 'Hume on Causality & Nietzsche on "the errors of philosophy."' See in addition, Hankinson, *Cause and Explanation in Ancient Greek Thought*.

86 See my essay, 'Die Naturkunde der Griechischen Bronze im Spiegel des Lebens' a more developed version of the lecture the late John Cleary kindly invited me to offer in Maynooth, published as 'Greek Bronze: Holding a Mirror to Life.'.

87 See here both Manfred Barthel's popular account: *Die Enkel des Archimedes* as well as Horst Bredekamp's *Antikensehnsucht und Maschinenglauben* an approach at once stylistically disenchantment-oriented as well as esoteric. And see too, Ludio Russo's *The Forgotten Revolution* which had originally appeared in Italian in 1996. On Greek science as indicated in a broader sense, see not only Szabó, *Das geozentrische Weltbild* but also Kahn's *Anaximander and the Origins of Greek Cosmology* in addition to Kraft, *Geschichte der Naturwissenschaft* I.

then too because of the central importance of music and architecture, we could note Nietzsche's references to architecture and if I do not have the space to review these, it is still important to note Nietzsche's sustained engagement with what he called 'monumental' history in his writings on history, including what turn out to be fairly metaphors of decadence, falling statues beyond Aristotle's famous reference and paralleling the investigations of Robert Hahn and others[88].

In the present context what is essential is Nietzsche's discovery that the text of ancient Greek tragedy ought to be read, ought to be tuned, one might say, in effect as its own musical score. Thus for Nietzsche, a reading of ancient Greek lyric poetry and tragedy would not require what did not exist (and Nietzsche would argue that it would not have existed as such):qua second system of writing to match the first system, what would be in effect the missing musical score. For Nietzsche the ancients having invented the spirit of phonetic writing needed no second system and only came to require such a scoring later, after what he called the death (the suicide). Thus Nietzsche argued that this second system, musical notation as such, was a later, Alexandrian invention as he spoke it. His point, a point made with a sparseness an ancient would have admired, is that reading the Greek text of musical drama - that is, reading tragedy in the age of Aeschylus or Sophocles - would be, in effect, (sight) reading music, (See Nietzsche's 1870 essay, "Greek Music Drama[89]" and, yet more explicitly, "On Music and Words[90]."

Contra the typical scholarly division of Nietzsche's philosophy into three distinct evolutionary periods, I argue that it is important to note that Nietzsche never forsakes the themes of his first book, telling us repeatedly as he does that he sought to raise the question of tragedy (as he sought to raise the question of science) as a question[91]. Thus Nietzsche traces the 'birth' of tragedy out of the spirit of music, i.e., in terms of the performative working of the tragic work of art in terms of the tragic cult and hence with respect to the political and religious economy of ancient Greek society. It was in this context that Nietzsche reviewed the question of the role of the chorus and the tragic spectator[92].

Nietzsche's illustrative philological notes (explicitly didactic as these were prepared for his courses at Basel) emphasize that for the Greeks reading was a fundamentally sounded out, a 'spirited' phenomenon and the tragic poet's poetic compositions accordingly would be composed as much for the ear as for the eye (Nietzsche's Zarathustra speaks of 'listening' with one's eyes)[93]. Thus Nietzsche invokes a different readerly praxis (or performative technique) in his *On the Theory of Quantitational Rhythm*, ranged under the heading '*Arsis-Thesis*' itself a conventional distinction Nietzsche dates to Horace and which Williams traces to Bacheios (following Westphal) and differentiated into two kinds of quantifying rhythm, marked either with the hand or the foot, 'by which one indicates the tact interval: percussiones,' in other words, keeping time by 'striking time[94].' As

88 See here Hahn, *Anaximander and the Architects* and see too Couprie, Hahn, and Naddaf, Anaximander in Context as well as McKewen, Socrates Ancestor.

89 Nietzsche, '*Erster Vortrag: Das griechische Musikdrama.*'

90 Nietzsche, 'On Music and Words.' Carl Dahlhaus has emphasized Nietzsche's importance in connection with Beethoven, including a translation of Nietzsche's '*Über Musik und Wort*' in his Between Romanticism and Modernism, pp. 103-119. In addition, Dahlhaus's then assistant in Berlin, the Stanford musicologist, Stephen Hinton, is also useful. See Hinton's '*Not Which Tones*? The Crux of Beethoven's Ninth,' reviewing (as Ian Bent also takes up) this thoroughtly hermeneutic relection in this manner. For Hinton, 'the beginning of the baritone recitative, O Freunde, nicht diese Töne!' (Ibid.) underlines Beethoven's 'irony' in terms of the very notion of tone as such.

91 Babich, 'Nietzsche's Philology and Nietzsche's Science.'

92 See, again, Babich, 'The Science of Words or Philology.'

93 Nietzsche, *Griechische Lyriker*, p. 375.

94 Nietzsche, *Greichische Rhythmik*, p. 102. See for further discussion Bornmann, '*Nietzsches metrische Studien*,' as well as the late Kremer-Marietti's '*Rhétorique et Rythmique chez Nietzsche*.' Kremer-Marietti refers to Marcel Jousse, *L'anthropologie du geste* as this develops in Jousse, *Les Récitatifs rythmiques*. See in, English: Jousse, *The Oral Style*. As Kremer-Marietti informs us in her footnote 5, Father Jousse gave a course in 1935 at the Sorbonne on the theme of the psychology of the parable in the Palestinian style of orality which was subsequently taken up by Boucly in his '*La mimique hébraïque et la rythmo-pédagogie vivante*.' See further on Jousse and including a valuable bibliography, Sienaert, 'Marcel Jousse. In addition to Kremer-Marietti's *Nietzsche et la rhétorique*, see Porter's '*Being on time*' in

Nietzsche explains - and it is just to his purpose here to be repetitive (as is Westphal and others writing on the same theme) - there are two distinct styles or 'arts' of keeping time: again, one for visual indication, 'for the eye using the hand,' and the other 'for the ear with an audible tap of the hand, finger or foot[95].' Nietzsche here invokes the standard for the Aristoxenian order of time, but goes beyond it with his emphasis (this would be the underlined hint in his *Theory of Quantitational* [*Quantitierenden*] Rhythm in figure 3), that what ultimately aids us in this regard is language or usage: '*Sprachgebrauch*[96].'

Here pitch rather than accent is at stake for Nietzsche, a point which the musicological classicist, Audry Williams came very close to articulating only to lose it at once and again, more than a century ago. For Williams, 'the old Attic refinements were Nietzsche emphasized time-measurement and to that he added a clear denial of 'accent or stress' as William here speaks of it, and what he usually calls *ictus*, as Nietzsche either declares that there is no *ictus* or, else -- and distinct from the Latin stress *ictus* -- that the Greeks had in its place an *ictus* we can barely imagine, that is the *ictus* of pitch or tone: the spirit or sound of music.

For many Wagnerians this can seem to be little more than a repetition of the same point to which Wagner appears to lay his own claim, and I quote Williams citing Wagner here from his *Oper and Drama* on the contrast between the emphases of modern and ancient Greek expression:

> 'They had only our rapid speech-accent in their ear, when they invented the measure by which two shorts invariably equal one long. The explanation of Greek metres would easily have occurred to them if they had had in their ear for the so-called long the sustained notes of musical measure, by which the length of words can be varied in melody[97].'

But Wagner's musical explanation needs more precision. Thus however Nietzsche was inspired by Wagner's attention to this very significant point, what is at stake is not a "musical measure" as a matter of melodic variation but quantitational or quantifying measure or time, and given that distinction one can bring Nietzsche and Wagner together. Williams himself is compelled to object as a classist specializing in rhythm well might and as Nietzsche certainly would also have objected, that

> 'Wagner is mistaken when he attributes the invention of the rule of 'two shorts equal one long' to 'our prosodists and metricists.' It goes back for nearly eighteen centuries, having been invented by Latin grammarians of early imperial times, who knew little of, and cared

addition to Sauvanet, '*Nietzsche, philosophe-musicien de l'éternel retour*,' Dufour, '*La physiologie de la musique chez Nietzsche*,' in addition to Dufour's *L'esthétique musicale de Nietzsche* and Günther, *Rhythmus beim frühen Nietzsche*. Corbier's '*Alogia et Eurythmie chez Nietzsche*' offers a useful reading that detours into an emphasis on dance, important for Nietzsche but which (as Nietzsche emphasizes) carries the risks of our own associations. Thus Nietzsche emphasizes the narrowness of the Greek stage, a constraint that also set the actors, as it were in a kind of 'relief,' and emphasizing the dance in question not as freely moving, as we assume today, but as more a 'beautiful walking than a whirling dance.' Nietzsche, *Zur Theorie der quantitirenden Rhythmik*, p. 270. Here one is reminded of Augustine's definition of rhythm as *ars bene movendi* (the science of beautiful movement).

95 Nietzsche, *Griechische Rhythmik*, p. 102.

96 C. F. Audry Williams, *The Aristoxenian Theory of Musical Rhythm*, pp. x-xi. Williams gives an account of some of the efforts of the German philologists of the generation just before Nietzsche and he is especially struck by the same Rudolph Westphal to whom Nietzsche was indebted for his own advances beyond him. It is worth reading this text (along with that his French contemporaries as he himself indicates) because the author, in the historical hermeneutic spirit emphasized by Butterfield in his *Whig Interprtation of History*, is simply closer to the debates out of which Nietzsche spoke, even if Nietzsche's emphasis on pitch would not come to be the received view (the received view looks, as Westphal did, for a separate musical notation). Williams also offers a comprehensive list of Greek theorists of rhythm, pp. 3ff and see too 9ff. along with an account of the evolution of tragedy from the dithyrambic goat song and the old comedy or Bacchic revel in concert with the introduction of the actor and the chorus with Thespis, 'The Father of Greek Tragedy.' (Ibid., p. 15.) Like Nietzsche, Williams emphasizes the musical: 'The drama, whether Tragedy or Comedy, was set to music throughout. The choruses took the form of a series of strophes and antistrophes, and the dialogue was carried on in melody, accompanied by the instruments. … The whole of the drama, in fact, whether spoken or sung, was uttered in rhythmical measure: there was nothing equivalent to our recitative, with its freedom from formal construction.' Ibid., p. 18. Williams uses discussions of contemporary music, alluding to Wagner (whom Williams explicitly allies to Aristoxenus, pp. 161ff.), but especially illustrated with reference to Bach and Mozart as well as an extended analysis of Beethoven, see esp. his discussion of the rhythmic structures of Beethoven's Ninth, pp. 136ff. Compare in general on Aristoxenus, Angèle Kremer-Marietti, '*Rhétorique et Rythmique chez Nietzsche*.' And see, once again, Bornmann, '*Nietzsches metrische Studien*.'

97 Williams, *The Aristoxenian Theory of Musical Rhythm*, pp. x-xi.

less for, musical rhythm [98].'

Nietzsche's concern is the relation of music and word as the literal musicality of the Greek word and for this the phenomenology of media is (or could be) key. Just as the late Friedrich Kittler remarks, Nietzsche was a philologist. "A rarity," as Kittler importantly takes time to emphasize - such a distinction would also have characterized Heidegger as much as it characterized my own teacher, Hans-Georg Gadamer - 'among philosophers[99].' And as the philologist-philosopher Nietzsche points out our 'lack of philology' tends to be fatal for us as philosophers (and it goes without saying that the philologists are hardly excluded from this judgment). Both Nietzsche and Kittler emphasize the importance not merely of reading but reading hermeneutically, phenomenologically and, as I argue, in the case of ancient Greek, especially in the case of ancient tragedy, such a hermeneutico-phenomenological reading presupposes an attention to sound as well; as to the socio-political culture, that is the very *alien*, as Nietzsche never ceased to underscore for us, context of ancient Greek society in which the tragic work of art was also and always a divine service and one that as the late Nicole Loraux also foregrounded as key to her own work, took place not interior to the polis but without, and not for a limited hour of the day or the evening, an event scheduled as part of routine or everyday life, but beyond the everyday, beyond the polis, literally *hypsipolis-apolis*, to quote the same passage from Sophocles that Heidegger cites in his *Introduction to Metaphysics* writing on the uncanny as such.

Overall and in addition to question of music and technology, including radio broadcast but also the techniques of musical production and performance practice, *The Hallelujah Effect* also explores questions of desire (including some radical reflections on gender and feminism that are unlikely to sit well with most gender scholars and at least some feminists). The concluding chapters reflect on the meaning of music as considered between antiquity and the present day, as on Nietzsche's reflections on the birth and the perishing of tragedy (why did the Greeks have tragedy? why did they enjoy it? why would anyone? what happened to it?) related in turn to the question of the role of Nietzsche's own comments on Beethoven (rather than, as noted to start with, the Wagner who continues to serve as the singular reference point for the great majority of readings of) Nietzsche's first book on tragedy).

Why Beethoven? Isn't this silly? Don't we all already know all the answers here when it comes to Nietzsche? Wasn't he smitten with Wagner? Or alternately, or ultimately, (really) with Bizet? And so on[100].

I have argued in spite of a scholarly tendency to read Nietzsche in connection with Wagner, even as a pale shadow of Wagner as many do, it is not Wagner who must be counted as Nietzsche's main reference in his thinking on tragedy, but Beethoven.

Thus what captivates Nietzsche in what he understands as a 'birth[101]' of an art form *out of the Spirit of Music* is Beethoven - and nothing less iconically (one could say) 'Beethoven' than the very choral fourth movement of the Ninth Symphony[102]. This question must be raised in the context of Nietzsche's earlier discovery of the role of ictus in ancient Greek (again Nietzsche claims that there is no stress ictus but and much rather only a pitch ictus) and I have noted elsewhere that Nietzsche's explorations of quantitative rhythm have served in

98 Ibid.

99 See Kittler, 'We are programmable.' Kittler, 'Interview,'

100 Studies of this favoring, for which Nietzsche himself is the obvious source (he declares himself contra Wagner and likewise declares himself pro Bizet or others) may be dated to the beginning of Nietzsche scholarship and even before the philosopher's death.

101 The Nietzsche of genealogical thinking repeats this notion of birth and distinguishes it in his '*On Music and Words*.'

102 At this point, as at any other, Wagnerians, enthusiasts and scholars, will point out that Wagner wrote on Beethoven even before Nietzsche was born. Which is quite correct and which I scarcely mean to gainsay here. Quite to the contrary. Hence I call attention to what the focus on Wagner's Beethoven has brushed aside and that is Nietzsche's Beethoven and also, but I only raise this as a theme for possible research, the relevance of Nietzsche's own influence on Wagner, as this might perhaps begin to be seen in Wagner's Beethoven. Nietzsche would, to be sure, publish his own text with the same publisher, and include a dedicated to Wagner as 'Vorkämpfer,' a word choice and constellation to which Tracy B. Strong has called some attention. See, for example, Strong, '*Philosophy and the Politics of Cultural Revolution*.'

the interim if not to make Nietzsche's thought on the ictus a household word then certainly the basis for the standard pronunciation of Ancient Greek, as this has already been mentioned in connection with musical phrasing and just this constellation is rarely considered - even by musicological experts and even with reference to the same Hugo Riemann on phrasing in music and about whom Nietzsche writes, more rather than less insistently, in his correspondence with the musician Carl Fuchs towards the end of the 1880's - which was at the same time the end of Nietzsche's creative or productive writerly life[103]. One other way to put this question would be to note that for the Greeks there would be no way to adapt a song in the way that Cohen's *Hallelujah* has proven to be so adaptable and malleable that it serves as many argue, as set music to what can seem to be everything in filmic and television culture, weepy and romantic, high and low, including a YouTube wedding, mass surprise, and a range of parodies too[104].

To raise the larger question of what Nietzsche meant by the spirit of music as the origin of the tragic artform permits us to dispense, as he does, the assumption with that the tragic artwork is generated by life's tragedies or its more dismal aspects. Instead Nietzsche has recourse to musical compositional devices, meaning dissonance; and I point out that Nietzsche, who saw himself as a composer, knew Albrechtsberger's Gründliche Anweisung zur Composition mit deutlichen und *ausführlichen Exempeln, zum Selbstunterrichte*[105]as well as the studies in compositional theories and harmony attributed to Beethoven himself in Henry Hugo Pierson's edition of Ludwig van Beethoven's *Studien im Generalbass, Contrapunkt und in der Compositionslehre*, most particularly considering the extensive contributions on dissonance to be found in this last[106]. I am inclined to think that it was Nietzsche's familiarity with both Albrechtsberger and Pierson/Beethoven which rendered him as amenable to the otherwise devastating criticisms of his own musical compositions by Hans von Bülow who in due course referred in his critique both to points made by Nietzsche himself in his first book and as well as the imperative prerequisite demanding that one know the grammar of composition if one wishes to compose music.

As a 'closeted' musician, as considered from the perspective of music scholars, Nietzsche conceived the Greek notion of diaphony as *Dissonanz* (which in the 19th century was translated into English as 'discord') as a musical term, corresponding to his dynamic account of the 'science of aesthetics[107].' As an 'out' classicist[108], Nietzsche used the term dissonance musicophilologically, as already suggested above and with reference to Pythagoras,

103 Nietzsche, letter to Carl Fuchs 26 August 1888, *Samtliche Briefe*, Vol. 8, pp. 399ff, and see too, pp. 403ff. See the last chapter of Babich *The Hallelujah Effect* for further references and discussion.

104 In addition to Ray Kelly's adaptation of Cohen's *Hallelujah* there are a number of other more parodistic versions, of which Michael is worth mentioning if only because of the beauty not of his lyric inventions, the lyrics of the song 'fail' as it were already in the title of Michael Hermiston's '*Hall-epic fail-ujah*.'

105 Johann Georg Albrechtsberger, *Gründliche Anweisung zur Composition mit deutlichen und ausführlichen Exempeln, zum Selbstunterrichte, erläutert; und mit einem Anhange: Von der Beschaffenheit und Anwendung aller jetzt üblichen musikalischen Instrumente.*

106 See Beethoven himself here in his notes from his studies with Albrechtsberger as we may read these and as Nietzsche would have known them in Pierson's 1853 edition of Seyfried's 1832 edition of Ludwig van Beethovens *Studien im Generalbass, Contrapunkt und in der Compositionslehre aus dessen Handschriftlichen Nachlass gesammelt und herausgegeben von Ignaz Xaver von Seyfried*, throughout but especially p. 130. In English as Studies in Thorough-bass, Counterpoint and the Art of Scientific Composition. Collected from the Autograph Posthumous Manuscripts of the Great Composer and first published together with Biographical Notices. It should be said, of course that in the interim we have gone to not reading Seyfried at all, as decreed by Gustav Nottebohm, Beethoveniana, pp. 175ff. Nottebohm's specific complaints are that Seyfried is incomplete, in places inaccurate, that he transcribes some of Beethoven's studies along with Albrechtsberger and Fux and others, without indicating which is which. Indeed as Nottebohm concludes (coincidentally using a source scholarship style convention not unlike the rule used to exclude material in Nietzsche's [similarly constituted or invented] Will to Power, 'Es ist also, als Ganzes genommen, falsch.' Nottebohm, Beethoveniana, p. 203. In the impatient scuttlebutt of modern history, this has entailed that todays readers do not read Seyfried at all and that Nottebohm's criticisms have the standing of a received truth. I cannot but leave this to one side, however the facts may stand or fall (and my point here is that this is unquestioned) because the Nietzsche of The Birth of Tragedy would only have had reference to Seyfried in any case. See the last chapter of Babich The Hallelujah Effect for further references and discussion.

107 Nietzsche, BT §1

108 See Babich, *Words in Blood, Like Flowers*, pp. 37f as well as Babich, 'The Science of Words or Philology'.

Aristoxenos, and of course Heraclitus and Empedocles.

As Nietzsche himself observed, the trick of dissonance which he also called 'pain' (in, to be sure, a musical context) is precisely its distinction from consonance and the elusive key to harmony. This he expresses in the complicated formula of the becoming-human-of-dissonance. In this context, rhythmic-theoretical (and historical), dissonance for Nietzsche is not a heralding of modern music, with its famous 'emancipation' of dissonance[109], as Adorno (better than most) knew this emancipatory impetus, but rather the quite practical musical notion of dissonance and harmony. This is the way his gnomic notion of the '*Menschwerdung der Dissonanz* that is to say, the becoming-human-of-dissonance, phrased in a single word, functions in Nietzsche's text: 'and what else is the human?' he asks, understood together with his earlier investigations into Greek rhythm and meter.

Thus beginning with an analysis of a single song, for which analysis I needed and benefitted from a sustained email exchange with Ernest McClain in my effort to analyse a singular (and singularizing) video performance of Leonard Cohen's *Hallelujah* sung by kd lang together with a extended analysis of its dissemination on YouTube and on Facebook, *The Hallelujah Effect* also sheds light on our network culture. Along with this, as we have seen there is Adorno on the space of music and sound and Nietzsche urging us to learn to "read with our ears," as the Greeks did who invented the very first technology for reproducing sound in their invention of the phonetic alphabet, which Nietzsche himself understood in terms of the 'spirit' of music.

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109 See for a useful discussion: Stephen Hinton, 'The Emancipation of Dissonance: Schoenberg's Two Practices of Composition.'

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Figure 1. The Hallelujah Effect.Reflections on radio, YouTube and Adorno Slide for a Presentation at the Society for phenomenology and Media.

Figure 2. Telefunken 340W 'Katzenkopf', Photo: Ralf Kläs.

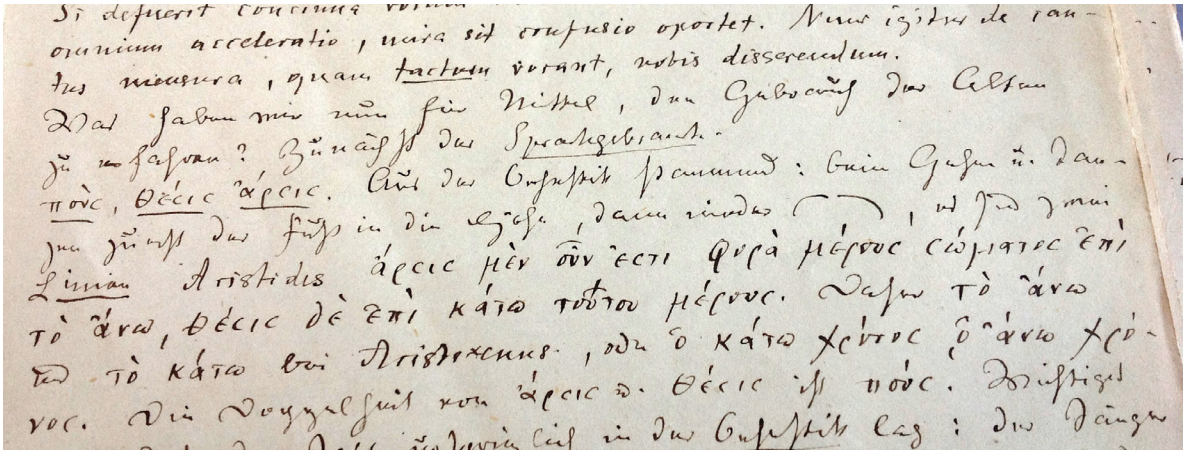


Figure 3. Excerpt from Nietzsche's unpublished notes on Quantitational Rhythm. Author's Photograph.

201-212

McCLAIN'S MATRICES:
An in-depth study of Ernest McClain's Inverse Symmetry

Pete DELLO

> 'The universe is one, and it began to come into being from the centre, and from the centre upwards at the same intervals as those below. For the parts above from the centre are in inverse relationship to those below. For the centre is to what is below as it is to what is above, and so with all the rest; for both stand in the same relationship to the centre, except in so far as their positions are reversed.'
>
> *Philolaus*, fragment.[1]

1. Prelude: All in a humming string.

A musician is a bit like a dog. Whatever era or society in which he finds himself, he will follow his instincts and always meander around strings and pipes and things to bang; and it will always be vibrations that fascinate him.

Any musician, ancient or modern, whoever twanged a string, as long as they had an ounce of curiosity would very quickly be struck by certain interesting occurrences. For strings, if they are reasonably taut, make musical sounds; and, if touched at certain places lightly enough as not to stop the vibrations, other musical sounds might be teased from them. He will think he has discovered this for himself but untold multitudes before him have already learnt the lore. All guitarists know about the harmonic that lives at the midpoint of any 'humming' string. If gently touched here a new note will sound that is higher pitch than the first but in every other respect identical. This note we have learnt to call the octave although it has more of the two about it than the eight, for this gentle touch no more than a slight tap, creates two standing waves - pockets of vibration - along the string, exactly equal in length to each other.

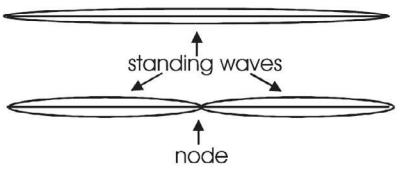


Figure 1.

The standing waves are separated by nodes, points of rest where nothing vibrates at all, yet if one of the standing waves is dampened the other is somehow informed through this still point and all sound and vibration cease. Magic like this might stimulate a musician's curiosity and lead him to discover all the fundamentals of music but for the philosophically minded it would be the beginning of what Aristotle called 'wonder and astonishment'. Certain things would fascinate him that might pass a practising musician by, for here is a direct link between number and a psychological event, - between order and the inner appreciation of order - number's aural validation.

Further investigations along the same lines would quickly show that the most sonorous sounds coincide with the simplest divisions, and that, moreover, these divisions coincide with certain numbers which have come to be known as the 'musical primes', thus establishing the basis of harmonics. If, for instance, a string were to be touched at a point one third along its length, it would produce a pitch exactly a perfect fifth above the octave; whereas a division of a fifth would produce a major third above the second octave of the open string. It would soon be apparent that divisions of a half, a third and a fifth and their multiples, (that is, generated by primes 2, 3 and 5) always produce sounds that the human ear deems 'musical'.

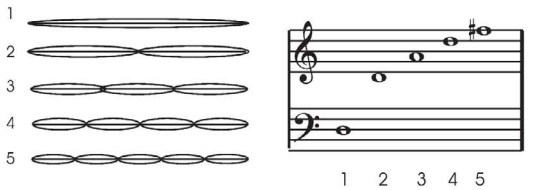


Figure 2

1 Freeman, K., *Ancilla to the Pre-Socratic Philosophers*, p.76.

It is not difficult to detect a philosophy embedded in the results of such experiments: that a string, itself merely a lifeless length of material, becomes possessed by some mysterious *animus* when the peg tightens and the string becomes taut. This *animus* is of course the very soul of harmony, and it's mysterious because it appears to 'know' things. We might even call it numerate though it is interested only in computing whole numbers – *integers*. There is no room in the domain of this vibrating string for remainders or leftovers (who ever heard of half a vibration?); and just as all things on the great cosmic string are fitted tightly into a celestial harmony where each would be in connection with each - no hint of separate existence - wasn't it also a fact that when any 'earthly' string is affected in one place implications would be felt along its entire length?

Touched at the fifth part five standing waves each one fifth of its whole length would instantly appear, calculated by this animus that only tolerates exactitude, and prompting the fifth harmonic to sound as clearly as the material and the tautness of the string would allow.

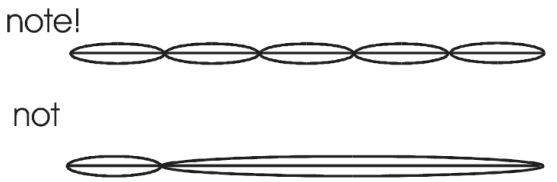


Figure 3

Now I know what you are thinking - what would happen if you created a portion that would not fit exactly into the length of the string, say 4/9$^{th's}$? Here the animus shows a cunning numeracy and finds another route to avoiding remainders - by instantly creating 9 equally spaced standing waves. Uncannily it seems to know about lowest common denominators and it cannot be tricked.

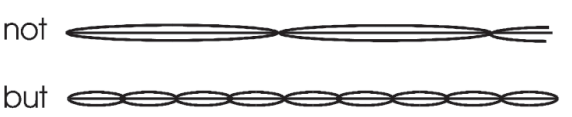


Figure 4

So two important harmonic principles have revealed themselves: equality and *completeness*. The harmonic animus seeks and finds elegant solutions to all things which stand in the way of these principles, mending fractions, aligning misalignments, always making whole what seems to have fallen apart. Of course, these higher numbers represent acute pitches and, as said, their clarity is dependent upon non-mathematical factors such as the quality of the string and its tautness; but the animus itself is perfect and will 'pour itself' even into an elastic band tightened around a matchbox – I know, I've demonstrated it publicly.

Although Ernest spends little time talking of vibrating strings, the monochord provided for him and the ancient musicians, a natural foundation for harmonics.

2. The sexagesimal system

That then is one source, but there is another, discovered by the Sumerians some time in the third millennium BCE. - the sexagesimal or base 60 counting system. Aristotle remarked that where leisure time is available man makes the greatest progress in both arts and sciences[2]. Although he cited the Egyptians he might also have had in mind that Mesopotamian land, then so fertile, skirting the environs of the Tigris and Euphrates. It was here that the sexagesimal system first made its presence felt. But nothing comes from nothing. The evergrowing complexities of trade made crucial a simplification of the myriad of measuring systems, forcing the powerful King Shulgi to institute a 'think tank' of scribes to solve the problem[3]. It seems that the sexagesimal system, a system that scribes had already been experimenting with, was this solution. Base 60 arithmetic, so rich in factors and sporting the first known place value system, provided a perfect interface for all other forms of measurement. One measure could be translated into base 60 and back out again into another different measure. So final calculations would never be made in sexagesimals but in whatever system was being interfaced into it. This system was always too sophisticated for mere weights and measures and the scribes that achieved this economic game changer were hardly going to be satisfied to rest on their laurels.

2 Aristotle, *Metaphysics*, 981b.
3 Hoyrup, J., *In Measure, Number and Weight*, p.78.

So these mathematicians, who had previously been at the beck and call of priests and merchants, became an important class in their own right, claiming the leisure time to experiment and innovate, just as later in Plato's Academy and other Greek institutions, mathematicians, musicians, astronomers were able to pursue their vocations, at their own pace and according to their own whims, to the great enrichment of succeeding generations.

How was it that these sexagesimal investigations proved so crucial for music? Here we must resort to our imaginations. It would not take much for a musically minded scribe, knowledgeable on the harmonics of the monochord, to quickly see its shortcomings. The natural harmonics of the vibrating string do not give us scales and musicians need scales for melodic formation. The problem is simple. We have this vibrating fundamental which aches to be the starting note of a scale, and yet the next harmonic turns out to be the end-note, over before it has started! How to access that empty octave becomes a scribal problem and, as we shall see, a sexagesimal solution is close to hand.

Let us imagine a train of thought that may have occurred. At the beginning of this article the musical primes were mentioned: 2, 3 and 5. Our scribe would have perhaps been excited when he realised that these are the three primes that also lay at the heart of the sexagesimal system. If our empty octave were 30:60 rather than 1:2, all the regular[4] numbers of this 'octave' could be arrayed:

30 32 36 40 45 48 [50][5] 54 60

If a fingerboard of a lute were marked out with frets or movable bridges according to these numbers, that is, from the middle of the string to its open 'nut', the scribe's mathematical instinct would have told him that the result would be musical for they were all multiples of primes 2, 3 and 5, those beloved by the human ear.

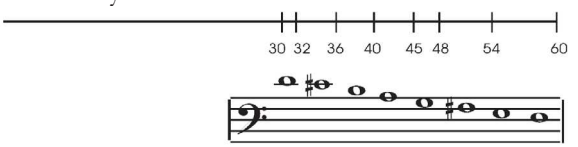


Figure 5

The major scale is very familiar to us, yet I dare say that plucking these notes from 30 to 60 would have had as profound an effect on our scribal musician as the octave emerging from the first harmonic when heard for the first time. His musical instinct would have soon unravelled the slight irregularities in the scale steps, what we call tones (t) and semitones (s), and observing this pattern, displayed as a double octave, his newly acquired reciprocal thinking would tell him that the pattern would take a rotation and therefore the possibility of a reciprocal scales

s t t t s t t s t t t s t t

Reciprocal scales

s t t t s t t>s t t t s t t

s t t t s t t s t t t s<t t

This arrangement of intervals has come to be known as the diatonic system. Any sequence of notes taken from this system, either way round, will fit somewhere on this double octave pattern and therefore be diatonic. The result will always be 'musical', so much so in fact that we could be forgiven for thinking that it was hardwired in our minds. Whether this be true or not, it arises from the simple 'regular' numbers of the sexagesimal system and its pure ratios have become known as 'just' intonation.

Before we continue I would like to discuss the matter of reciprocity a little, otherwise it will be difficult to understand Ernest's working methods when we approach the matrices. He was indeed the last ancient Sumerian as far as I am concerned, and nobody could manipulate note and number quite as he could.

I maintain that his deep admiration for these ancient scribes was well deserved. Their discoveries were a civilising influence on mankind that is still felt in our modern daily lives. Their 60 base system took over the world and we still base our astronomy, geometry and time cycles upon it. But its most subtle refinement was its *place value system*, giving mathematicians for the first time the ability to tackle reciprocals. Ernest's musical matrices and mandalas, which we are about to study, simply would not have worked without the element of rotation[6] and they cannot really be

4 'Regular numbers' refers to numbers of the 60 base system. i.e. squares and multiples of the primes 2, 3 and 5.

5 50, though a regular number, has no role in this scale.

6 In this article reciprocity, rotation/counter rotation

understood until the reciprocal nature of rotation is itself fully understood. It's simple really. All you need to know is that reciprocity occurs when one, or a number standing for unity, forms the geometric mean between two numbers. The reciprocal of an integer is always its unit fraction, but in harmonics the geometric mean must not be forgotten. So though the reciprocal of 2 is ½ it is best to think: 2 – 1 – ½. If you need a reciprocal of a fraction just swap numerator and denominator, but don't forget the mean between, e.g., 6/5 – 1 – 5/6. And, most importantly in a harmonic relation, though the geometric mean might not actually be 1, it at least represents 1. So if 30 is the mean then 20 becomes the reciprocal of 45, expressed as ratios 45:30:20. However, without understanding the role of 30 as the mean, it is easy to get confused; so care is needed. Here is a very simple visual description of this system:

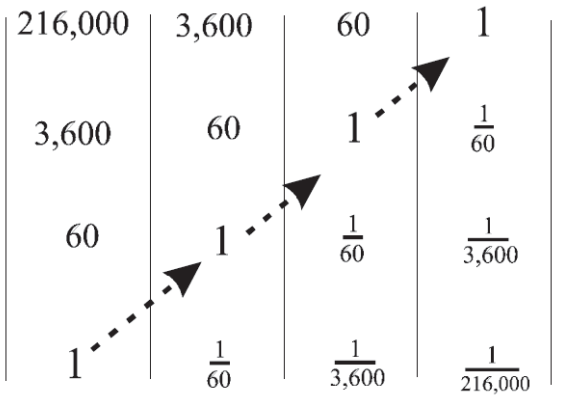


Figure 6.

Another principle of harmony that runs through all Ernest's matrices and other diagrams is that, unless circumstances forbid, relations are always expressed in their simplest form. If the harmony 6:5:4 loses its 5 for some reason, 6:4 returns to 3:2. Forgive me for sounding like a textbook here, but these principles are fundamental.

We saw reciprocal relations arising on the monochord between the number of standing waves and their lengths. This is a simple reciprocity that seems to be telling us of further, more complex inverse symmetries that infuse the whole underlying structure of harmony. This finds its most perfect expression in the Sumerian mathematical and musical mind.

The rotated scale, hypothetically conceived by our scribe, because it uses the same intervals albeit in reverse, would also have the same numbers but in reverse. But these would be unaligned to string length in the form given; so, commencing from the same note **D** but in the reverse direction, it would be simple to unravel these intervals sonically (by ear). A semitone step would take it to E♭ and continuing step by step as below:

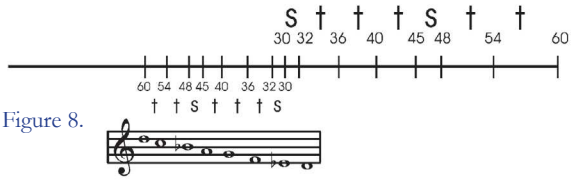


Figure 8.

In this diagram the intervals (T and S) are more important than the numbers as the scribe would have played the rotated scale 'by ear'. These notes could not be marked on the fret board using the simple sexagesimal numbers 30 to 60 although it is not apparent immediately why. We will not rush towards solving this rotational conundrum caused by two competing calibration methods, but we should at least set them down unaligned but side by side:

D	e♭	f	G	A	b♭	c	D
30	32	36	40	45	48	54	60
D	c♯	b	A	G	f♯	e	D
	s	t	t	t	s	t	t

Now we stand back to regard the numbers and their identical step patterns, hopefully in the way our lute playing scribe would have done many millennia ago. It is hard trying to imagine his mind set but he would have known certain things for sure. If he lengthened a vibrating string, all things being equal, it would drop in pitch. He would also have known that if this vibrating string were shortened, the pitch would rise. Of course this would be experience more than science - *techne* more than *episteme*, for he could not have know the results of Mersenne's experiments which taught us that string-length varied inversely with the frequency of vibrations. Yet even this he may have intuited, as Archytas the Pythagorean did after him[7].

and inverse symmetry mean essentially the same thing.

7 Barker, A., *Archytas, - Fragment 1. Greek Musical Writings*, p.41.

But though neither would have had the technology to have backed up their hunches, certain principles would have been crystal clear:

- if you double the length of a vibrating string it will drop an octave in pitch.
- if you halve that same length it will rise an octave.

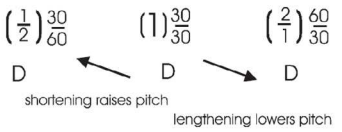


Figure 9.

Of course, this is how we would write it - I have no idea of what method or notation he would have used as they tended not to show their working out.[8] Surely, however, on reaching this point he would have seen a method of alignment for all the numbers and notes, for in the same way that, if he lengthened the string by 32/30 (i.e. 1/15th) it would drop by what we call a semitone in pitch, similar shortening would raise it a semitone. In other words, 30/32 > 30/30 > 32/30 describes the aligned status of eb, D and c♯. This for him would have surely been a quite straightforward though a lot of 'flatfooted' calculation would be involved[9]. I do not propose to go down this route for that is what the matrices are all about. They do the work for you in such an intuitive way that you come to understand it completely.

3. Lambda and Matrix

The term matrix describes two kinds. They are rather similar in appearance to Platonic Lambdas but differ in three distinct ways.

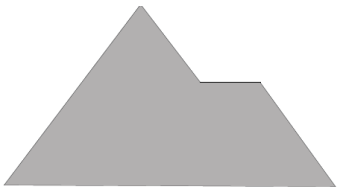


Figure 11.

- The lambda was portrayed by Plato's Academy as a pyramid whereas the matrix characteristically is a ziggurat, or stepped pyramid.
- the lambda, as does the matrix, presents itself as a a field of inverse symmetry; but where Plato deals with Pythagorean primes of 2 and 3, Ernest concentrates on primes 3 and 5, brilliantly using 2 as a kind of 'octave regulation'. It is this regulation that creates the steps in its right side.
- Also, whereas in the lambda everything fountains down from 1 at the summit, Ernest turns his matrix on its side, placing the 1 as the cornerstone at the left vertex, as did Nicomachus, but rising up rather than down.

Let us have a look at them:

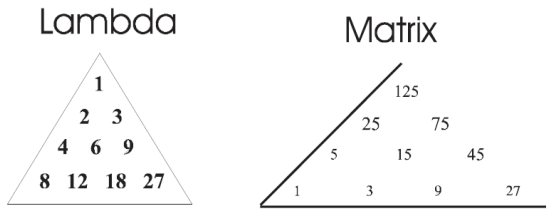


Figure 12.

Here we see the Lambda with its transversal $(3/2)^2$ symmetry included. I mentioned that there were two forms of the matrix. In the first, shown here, prime 2 is absent, and one sees the pure inverse symmetry of the matrix field without interference. It is very important to see the distinction between this type and the matrix which includes prime 2, for the latter interrupts the inverse symmetry quite markedly in its function to limit all the elements within it to an octave. Because it only contains the 'male' odd primes of 3 and 5 and their multiples, I suggested to Ernest that we should rename this type Patrix. He thought it was cute idea except that 'patrix' wasn't really a word. Well it is now!

So the patrix has no steps on its side because it is in theory unlimited, though Ernest often gives it the same size and shape as the natrix it accompanies. This can be seen in Chart 11 of his *The Myth of Invariance*.

The steps arise simply when the limit imposed by the octave becomes a natural cut-off point. So the kind of limit we have been discussing, the sexagesimal 30/60 octave, would exclude numbers that exceed 60. That is the shape-shifter that lurks in every matrix.

8 Robson, E., Mathematics in Ancient Iraq, '… almost all calculations were performed mentally … good scribes never showed their working.', p.78.

9 See Appendix.

But what about numbers below the limit? Are they to be cut off also?

The answer is no, because they can be doubled into the octave. For example, 3, on the bottom line of the 'patrix' doubles to 6 - 12 - 24 - and finally 48, which is within the 30/60 octave.

Look at these two diagrams which show how the numbers are first doubled into the octave, and after that are shuffled into scale order. The 'patrix' doesn't show things in scale order but rather presents the deep structure of music itself, before scales are even conceived. The matrix does both.

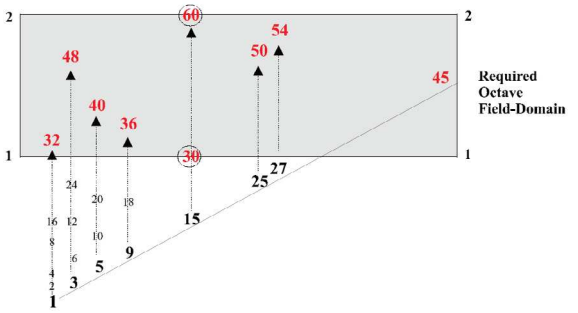


Figure 13a.

The above diagram explains the sexagesimal doubling process, as the prime 2 begins to do its work. The diagram below brings everything into numerical and musical order:

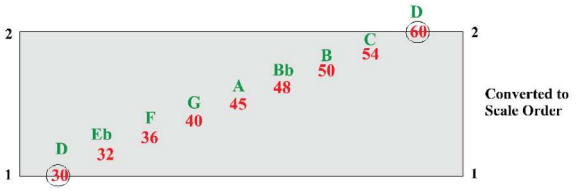


Figure 13b.

So this is part of the story the matrix tells. As a metaphor it illustrates the transitory world of scale formation, the song or melody, as it were, but it also unfolds the principles that shape the song. All this is held in a perfect harmonic unity - a study not of the many in separation but of the many in the service of the one - a truly Platonic conception.

The simplest example of a ziggurat or stepped pyramid matrix, and perhaps the most obvious for us to examine, is the 30/60 matrix as it contains all the numbers and notes that we have been studying in the company of our musical scribe. We must not forget him, for I want to show that it was possible for the Sumerians to have constructed something like the matrix though obviously not in every detail. Whether they actually did is not really of primary interest, but I truly believe they understood its principles, as soon as they had got to grips with inverse symmetry. There is, in my opinion, no better way to study the way the ancients thought mathematically and musically than through the perspicuity of matrix technology. I hope to show the potential of this in future papers but here my aim is to try to show people Ernest's methods, and hence how to become handy with matrices and inverse symmetry. We today, in our linear, time-ridden society have lost the art of what Philolaus described as 'starting from the centre'[10].

The 30/60 matrix is not difficult to understand if the previous two diagrams have been understood:

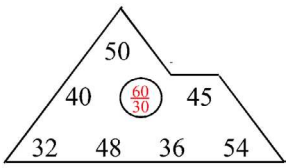


Figure 14.

As shown we double the smaller patrix numbers into the matrix and leave out all those that are greater than the limit. This is the way all matrices work. There are harmonic lessons here to be remembered. One is the principle that numbers when unobstructed always fall into their simplest formulations. You can see that this matrix could have played out within a 15/30 octave except for the presence of 45 in the second row. There is always at least one 'rogue male' that strains the octave to its limit.

Except for the rotation point, which remains **D** under rotation, all these numbers represent two pitches, from reciprocal scales. Now we can see how elegantly the matrix itself rotates to begin the alignment of the two scales. To avoid confusion I have omitted the numbers, but every rotated number is the reciprocal of the original.

The rotation, as Ernest always applies it, is to the matrix of string-length measurements. It could have been the sub-multiples, perhaps logically it should have been; but it doesn't really matter and

10 Levy, E., *A Theory of Harmony*, p.75, 'The world is One: it began to develop from the middle.' – but also see the more extensive introductory quote from Freeman.

certainly does not affect the final result.

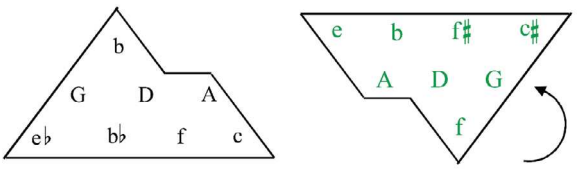


Figure 15.

The rotation, as Ernest always applies it, is to the matrix of string-length measurements. It could have been the sub-multiples, perhaps logically it should have been; but it doesn't really matter and certainly does not affect the final result.

(Now we can see that the unwanted 50 in the form of b and f really belong to reciprocal scales.)

Next, an important step towards final alignment of these scales sees the rotated scale placed upon the un-rotated at the point of rotation.

The numbers that are rotated become reciprocals but we don't have to worry about them for the matrix does all the work. (Generally in these diagrams letters or numbers in green represent reciprocals. Also in the following diagram the numbers are in red to ease visual congestion...)

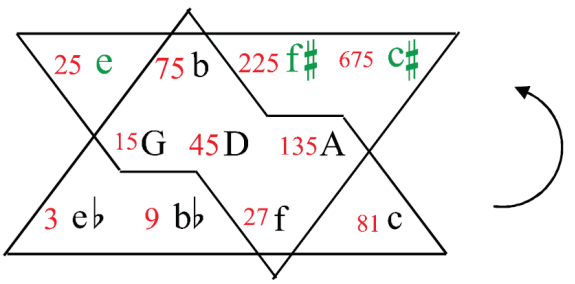


Figure 16.

Do not be alarmed that all the numbers have changed! This is simply a case of harmonic expansion, for it is seen that the previous rotation point can no longer accommodate the new numbers. In order to make clear what has happened I have reverted to patrix number values. But I want you to understand this thoroughly for it is quite an elegant example of our harmonic animus, that it is still calculating. The explanation is clear if we go to the left corner of the ziggurat and grab a piece of matrix DNA. This is like a single molecule of the matrix field; and remember that every part of this field is identical except for position; so this small molecule will allow us to get close up to its inverse symmetry.

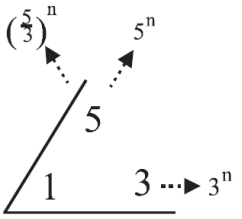


Figure 17.

We may subject this small fragment to the same kind of rotation as before. The numbers in green are reciprocals and therefore unaligned.

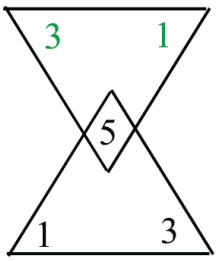


Figure 18[11].

Remembering our recent reciprocal investigations and using 5 as a rotation point we can give values to each reciprocal:

1) 3 x r3 = 52
r3 = 25/3
2) 1 x r1 = 52
r1 = 25

This gives:

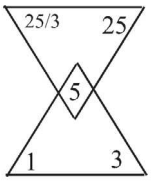


Figure 19.

But remembering our *animus*' dislike of anything other than whole numbers we must clear fractions by multiplying throughout by 3.

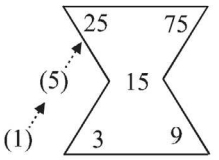


Figure 20.

And this opens a new column of 5's on the left bank.

11 The numbers in green are, as yet, unaligned reciprocals.

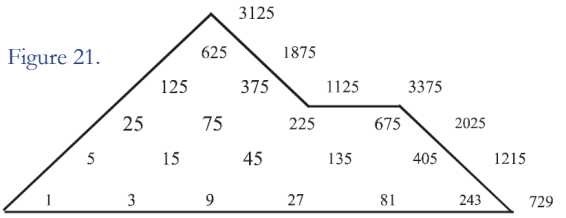
So all we are doing is utilizing Sumerian reciprocal technology, cutting edge at the time, and even today fascinating as an exercise of mind for all of us who prefer to let a calculator do all the work. (Remember a calculator never reveals to you the workings of your own mind!)

Everything we have done so far was within the range of a group of musically minded scribes, though they would have conceived it their own way.

This little exercise in reciprocation has solved the first problem of the rotated matrices. Because the left 'wing' protrudes beyond the unrotated matrix, an extra column of 5's has been created sloping upwards left to right, and this has had the effect of multiplying all the elements by 3. So the rotation point moves across from 15 (30/60) to 45/90. However we now have another 'rogue male' to contend with on the right 'wing' and this compels the rotation point to double 3 times to 360/720.

It is still useful to retain the patrix format while we search for any new numbers that may fit into out 360/720 octave limit and that being soon done we are in a position to present the matrix for a year of days and nights, which is to be found at the bottom of Chart 11 of *The Myth of Invariance*. For this chart I have extended the patrix, which should explain why the shape is as it is.

Figure 21.



Still to do is to double all notes that need doubling into the new octave using the same technique as before (figs. 13 a and b).

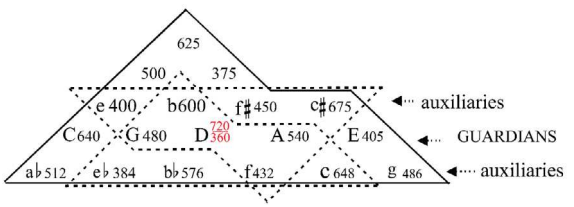


Figure 22.

I have taken the liberty of adding a few things. I have included the previous 30/60 matrix for its rotation as it may help to visualise the whole process. Also I have designated the transverse axis containing the rotation point as Guardians. The two rows on either side are 'auxiliaries'. This is of course terminology from Plato's *Republic* carried over from Ernest's *Pythagorean Plato*. These three rows have a special meaning in the matrix and to understand it we need to begin to study it in a different way.

4. The Philosophy of the Matrix

Man has always admired permanence. Sumerian kings and Hebrew saints were said to have had impossible longevity. That nothing can live forever even the ancients acknowledged; nobody can defeat time itself unless it is immortal (the sempiternal merely fights a draw).

Plato begins his description of a kind of perfection:

> 'It is very difficult for a state constituted thus [according to the ideal] to be moved from within. But since all that has been generated is subject to corruption, such a constitution too will not be able to endure always but will disintegrate.'
>
> *Republic* 546.[12]

Plato's sober appraisal here is of his Guardians' inability to rule the state forever, even in an ideal world - that tiny blemishes would creep in, accruing incrementally in the fullness of time.

In *The Pythagorean Plato*, his follow-up to *The Myth of Invariance*, Ernest symbolised the Guardians as those who watched over the state from the transversal axis of the matrix, which contained the rotation point (sometimes called 'throne'). This row is always marked by capital letters, in which the supreme ruler (perhaps in this case the philosopher king) is always seated at D.

Genetically and musically this 'powerline' is generated by Pythagorean prime threes, representing the highest property class. This prime provided intervals nearest to what we today would call equal temperament (ET) which for the ancients would have been an unattainable ideal. Today we have conquered the technology but, in the process, lost its significance. What was a perfect

12 Ficino's Latin translation rendered in English by Michael Allen, slightly amended by myself for the purposes that I am using it.

democracy for Bach later turned out to be a kind of indeterminate tonal communism inhabited by spurious hierarchies of its own, far removed from the aristocratic guardianship of an idealised Athens.

Equal temperament values each semitone at an even 100 cents and the entire octave at 1,200 cents. Pythagorean tuning, generated by prime 3's, comes astonishingly close to this ideal - a perfect fifth is 702 cents against the 700 of E.T. and a perfect fourth of 498 against E.T.'s 500. Therefore when all 12 chromatic tones have been generated in the Pythagorean way, the 13th only overlaps the octave by 24 (12x2) cents -- discernable, but not easily so in the flow of music. Here are two ways that Ernest presented this cycle of fifths/fourths:

A♭ < E♭ < B♭ < F < C < G < **D** > A > E > B > F♯ > C♯ > G♯

Alternative view:

D > A > E > B > F♯ > C♯ > G♯

A♭ < E♭ < B♭ < F < C < G < **D**

The overlap of A♭/G♯ is harmonically $3^{12}/2^{19}$ (531,441/524,288) commonly known as the Pythagorean Comma.

This reveals itself in a serpentine chain of perfect fifths/fourths within the confines of an octave.

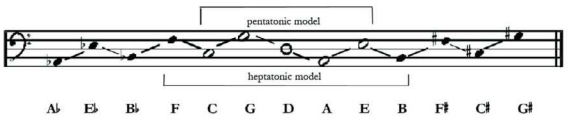


Figure 23.

Intervals constructed of Prime 5 have no such sterling qualities, being noted for their pleasure rather than their sobriety and stability. No one, apparently, can resist their transient beauty, though Plato himself was wary. To the ancient Greek the major third interval was not even given the accolade of a consonance. However it seems that the Sumerians and Babylonians were willing to succumb to its pleasant sonority and prime 5's as we have seen were liberally sprinkled throughout their sexigesimal scales. For this reason it is difficult to understand how Pythagoras, returning to Greece after his enforced sojourn with Babylonian priests showed himself immune to its charms by banning it from his tuning system. When Shakespeare wrote 'Lilies that fester smell far worse than weeds[13]' he could just has well have been describing the 'short lived' poignancy of the musical prime 5. But we are speaking harmonically here, for prime 3 harmonies cannot generate such poignancy as their prime 5 counterparts (the 81:64 major third is far too briny at 8 cents above ET than the 5:4 major third - even though it falls 14 cents below, our ear would always choose the latter). When untutored singers with 'a good ear' sing a major chord together, they naturally aim at the 4:5:6 'just' version. They would say that it 'just' sounds better.

But, looking again, the matrix seems to agree with Shakespeare. If we stacked 3 of these 5:4 intervals on top of each other we should, if they were true, obtain an octave[14]. Yet, sadly, we do not; and the answer lies in the first column of rising powers of 5 on the left ascent: $5^3 < 2^6$ (125:64) - a falling short that truly 'festers' in the ear as an approximation of an octave. Where did all that sonority go? In truth, prime 5, bewitches us to follow it into a tonal wilderness bringing to mind another quote from the word-master, 'all that glisters is not gold'.

The matrix tells all. The throne axis-line is shorn of prime 5 substitutions but below and above this line are but a single prime 5 removed and mark the limit of just intonation. One step beyond this in either direction is a step towards a musical Babel, a tonal no mans land, filled not so much with dissonance as with a cacophony - the 15:16 interval is the sharpest dissonance in music, but it's a true harmony. *Kakos* means 'bad' in Greek, and these intervals get worse and worse as they move from simple enharmonic obscurity (b♯ for c) into the quagmire of double and even triple sharps and flats. In the great Revelations matrix this musical nightmare extends to seven rows above and seven rows below. I wonder to myself whether outside rows are Dante's seventh circle of hell. It's just a thought. Ernest himself once told me that he wished he had not put all the 'non musical' notes in. I disagreed, and still do. They convey the fragility of civilisation, and what awaits when the music

13 Sonnet 94.

14 Three equal temperament major thirds exactly span an octave.

stops and civilised society is no longer valued[15].

So all pitches beyond the transverse row of the rotation point are marked in small case letters. It's taken me a long time to tell you this, but probably you guessed anyway. In *The Pythagorean Plato*, the lines immediately above and below this Pythagorean power line are Just rather than Pythagorean. This is, for example, where the 10:9 tone is to be found as opposed to the Pythagorean 9:8 on the middle row. Just intonation combines both prime 3 and prime 5 as we saw in the sexagesimal numbers. Civilisation exists only within the limits of these three lines, with the Platonic Guardians skirted by the Auxiliaries, who defend the ideal republic from the 'barbarians' of the Wild Wood without. This was the Greek 'musical' view of civilised society. We are still not agreed as to whether they had it right or not.

5. The Eternal Mother

I have spoken of the characters of the primes 3 and 5 but we also need to talk of prime 2, which in many ways is the most remarkable of numbers. The octave interval, 1:2, is in fact the only interval of equal temperament in the ancient octave, valued at exactly 1200 cents in modern currency. It is the prime that gives the matrix its name and meaning and as Greek *mater* or mother is the first 'female' or even number, after God as 1 - this, according to Pythagorean numerology. It is also philosophically the *dyad*, or, as Aristotle calls it, the 'doubler[16]'. For in the step from 1 to 2 all other numbers can be harmonically arranged in pattern; and, in an important sense, as harmonics constantly teaches us, it is also the last number, for beyond the 1:2 octave all is mere repetition at higher or lower pitches; no new musical sound is added beyond the octave that is not also represented within the octave.

Ptolemy pointed out the distinction between *di okto* - through 8 (notes) - and *diapason* - through all - the latter alluding to the whole pattern or Form[17]. That the octave as *diapason* is the model or form of all octaves is borne out by the matrix, for though it can contain a great multitude of octaves they are contingent upon a single diapason at the rotation point.

When I read Plato's account in *Timaeus* of the Nurse and Receptacle of All Becoming I was struck by its similarity to Ernest's matrix[18]. Plato describes a kind of prime material which is so devoid of characteristic that it was able to reflect all things without distortion. He adds:

> 'Manufacturers of scent contrive the same initial conditions when they make liquidswhich are to receive the scent as odourless as possible.'
>
> *Timaeus* 50e

If we consider the octave, it is a similar base solution, colourless in itself but fixing all other tonal colours without impediment to them. Aristotle must have been commenting on this passage when he writes:

> 'His [Plato's] conception of the other principle as a duality to the belief that numbers other than primes can be readily generated from it as from a plastic material.'
>
> *Metaphysics* 987b

The Greek word for 'plastic material' is *ekmasso* the same as used in Timaeus, and, interestingly, is sometimes translated as 'matrix'. I think both matrices, Ernest's and Plato's, point to the same thing, whether they come from different directions or not. This is Two-ness or the Dyadic Principle which expresses itself though music as the octave and is not unlike a like a womb (another Platonic image from the same passage), for all manner of things are brought forth from it.

This mystery of the Two was not lost on the Vedic poets. As described in *The Myth of Invariance*, the serpent Vrtra is (continually) slain by Indra. Ernest likens this to the coils of the monster, representing the sound continuum (think steel guitar!) being cut by Indra's sword in sections and creating the particular pitches. This is analogous to the Greek concept of *peras*, or limit, binding *apeiron* the unlimited.

15 'Just' intervals and their relative inaccuracy are encoded in what is known as the syntonic comma (81:80) and each row on the matrix is therefore set a syntonic comma apart. Note E as 405 on the axis row and 400 for e on the row above. 400/405 = 80:81. The need for harmonic limit is clear.

16 Aristotle, *Metaphysics* 987a. as usual deriding Pythagorean notions of which he had little interest or, if truth be told, understanding.

17 Ptolemy, *Harmonics*, Book 3 Chapter 1.

18 Plato, Timaeus, 50c

6. Examples of Matrices.

In this final section I would like to examine some of Ernest's matrices, mainly from *The Myth of Invariance* and subject them to the same analysis as I have been discussing throughout the article, and avoid in the main the mythological aspects, leaving it for the pleasure of the readers to tease these out for themselves.

The first matrix is from Chart 11 in *The Myth of Invariance:*

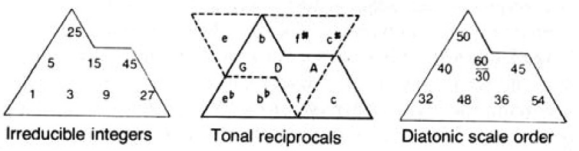


Figure 24.

These are all really the same matrix and I have already dealt with it in depth. I only want to comment on the three central elements, that is, on the transverse axis of rotation. This is the minimum requirement for a matrix.

These are the invariants in Just Intonation. G and D swap places under rotation but D retains its identity. Every guitarist will tell you that in the key of D that the 'three chord trick' are the chords D, G and A. More formally they are the tonic, subdominant and dominant triads (I, IV and V) forming the three 'natural' centres of gravity within the octave.

The next matrix is from the upper section of Chart 12:

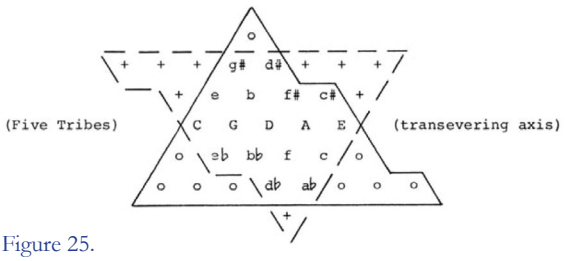


Figure 25.

The five-element throne axis of C-G-D-A-E form the simplest of scales, the pentatonic. In this it is similar to the 360/720 matrix on the lower part of Chart 11 (see fig. 22 above) but the throne in this larger 60^2 (1800/3600) matrix is on the central (third) row rather than the second. Larger numbers generally mean larger matrices; but Ernest looks for the most economic way to get the message across which is, in both cases, the scale that he often associates with the blissful youth of mankind, the Garden of Eden, which, up until the time of 'man's first disobedience', was without discord. When a child approaches a piano it doesn't take him too long to discover the pentatonic 'black keys' which he can play to his hearts content never 'hitting a wrong note'. (Parents may not always agree!)

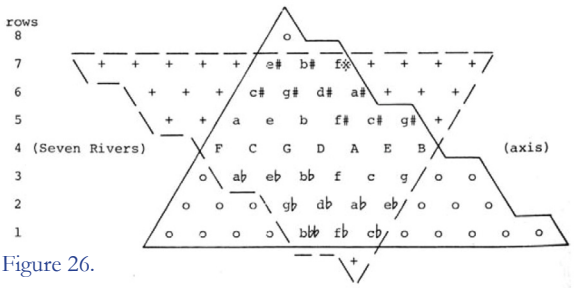


Figure 26.

In figure 26, we are working along the sexagesimal floating place system. This is the matrix for 60^3 or 216,000 which is officially the limit, though there is evidence that they took it beyond to 60^4 (12,960,000) - certainly Plato thought so. There is an addition of two extra elements along the axis, giving F-C-G-D-A-E-B, and -- with the arising of the tritone F/B -- the possibility of discord never known in Eden! For this loss there is the gain of seven note scales and modes and a fully functioning diatonic system, although those harmonies made so sonorous by the admixture of of prime 5 are not available to prime three Pythagorean temperament. Consequently major and minor chords will have a slight dissonance to them and perhaps this is why the Greeks, who used this system, (officially) rejected them as concords.

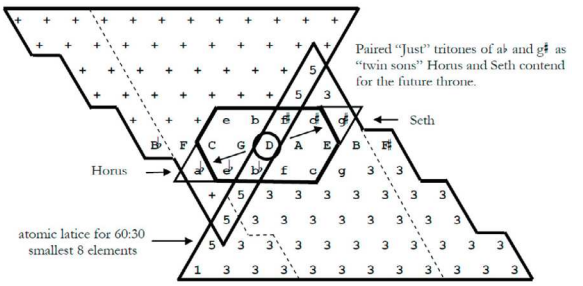


Figure 27.

These final two matrices are from Ernest's *Narmer* article in ICONEA 2009.

The axis row here is less important as the main point of figure 27 is to establish the tritones a♭ and g♯ in prime 5 Just Temperament, as did the

Pythagorean Comma for prime 3 Pythagorean Tuning. The rising 5's and linear 3's are, I believe, just a bit of 'McClain shorthand'. The 'squashed' hexagon at the centre is often described by him as an Egyptian cartouche hieroglyph often found on tombs, and containing specific data. The harmonic data here seems to be the wedding of the heptatonic (seven-note) Just intonation with the Pythagorean pentatonic. Beyond the cartouche at top far right and bottom far left, as Ernest points out, are the tritone reciprocals, harmonically valued at 32/45 and 45/64. As the tritone is also the proportionate middle of the octave, these are also square root approximations, one just slightly below, and the other slightly exceeding, the square root of two.

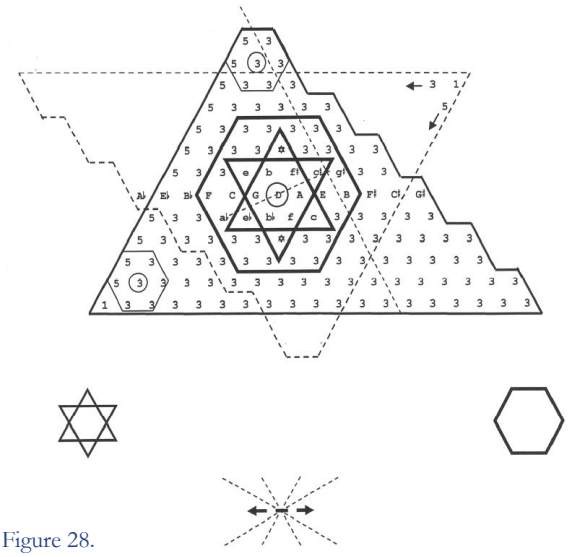


Figure 28.

Figure 28 is the matrix for 777,600,000 which Ernest calls the YHWH matrix interpreted from gematria. What is harmonically interesting here is that the un-rotated matrix only contains one element of the Pythagorean Comma G♯, but under rotation it aquires the A♭ also. So we have a rotation axis of all 13 tones of prime 3 Pythagorean tuning to match the prime 5 just equivalent of the last matrix.

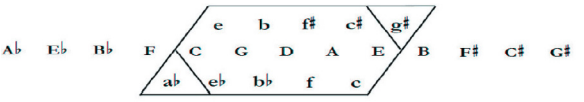


Figure 29.

This figure is 'distilled' from the same matrix but given a different reading, for you see here both Just and Pythagorean full chromatic scales coalescing with each other.

This then completes my analysis of Ernest's tremendous achievement. I have tried in some small way to explain both their harmonic basis and what I consider their philosophic basis; and I only hope that it makes further reading of his works a little more interesting for those who could not 'get' his numbers. I know there are a few of you.

Appendix

The Flat-Footed Method:

720 648 576 540 480 432 384 360

16 --------15 10 -------- 9 -------- 8

10 --------- 9 --------- 8 9 --------- 8 16 -------- 15

60 54 48 45 40 36 32 30 (x12)

9 ------- 10 15 -------- 16 8 ---------- 9

8 ---------- 9 8 --------- 9 -------- 10 15 ------- 16

(9x8 10x8 10x9) (12x8 12x9 12x10)

(15x6 16x6) (8x15 9x15 9x16)

72 80 90 96 108 120 135 144 (x5)

360 400 450 480 540 600 675 720

Multiplication by 12 and 5 in Platonic terms 'render them conversable' or aligned to the same harmony.

The Final Integration:

360 384 400 432 450 480 540 576 600 648 675 720

rising

D eb e f f# G A bb b c c# D

falling

D c# c b bb A G f# f e eb D

The just chromatic scale with all 12 notes aligned with each other either way as reciprocals of each other. The 13th note A♭/G♯ must be obtained by other means.

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SEVEN, YES BUT...
or the Truth about Babylonian Music Theory

Richard DUMBRILL[1]

'In describing non-western music, be it Oriental or primitive, one must strictly refrain from misusing incongruous concepts of western music. The terminology that has been learned in music school applies to the harmonic structure of music and is inappropriate, indeed misleading and distorting in descriptions of non-harmonic, non-western music'
Curt Sachs, *The Wellsprings of Music*[2]

Introduction

The title of this article was not meant to benefit from the current world trend of 'untruths' with which we are saturated[3]. It is coincidental but nevertheless opportune. Truly, there has been a chain of 'untruths' about the history of Babylonian music theory for the past fifty-seven years, and growing strong.

I will address the matter, its causes, consequences and remedy.

The tablets examined in this article are the oldest texts of music theory ever found anywhere in the world. They were published from the early 1960s onward as the corpus increased when new texts were discovered[4] (mainly in the museums where they were kept). Authors approved each other's interpretations with meaningless addenda. According to their authors, the Babylonian scale could only be ascending, tense diatonic[5],heptatonic and octavial because for them, 'it could not be anything else'.

Then, in 1994, a paper[6] fuelled by the new reading of a verb, turned the world upside-down and all, or most, agreed that the scales were descending[7].

Another paper[8] claimed, extraordinarily, that the intervals listed on a tablet were to be played simultaneously, a view resting on no evidence, as there is, to my knowledge, no known comparable system, anywhere in the world, past and present. This dogma met with the horns of dilemma with Kilmer's interpretation of the Hurrian "hymn[9]": Which of the two notes to sing?

The problem was solved, laboriously, when Kilmer 'spin-doctored' the matter and decreed that the paired pitches, were the accompaniment of the hymn, and that either bass or treble pitch of

1 Richard Dumbrill is the leading expert in Near-Eastern Archaeomusicology to which he has devoted all his working life. He is the Director of ICONEA, a research Group at the University of London, consultant to the British Museum and member of the scientific Board of MUSICOMED, a research team at the Université de la Sorbonne, Paris. His expertise is recognized among the highest institutions, from Oxford to Harvard and Yale, to China, to Iraq, Palestine, Lebanon, Syria, etc. He has published intensively and his The Archaeomusicology of the Ancient Near East is the textbook for universities worldwide. He is the editor of ICONEA, London; co-editor of NEMO-Online, Beirut and London; and co-editor of ARAM, Oxford.

2 [Sachs, 1962, p. 49].

3 Barry, D., quoting advisor to President Trump, who used 'alternative fact' to describe assertions by the White House: 'In a Swirl of "untruths" and "falsehoods", calling a lie a lie. *The New York Times*, Jan 25, 2017, https://www.nytimes.com/2017/01/ 25/business/media/donald-trump-lie-media.html.

4 *nabnītu* xxxii; CBS 10996; U.7/80; YBC 11381; CBS 1766; H6 (RS13.30 + 15.49 + 17.387). These references are well known to Assyriologists. They refer to various collections: CBS = *Catalogue of Babylonian Section of the University Museum, Philadelphia, Pennsylvania*; YBC stands for *Yale Babylonian Collection*; H refers to the tablets of Ras Shamra. *nabnītu* xxxii is the name of a series of tablets. The tablet is also known as UET VII 126, UET standing for *Ur Excavation Texts, volume VII*, Plate 126.

5 It is clear that these authors did not know of other diatonisms. Obviously, they meant that it was 'ditonic diatonism' to which they referred as it addresses the only type in Western theory. Beyhom, in his Hypothesis [Beyhom, 2017] uses 'ditonic' to differentiate tense diatonism which has two Pythagorean tones in the Just fourth, from other types of diatonism.

6 [Gurney, 1994, p. 101–106].

7 A descending scale is not the contrary of a given scale, i.e. ascending *c-d-e-f-g-a-b-c*, is not *c-b-a-g-f-e-d-c*, descending. But it is its musical reciprocal: descending *c-b-a-g-f-e-d-c* is ascending *f-g-a-b-c-d-e-f*.

8 [Duchesne-Guillemin, 1963].

9 See Chapter VI, H6: The 'proof of the pudding'?

the dyad could be chosen to make up the melody, a rather bizarre method. It usually is the melody which commands the accompaniment, and not the contrary[10].

Furthermore, the colophon does not mention any instrument, a fact dismissed by Kilmer.

Many assumptions laid unfounded, with their authors dismissing, and even censoring every article challenging their views[11]. Regardless, these flaws crept into encyclopedias and other publications[12].

For their analysis of Ancient Near-Eastern musicology, these scholars, mostly, had used Western musicological tools. Most systems can be explained by, and made to fit heptatonism, for example, by squeezing, metaphorically, pitches on and between the lines of the stave, by explaining scale constructions with elusive alternations of fourths and fifths. But none of these experts were willing to accept that other methods also do exist.

Their insistence at force-fitting a musical system into the Western model, and in this case with the 'unconscious' aim at acculturating Semitic[13] musicology under the Occidental yoke, is nothing but a last breath, it is hoped, of supremacist musicology. It is one of the greatest oversights in the history of music. It came from the methodology (or rather of its absence) of certain Assyriologists and of their determination at spearheading 'their discovery' by means of unsuitable Western models. A bit like translating Old-Babylonian with a grammar of Mandarin.

The manner in which systems are constructed, whether consciously or not[14], are part of the culture of a people and must be unveiled with the utmost respect and without linkage to theories of later cultures as this would lead to colonialist unification.

This article is the consequence of my determined endeavor at academically fostering the proof of the evidence against unproven presumptive inference, and more significantly to assert, scientifically, that heptatonism[15] – which is not universal – is by no means engraved onto mankind's unconscious. It is a structure, among others, which eventually hatched in the Near-East, as part and consequence of another or other systems, but not as a new, independent and exclusive concept.

This work is intended for a general readership. Therefore, Assyriologists and musicologist may find some of my explanations facile. I have avoided diacritic signs for Akkadian and Arabic transliterations whenever possible. I have used the English language notation c-d-e-f-g-a-b so that readers may have an approximation of the musical sets and sub-sets described on the basis that Babylonian intonation, while different, is sufficiently close to our Western practice. I have avoided as often as possible mentioning musical ratios as, while meaningless to many, they are subjectivist tools unsuited to the epistemology of Babylonian musicology.

Most obsolete musical terms are replaced with neologisms which will be explained whenever they appear or whenever necessary.

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10 [Hagel, 2005]. Here, Hagel authoritatively writes that Babylonians only could notate accompaniment but not melody! I quote [p. 290]: 'It is significant that this system was not orientated towards melody, as was Ancient Greek notation and music theory, but to instrumental practice'.

11 Madame le Docteur Marcelle Duchesne-Guillemin warned me, (in diplomatic terms, in a private correspondence) against publishing anything contradicting the current interpretations of Sumero-Babylonian music theory.

12 See for example [Kilmer and Mirelman, 2001].

13 I am using the term 'Semitic' in its etymological meaning and not relating, exclusively to the Jewish people as it is nowadays. 'Anti-Jewism' would be more appropriate since, for example, the term 'anti-islamism' is liberally used.

14 'Qu'il ait fallu en quelque sorte ce quelque chose qu'est l'analyse, et qui est venu nous annoncer qu'il y a du savoir qui ne se sait pas, et que c'est à proprement parler un savoir qui se supporte du signifiant comme tel […]' – Jacques Lacan, *Séminaire 20*, *Encore, séance du 20 mars 1973*, in [Lacan, 1975], available in audio as http://www.valas.fr/IMG/mp3/lacan-encore-20-mars-1973.mp3. This loosely translates as: 'Analysis has come to announce that there is a sort of knowledge that is not known and which is based on the signifier as such […]' – [Lacan, 1999]. There is an on-going dispute between two schools of thought, the first conditioned by dogmatic *a prioricity* (see [Field, 1998]) which sustains the irrational belief in the universality of Western diatonism(s). This position finds reasonable, firstly to infer anything without any empirical evidence, infallibly, because in this case nothing can be taken as evidence against it, and that therefore (undefined) diatonism must be the consequence of 'just intervals theory', or of the 'theory of resonance', or for the reason that 'it cannot be otherwise than it is'.

15 There are various cultures where instruments sets are tuned in precise intervals without any construction and unconsciously memorized – [Sachs, 1962, p. 103] for a detailed tuning method.

Whenever possible, I have avoided naming researchers in the body of the text. They are acknowledged in footnotes.

My usage of the following terms: dyads (2), triads (3), tetrads (4), pentads (5), hexads (6), heptads (7), octads (8), and enneads (9), etc., define 'containing intervals' having pitches inside them, i.e. C-d-e-f-G, where C and G are the boundaries of the container and d-e-f, the infixed pitches.

They differ from seconds, thirds, fourths, fifths, sixths, sevenths (heptachords), eights (octaves), etc., which are empty cells used in heptatonic tuning constructions, or for general theoretical and practical purposes.

I use the terms 'infix' to qualify pitches placed within intervals of triads and pentads. Intervals larger than pentads are made up of smaller intervals, for example a hexad is made of two conjunct intervals, a triad and a tetrad. Pitches placed before the principal infix or 'nucleus', are called infrafixes, and those above are suprafixes.

It is the many possible locations of infixes, diverging from constructed pitches (i.e. such as pitches resulting from the alternation of just fifths and just fourths) which define the cultural source of a given set.

These structures erroneously became known as 'modes', a term which only appeared during the Dark Ages of the Christian West and are only suitable for ecclesiastical types.

The theory of music is a science developed by, and made up for the amusement of the musicologist and is of little concern to the musician. However, Mesopotamian musicology is unique because its earliest reporters – the scribes – laid the fundaments of theory from their meticulous observation of the lyre, probably, and of its strings, and comments from the musician's mouth.

As such, it has drawn the most accurate portrait of pre- and early literate music, a feat never achieved before and since, in the long history of music.

I - *nabnītu*[16] XXXII: Setting the strings.

This text was excavated by Sir Leonard Wooley at Ur, Southern Iraq, in the late 1920s[17]. It dates from the middle of the first millennium BC and might be the copy of an older text, perhaps Old-Babylonian, from the early second millennium BC, and possibly earlier, I think, because of musicological and philological hints. It is a bilingual lexical text where the left column is written in Sumerian and the right, its translation, in Babylonian (Table 1). Most importantly, the text also reveals, in a second layer of meaning, an implied tuning pattern for a structure made of two conjunct pentads, amounting to an enneadic set or scale of nine pitches.

There are ten lines. The tenth says 'nine strings'. This indicates, I contend, inconspicuous indications for the harmonic interaction of nine strings. It has been advanced that the Sumerian word 'sa = string', Akkadian '*pītnu*'[18] (with qualifiers such as 'di', '2.a.ga.gul[19]', etc.) excluded the pitch to which a string was tuned. I would find it illogical that a Babylonian theoretician segregated the pitch of a string from its name in his demonstration, which otherwise would be pointless. Therefore, the word 'pitch' is a substitute for 'string', and reciprocally. The practice remains today, as the 'e' string of a violin is called the 'chanterelle' in French.

In the English language, the 'e'; the 'a'; the 'd' and the 'g' strings of the violin are tuned to 'e', 'a', 'd', and 'g' respectively. In Bach's 'g'-string Air, it is the string and the piece which take the name of the pitch. May I remind the reader that the seven strings of the Greek lyre had names which became synonymous to the pitches of the scale, in descending order[20]. There is no reason why this would not have been inherited from a Babylonian precursor, but it is even more surprising that

16 The word translates as either 1) offspring, progeny, product, living creature, 2) habitat, place of growth, 3) living creature, 4) appearance, stature, features. *Chicago Assyrian Dictionary, CAD* henceforth, Vol. 'n'.

17 [Gurney, 1974], Pl.74.

18 CAD, Vol. 'p'.

19 Sumerian sa.di; sa.2.a.ga.gul. 'sa' = 'string/pitch', 'di' means 'foremost, prime'. 2 a.ga.gul, means 'second of behind'.

20 This will be evident to the Hellenist since the names of the strings were also the names of the notes. This fact is given in most books about Greek music, for example in [West, 1992], p. 64.

scholars did not make this parallel.

These nine strings (therefore pitches), are consistently mentioned in texts from the second to the first millennium BC. This means that for two thousand years, and perhaps more, a nine-pitch system was known. However, I do not suggest that a nine pitch or enneadic (bi-pentadic)[21]scale was the only one in practice during that period. I am of the opinion that there were concurrent structures. Sumer and Babylon, had different counting systems for different things and therefore it would not be dazing should music, too, have conformed to different ones. Additionally, there would have been various regional styles adding to the sound palette. These regionalisms persist to this day in rare countries which have not yet been polluted by equal temperament, or where regionalisms are protected.

After extrapolation, I will propose that the interval between strings 1 and 2 of the front has the same value as the interval between strings 2 and 1 of the back. The interval between strings 2 and 3 of the front has the same value as the interval between strings 3 and 2 of the back. The interval between strings 3 and 4 of the front has the same value as the interval between strings 4 and 3 of the back and finally, the interval between strings 4 and 5 of the front has the same value as the interval between strings 5 and 4 of the back. Therefore, the intervals between strings 1^{f}-3^{f} (^{f} = front; ^{b} = back) and 3^{b}-1^{b} are equal; between strings 1^{f}-4^{f} and 4^{b}-1^{b} are equal and between 1^{f}-5 and 5-1^{b} are also equal. This is probably why the strings were recorded in this palindromic manner.

The nine strings should be read as 1^{f}-2^{f}-3^{f}-4^{f}-5-4^{b}-3^{b}-2^{b}-1^{b} but never 1-2-3-4-5-6-7-8-9, as most scholars did, because this would imply that the scale is heptatonic, with two added pitches, which it is not. The scale is made up of two conjunct pentads[22], such as: a-g-f-e-d/d-c-b-a-g.

The pattern can be simplified as:

1 2 3 4 **5** 4 3 2 1

With 5, in red, as pitch of conjunction.

Strings 3 and 4 (green) of the front have terms to qualify them. These Sumerian qualifications vary in their Akkadian version. The reason for this will be explained later as it is essential to Babylonian theory. Another important philological detail is that the first string is called 'sa.di' in Sumerian and so is the fifth string called 'sa.di.5', with added '5'. If 'di' means 'prime' as well as 'first', then 'di' emphasizes the value (in the theory) of strings one and five (1-5-1) because they are the boundaries of the system. The Babylonian translation does not reflect this.

Modern Western music uses the equal temperament system (ET)[23] where tones and semitones measure 200 and 100 cents respectively[24]. They are ascending, heptatonic, octavial, and (tense) diatonic, for example: c-d-e-f-g-a-b-c, a scale of C major. They are made up of tones and semi-tones arranged in a strict sequential order. For the purpose of tonal appreciation, the symmetry in *nabnītu* xxxii, can be played with out modern scale extended to nine pitches, for example: g-a-b-c-d-e-f-g-a, or its descending form: a-g-f-e-d-c-b-a-g. But it must be borne in mind that this translation is only approximate because it is constructed from a different method. To the untrained ear, the scales played one after the other would sound very similar, but would reveal differences when played together.

In figure below, columns in grey indicate tone[25]

21 As we shall see later, an ennead or set of nine contiguous pitches (tense diatonic) is made up of two pentadic subsets.

22 The numbering of the strings from one to nine led to the conclusion that it was heptatonic, with strings eight and nine being at the octave of strings one and two, but since the set is made up of two conjunct pentads, neither pentad can accommodate octaves.

23 Composers do not imagine their music in Equal Temperament. It is far removed from their creativity. However, in order to make their music playable, the transposition of the imagined music is written with it. My concern is that computer programs used by modern composers, have forced their creations into an ET infrastructure, not unlike composing 'at the piano' has, in its time, contributed to the melodic enslavement to the harmonic master.

24 The cent is a logarithmic unit used for measuring musical intervals. Twelve-tone equal temperament divides the octave into 12 semitones of 100 cents each. Cents are used to quantify or to compare intervals. Alexander J. Ellis based the measure on the acoustic logarithms decimal semi-tone system developed by the French mathematician Gaspard de Prony in the 1830s. See [Ellis, 1876] – notably p. 9-11 – and, for more information on Prony, [Anon. "Gaspard de Prony", 2016].

25 The word 'tone' is a term used to designate an unqualified interval, i.e., an interval which can be Just, Pythagorean, ET, etc. Sachs writes [1962, p. 60–61] that 'tonic' has six different meanings. 1) As an adjective used as a noun, it is the main gravitational pole of a harmonized or harmonizable

intervals; yellow, semi-tone intervals. The red column is the axis of symmetry of the system.

Although they would have had the mathematical ability[26], it is very improbable that Babylonians used the equal temperament. The size of their intervals would have differed slightly, but significantly, from our Western systems. It is my opinion that they used Just Intonation because it is the most natural manner to produce and appreciate intervals, at least in theoretical musicology, but it would certainly not have been an inflexible rule[27].

A Just Intonation[28] fifth measures 702 cents, (expressed by the ratio of 3:2)[29] (=701.955001 cents); a Just Intonation fourth measures 498 cents, (expressed by the ratio 4:3) (=498.044999 cents), etc. In the Equal Temperament fifths measure 700 and fourths 500 cents, respectively.

From this basis, it is possible to uild up an estimation of how the generative[30] Babylonian set might have sounded, but first, I shall describe the implied tuning process.

Firstly, the central string is tuned to an appropriate pitch. This will depend on the quality of the string. From my own experiments with sheep gut strings, such a string sounds its best when stretched at about 80% of its breaking point.

Therefore, it is possible to make an estimation of pitch in relation to the type of string used (gut of sheep, of fallow deer, of cow, of bull, etc.) and its length.

When the pitch of the central string is stable, (that is when it does not stretch any longer under a given tension) both the first string of the front and the last are tuned a just fifth away from the central string. The continuation of the process is explained in figure 2.

A calculation of strings parameters: length; tension; weight; section; mass, was made in order to find the most appropriate gages and tensions for stringing a lyre. I chose my 2008 replication of the silver lyre of Ur as model although it has eleven strings. I used Taylor's Equation: T=M(2L F)2, where T is the tension of the string; M the linear mass; F is the Frequency and L the length of the string.

The strings which came from the calculations were inharmonious to the organology of the lyre. They all sounded dull and could not possibly have been used some five thousand years ago, or at any time, for that lyre. I rejected them and worked with some basic "intuitive" logic: eleven twisted strands of sheep gut for bass string 11; ten strands for string 10; nine strands for string 9, eight strands for string 8, seven strands for string 7; six strands for sting 6; five strands for string 5; four strands for

melody. The original Greek noun, tonos (and hence, via Latin and Old French, our 'tone' is related to 'tension' and means, 2) acoustically speaking, any regular sound as opposed to irregular noises; 3) the pitch, vibration number, or frequency of such a sound, say C or C sharp; 4) its colour or timbre, warm or cool; 5) a melody pattern (like 'psalm-tone', and 6) the distance or interval of a major second.

26 [Fowler and Robson, 1998] explain, in the abstract, p. 366: 'We consider several aspects of the role and evaluation of the four- sexagesimal-place approximation to √2 on the well-known Old- Babylonian tablet YBC 7289. By referring to what is known about OB school texts, we show that this text is most probably a school exercise by a trainee scribe who got the approximation from a coefficient list. These coefficient lists are briefly described, with their use in geometrical problems. We consider other texts involving square roots and derive an algorithm for evaluating them, which complies with all known OB exampl s, from a simple geometrical construction of the type that seems to underlie many other OB procedures'. Therefore, they would have been able to calculate an equal temperament scale. However, there must be a distinction between the ability as an unconscious knowledge, (unknown known) and the need to apply such a concept when the application is possible because of the ability. 'Si, avec un si, on peut mettre Paris dans une bouteille, on doit pouvoir aussi, avec un si bémol ou naturel, mettre une contrebasse dans un porte-document ou un hélicon dans un carton à chapeau' – [Dac, 1981] (this quote is also available at http://dicocitations.lemonde.fr/ citations/citation-29012.php).

27 There is a great variety of musical intonations in World Music, all with different interval values although intervals of Just fifths, principally, and fourths appear to be constant actors, though often approximate. Some ethnomusicologists claim that the octave is the predominant interval. It is predominant, indeed, but only in systems in which it is predominant by design and not by chance. For further reading: [Beyhom, 2010b ; 2017].

28 Just intonation is a musical tuning in which the frequencies of notes are related by ratios or quantifications of small whole numbers. Any interval tuned in this way is called a Just Interval. Pure intervals correspond to the vibrational patterns found in physical objects which correlate to human perception. The two notes in any just interval are members of the same harmonic series.

29 Ratios of string length and ratios of frequency stand in reciprocal relationship to each other: 3/4= string length and 4/3 = frequency.

30 A generative scale is the result of a construction from which other scales are derived. The descending scale a-g-f-e-d-c-b-a-g is constructed from the alternation of fifths and fourths.

string 5; 4 strands for string 4; 3 strands for string 3; 2 strands for string 2 and one strand for string 1.

This intuitive method proved to be the best for the lyre which now sounds at its best.

Therefore, while Taylor's equation is correct for the calculation of strings for tense diatonic harps, it is totally unsuited to ancient lyres. The number 64.8 in figure 3 above (line VIII: 81; 72; 64.8; 60, etc.), which came from my hypothetic tuning in fifths and fourths, presented a problem as it needs to be multiplied by ten to become a whole regular number (64.8 10 = 648).

It could be argued that they multiplied all of their regular numbers by ten (810; 720; 648; 600, etc.) as it was done later during the Western Renaissance, and later, but I do not think they did[31].

However, 64.8 is the value for string 3 and therefore its qualification of 'third thin string' hitherto obscure, is now explicit, due to its abnormality.

It is rectified with 60, the fourth string, 'corrected/created' by the god Ea/ENKI, the god of music, whose qualification is thereby understood.

64.8 in relation to 45 delineates a 'dissonance[32]' of 631 cents (versus 612 cents, the Just Intonation 'tritone' made of three just tones of 204 cents each).

It is this 'dissonance', consequence of the introduction of the semi-tone, which off-balanced an otherwise perfect pentatonic system: a-g-e-d-c-a-g/g-a-c-d-e-g-a which became enneadic/bi-pentadic a-g-F-e-d-c-B-a-g/g-a-B-c-d-e-F-g-a. 64.8 was eventually adjusted with the contraction of the fifth 81:54 to 80:54 = 702 to 680 cents as shown in figure 4, line VIII.

At present, my hypothesis is all we had to speculate about Babylonian intonation. I hoped that this would be useful as basis for more punctilious research. To that end I decided to look for mathematical cuneiform texts having series of regular numbers.

About ten years ago, I 're-discovered' four tablets found in the early 1900s at the Temple Library of Nippur[33]. They date from 2300-2200 BC. They have a series of numbers from 1 to 81. They all are regular numbers taken from the Babylonian sexagesimal system, or base 60 arithmetics, and evenly divide powers of 60.

For instance, $60^2 = 3600 = 48 \times 75$, so both 48 and 75 are divisors of a power of 60. They are numbers with only prime divisors 2, 3, and 5. In music theory, the Just intonation of the tense diatonic scale involves regular numbers: the pitches in a scale have frequencies proportional to the numbers in the sequence given above from 1 to 81[34].

Thus, for an instrument tuned in this manner, all pitches are regular numbers, therefore, harmonics of a single fundamental frequency.

This scale is called a 5-limit tuning, meaning that the interval between any two pitches can be described as a product $2^i3^j5^k$ of powers of the prime numbers up to 5, or equivalently as a ratio of regular numbers.

These numbers agree with my views. They are printed in red in figures 3 and 4. There is no formal evidence that they were used for musical purposes. However, they end with 80 and 81. This means that the interval between them, later named by the Greek word κόμμα (kómma) from κόπτω (kóptō,

31 I believe that they did not quantify their pitches beyond 81 (in their theory) because this is the last number in the Nippur tablets, with penultimate 80. Since 81-80 = 1 (81/80=1.0125; 80/81 = 0.98765432...) and that the ratio of 81:80 = 21.506290 cents. This is the comma of Didymus, also called syntonic comma, chromatic diesis, Ptolemaic comma, or the wrongly qualified diatonic comma, which is a small interval between two musical notes, equal to the frequency ratio 81:80 (21.51 cents). In later Greek theory, this comma is referred to as the 'comma of Didymus' because it is the amount by which Didymus would have corrected the Pythagorean major third (81:64, around 407.82 cents) to a just major third (5:4, around 386.31 cents). The quantification of 81-80 = 1, producing the smallest interval in the Nippur list, would have ended the series, logically.

32 The term dissonance is inappropriate. Babylonians used the terms '*la zaku*' which roughly translates as 'unclear', but unclear does not mean dissonant. Therefore, although Babylonians found that interval 'strange' it had not reached the qualification of 'dissonant'.

33 [Hilprecht, 1906, p. 21, Pl. 10, 11, 12 and V].

34 Regular numbers are numbers that evenly divide powers of 60 (or, equivalently powers of 30). As an example, $60^2 = 3600 = 48 \times 75$, so both 48 and 75 are divisors of a power of 60. Thus, they are regular numbers. Equivalently, they are the numbers whose only prime divisors are 2, 3, and 5. Regular numbers from 1 to 81 are: 1; 2; 3; 4; 5; 6; 8; 9; 10; 12; 15; 16; 18; 20; 24; 25; 27; 30; 32; 36; 40; 45; 48; 50; 54; 60; 64; 72; 80; 81.

'I cut'), was already known in Babylon over 4000 years ago.

This strongly reinforces my opinion that the Nippur Tablets were used as the basis for pitch quantification theory, although probably not exclusively.

But the question is how they could have associated these numbers with the harmonic series is difficult to understand.

The Nippur regular numbers could also have been used as practical string length standards, essential to the instrument maker who would have used them as speaking lengths of string and lengths of air columns of wind instruments. (Fig. 5)

These measurements might suggest a standard 'Babylonian relative tuning[35]'. They can also be read as units of frequency, but the likelihood that they understood the concept is most improbable. However, we must never underestimate Babylonian scholarship. Frequency might not have been conceptualized as we understand it, but it might have been sensed.

If a string is plucked and if the tip of a finger is lightly placed at about the middle of the string, its vibrations are felt.

It is probable that they would have noted that the higher the pitch, the faster the vibrations, and reciprocally, but they would not have been able to count them. However, from their expert usage of reciprocals, they might have perceived that the reciprocals of string lengths equated to the number of their vibrations.

This text describes a bi-pentadic (enneadic) set. Strings gave their names to pitches. Therefore, they could have notated a melody with them although there is no evidence that they did. The Babylonian set described in this tablet is a 'Just Intonation enneadic diatonic' system made up of two symmetric conjunct pentads. But it is not a heptatonic set enlarged by two degrees.

II - CBS 10996[36]: *Occidental or Oriental?*

This text is also Neo-Babylonian, perhaps a bit older than *nabnītu* xxxii. It was excavated at Nippur and first published in 1960[37]. It lists a series of names of intervals and numbers associated with them. Since numbers do not exceed seven, Kilmer and others thought that this was evidence of ascending heptatonism (Fig. 6)[38]. However, it was later proven that the system is descending[39].

Figure 7 is my graphic interpretation of the tablet. The top part (A) is Kilmer's erroneous reading of the text. It is, she claims, a pattern spanning seven steps numbered from the bottom, suggesting that '1' is the lowest pitch, and that therefore the structure is ascending. The bottom part (B) shows my reconstruction of an original and hypothetical tablet, forerunner of CBS 10996. It displays a regular pattern spanning thirteen steps starting with number '1' at the top, suggesting that '1' is the highest pitch. For the sake of clarity, let us agree that '1' = 'c'.

At the first line (I) of top part (A), Kilmer's interpretation, 1↑5 (*nīš tuḫri*) is c↑g. (since Kilmer sees it ascending).

At the first line (I) of the bottom part (B) of my reconstruction, 1↓5 = c↓f is descending.

Line II in (A) is descending 7↓5 = b↓g with Kilmer.

Line II in (B) is ascending 7↑5 (*šeru*). It is d↑f.

The rest of my graphic representation where part (B) is the reconstruction which would have been the triskaidecadic[40] source for part (A). The

35 Relative tuning is when an instrument is tuned to itself. An absolute tuning is when the instrument is tuned to a pitch common to an orchestral, national or a 'tentative universal pitch'. See [Young, 1955], and [Lloyd and Fould, 1949, p. 84 sq.].

36 [Gurney, 1974, v. VII (1973), Pl. 126].

37 [Kilmer, 1960]. In her paper, Kilmer does not write anything worth mentioning about music. In another article entitled 'The Strings of Musical Instruments: their Names, Numbers and significance', [Kilmer, 1965], she makes interesting philological remarks but no progress with musicology. The article has an appendix written by Duchesne-Guillemin who wrongly confirms that the scale is ascending. Another article by David Wulstan, 'The Earliest Musical Notation', [Wulstan, 1971], is also misguided. Another paper by Kilmer, 'The Discovery of an Ancient Mesopotamian Theory of Music', [Kilmer, 1971], confirms that she has concocted a whole theory resting on the flawed interpretation of one text only.

38 A scale of seven degrees as: c-d-e-f-g-a-b; d-e-f-g-a-b-c; e-f-g-a-b-c-d, etc.

39 The rising or falling of a system is only relevant to the theoretical process but is irrelevant to praxis.

40 From Ancient Greek τρεισκαίδεκα (treiskaídeka,

erratic arrangement of part (A), the original CBS 10996, was left unexplained even as recently as 2013[41].

The arrhythmic order of the intervals in CBS 10996 is the consequence of the adaptation of a larger system into a smaller one, of seven pitches, or for an instrument fitted with seven strings.

The scribe who wrote the text kept the original polarities[42] of the intervals as they were in the original text, in his adaptation. It explains the inconsistencies in the numbers. This proves, indubitably, that the scribe knew about the larger span which he was adapting (Figs 8 and 9), probably as an exercise, to an instrument with seven string/pitches.

Such an instrument would have been designed exclusively for music composed from a heptatonic system, obviously.

But I would like to be clear in my opinion that it was certainly not a catalogue of intervals that musicians would have used for writing compositions or playing pieces.

The two most puzzling questions are, firstly, why eminently intelligent Babylonian theoreticians could have devised such an incredibly ill-conceived method? In any literate and illiterate culture[43] in the world, past and present, music is notated, or memorized from successions of pitches.

That Babylonians would have been restricted to composing with dyads is inconceivable. Intervals of dyads cannot be used for melodic notation since the human voice can only sing one pitch at a time[44]; and secondly, why eminently intelligent contemporary scholars could not have seen that their interpretation of the Babylonian notation of melody was flawed[45], is impossible to understand.

But what is even more difficult to understand is that while early Greek theory, which is built up from the same basic principles[46], is undisputed, its Babylonian origins are deemed whimsical.

The numbers printed and encircled in red in Figure 9 are speculative. The enneadic system' is perfectly symmetric in a Just Intonation made from the alternation of just fifths and just fourths.

My reconstruction of CBS10996 in its expanded version in figures 8 and 9 assumes that the central pitch in the interval qablītu, is the axis of symmetry for the whole ystem in its original status.[47] This interval is made up of (27/25 [great limma]=133.237 cents) + (10/9 [minor tone of just intonation]=182.40 cents) + (9/8 [major tone]=203.91 cents) + (16/15 [just semi-tone]=111.73 cents)=631.28 cents, which is an approximate acute diminished fifth.

In case of the adjustment of 64.8 to 64, the intervals of which qablītu is made are (16/15 [just semi-tone]=111.73 cents) + (10/9 [minor tone of just intonation]=182.40 cents) + (9/8 [major tone] =203.91 cents) + (16/15 [just semi-tone]=111.73 cents)=609.78 cents, which is an approximate diminished fifth.

Both values which would be the Babylonian

thirteen), from τρεῖς (treîs, three) + καί (kaí, and) + δέκα (déka, ten). Hence triskaidecadic, adj.

41 [Mirelman, 2013, p. 46, fn. 6]: 'The order in which the dichord pairs are referred to here (e.g., '5-2' as opposed to '2-5') corresponds to the order in which they occur in the theory texts. The theory texts enumerate the dichords according to a pattern that is not consistently ascending or descending.'

42 The polarity of an interval is defined by which note comes first: c↑f (1↑5) suggests that c (1) is first played, followed by lower f (5). In CBS 10996, polarity is given in number and pitch order.

43 By literate I mean cultures which are musically literate/numerate. Musical literacy/numeracy is certainly not essential to music theory and practice. Oral usage of contiguous pitches is not the prerogative of the literate. Music existed a long time before the written language and it is obvious that the earliest attempts at writing down theory rested on orality.

44 However, there is a form of "polyphonic overtone singing" by which the singer can produce overtones, one at a time above the fundamental pitch, as well as undertones. This is known in various cultures such as in Inner-Mongolia, Tibet, etc. It is also called 'harmonic singing'. But this technique would not allow to sing two unrelated pitches at a time s in Kilmer's hypothesis. YouTube has many examples of this polyphonic overtone singing. See [Anon. 'Overtone singing', 2017].

45 [Hagel, 2005]. This article must be read in extenso to judge for oneself the ways by which Hellenistic supremacists attempt at segregating knowingly and deviously Greek from Oriental theory and praxis, in order to majorize the one and pejorize the other, respectively. This is done despite the evidence under the form of cuneiform texts which, to the contrary, proves that it was the Greeks who "borrowed" all they could from Babylonian scholarship. It would be laborious to list these cuneiform texts but the essential ones are studied in the present paper.

46 [West, 1994, p. 219–223].

47 'Original status' is the series of pitches resulting from a generative construction, in this case, as given in *nabnītu* xxxii.

counterpart of the later Greek tritone are made up of the four different intervals with which the ennead is built and not of three just tones in the Greek system.

However, neither form of *qablītu* is tritonic as both are made up of four intervals. It is conceivable that both forms were considered as dissonant. This would explain the Babylonians term '*la zaku*', meaning 'unclear', i.e. 'dissonant'[48] but it is not possible to determine its exact value as it was, as we shall see in the next text, based on tension and relaxation. I suggest that in Babylon the concept of dissonance was not clearly defined, or rather was not confined to a specific interval. In Babylonian theory, there is no known term for other forms of dissonance.

As noted, the aforementioned mathematical texts from the Nippur Temple Library have series of numbers from 1 to 81. They are all regular numbers taken from the Babylonian sexagesimal system, or base 60 arithmetics and evenly divide powers of 60[49].

The Nippur numbers agree with my hypothesis. They are the numbers with only prime divisors 2, 3, and 5. In music theory, the Just intonation of the 'diatonic' scale involves regular numbers: the pitches in a scale have frequencies proportional to the numbers in the sequence given above from (in our case) 36 to 81 or to 80.

81 is the first pitch of the generative descending scale (as derived from *nabnītu* xxxii).

It is preceded by 80. The ratio of 81:80 known as the aforementioned comma of Didymus[50], which is of great importance in musicology, was already known at Babylon.

To conclude, the manner in which this scale is symmetrically built as 1-2-3-4-5 and 5-4-3-2-1, means that it was made up of two conjunct pentads, i.e. two pentads sharing a common pitch 'D'. Therefore, 'two conjunct pentads' is a better description of what I call 'enneadic set'. This survived in Greece where 'two conjunct tetrads' is a better description of heptatonism.

48 See also fn. 32, p. 97.

49 For instance, $60^2 = 3600 = 48 \times 75$, so both 48 and 75 are divisors of a power of 60.

50 See fn. 31, p. 96.

III - U7/80 = UET VII, 74: *Not a tuning text.*

This text, among many others, is further evidence, of the remarkable creative genius of Babylonian scholarship. The method explained in this system was never equalled in any other civilization, as it translates a dynamic disposition into its thetical[51] form.

This third tablet dates from the Old-Babylonian period, about 1800 BC. It was unearthed by Sir Leonard Woolley at Ur[52] and was published about forty years later, in 1968, by Professor Gurney[53]. At that time no one had yet hypothesised that the scale might be descending. Despite my attempts at promoting the idea on the basis of Greek and Oriental models, I was ignored. Consequently, and despite having asked advice from Oxford musicologist David Wulstan, Gurney's paper was published with the premise that the system was ascending.

Then in 1982, the Syrian Raoul Gregory Vitale[54] also attempted at promoting a descending system but was likewise ignored. At last, in 1990, Assyriologist friend Th.J.H. Krispijn perceived a new reading of line 12 as *nu-su-ḫ[um*, a form of the verb *nasāḫum*, 'to tighten'. This new term *nasāḫum*, Sumerian gíd-i, or *nussuḫum*, Sumerian zi-zi, is the technical verb for 'to tighten' strings. Its antonym is *ne'ûm*, Sumerian tu-lu. Subsequently Gurney published another paper in 1994[55] where it was finally established that the Babylonian system was descending[56] on the basis that the strings must

51 Thetic means 'set down or stated positively or absolutely'. From Greek 'thetikos' = that can be placed <'tithenei' = to place. It describes sets translated from the dynamic layout in a disposition as is described with text U7/80. It is the opposite of static, from Greek 'dynamikos', 'powerful'. It describes the layout of pitches as in the GBS (Greater Babylonian System). Dynamic a-b-c-d-e-f-g-a is thetic c♯-d♯-e-f♯-g♯-a-b-c♯, in case the thetic is set on the scale of 'c' with accidentals added according to a given dynamic scale.

52 [Gurney, 1974], Pl. 74.

53 [Gurney, 1968].

54 [Gurney, 1974], Pl. 74.

55 [Gurney, 1968].

56 However, a descending system is not appropriate for lute types where the strings are 'stopped'. They require an ascending system because the system starts by an open string. Complementary pitches are produced by the position of finger tips along the neck of the instrument. This results in

be tightened in part one of the text while it was assumed that strings were to be loosened in the previous publication of 1968. So, it was philology which won the case for musicology: Assyriologists did not trust musicologists.

The Text: restoration, commentaries.

Transliteration of U.7/80

- [*šum-ma* ^{giš}ZÀ.MÍ *pi-i-tum-ma*]
1 [*e-e*]*m-b*[*u-bu-um la za-ku*]
2 *ša-al-š*[*a-am qa-at-na-am tu-na-sà-aḫ-ma*]
3 *e-em bu-bu-u*[*m iz-za-ku*]

4 *šum-ma* ^{giš}Z]À.MÍ *e-em-bu-bu-um-ma*]
5 *ki-it-mu-um* [*la za-ku*]
6 *re-bi úḫ-ri-im* [*tu-na-sà-aḫ-ma*]
7 *ki-it-mu-um i*[*z-za-ku*]

8 *šum-ma* ^{giš}ZÀ.MÍ *k*[*i-it-mu-um-ma*]
9 *i-šar-tum la za-*[*ka-at*]
10 *ša-mu-ša-am ù-úḫ-ri-a-a*[*m tu-na-sà-aḫ-ma*]
11 *i-šar-tum iz-za-*[*ku*]

12 *nu-su-ḫ*[*u-um*]

13 *šum-ma* ^{giš}Z]À.MÍ *i-šar-t*[*um-ma*]
14 *qa-ab-li-ta-am ta-al-pu-*[*ut*]
15 *ša-mu-ša-am ù-úḫ-ri-a-am te-*[*ni-e-ma*]
16 [^{giš}]ZÀ.MÍ *ki-it-mu-*[*um*]

17 [*šum*]*-ma* ^{giš}ZÀ.MÍ *ki-it-m*[*u-um-ma*]
18 [*i-ša*]*r-ta-am la za-ku-ta-am t*[*a-al-pu-ut*]
19 [*re-bi*] *úḫ-ri-im te-ni-e*?[*-ma*]
20 [^{giš} ZÀ.MÍ *e-em-bu-bu-um*]

Certain words in this transliteration have a final mimation, an '*m*' following the case ending of a word, i.e. *išartum* instead of *išartu*. This practice is typical of the Old-Babylonian period.

It was on this basis that Gurney translated the text, and reconstructed it partially by extrapolation as follows:

'First part
1. If[57] the harp is *išartum*
the 'unclear interval' between strings 5 and
2 is *qablītum*
(should be 5-1b)
tighten by a 'semi-tone[58]' string 5
the harp will be *qablītum*'

This first quatrain of the first part was reconstructed by Professor Gurney. My interpretation is that the set of *išartum* comes from the conjunction of pentads *nīš tuḫrim* and *qablītum* = c-b-a-g-f + f-e-d-c-b as explained in my reconstruction of CBS 10996. But the 'unclear interval' is not between strings '5' and '2 of the front' (although in theory it exists as an 'unclear' fourth at that position) but between '5' (*ḫa-am-šu*) and the '1 behind string' (*úḫ-ru-um*), an 'unclear' fifth.

Now, that it was strings '5' and '2 of the front' which located the 'unclear interval' would not have been written as such in the original text. It would have said that *qablītum* is *la zaku* which means, as mentioned before, 'unclear', i.e. unpleasant[59]. The substitution of string '1 of the behind' by '2 of the front' is the consequence of the erroneous reading of CBS 10996, and is used to suggest heptatonism.

'2. If the harp is *qablītum*
the 'unclear interval' between strings 1 and 5 is *nīš tuḫrim*
(correct)
tighten strings 1 and 8 (should be 1^{f}-2^{b}) the harp will be *nīš tuḫrim*'

The set of *qablītum* comes from the conjunction of pentads *qablītum* and *išartum* = f-e-d-c-b + b-a-g-f-e. Therefore, the 'unclear interval' is between strings '1 of the front' and '5'. The reconstruction says that strings 1 and 8 should be tuned-up by a 'semi-tone'. But it should be written that it is 'string 1 of the front' and 'string 2 of the back' which should be tuned-up.

'3. If the harp is *nīš tuḫrim*
the 'unclear interval' between strings 4 and
1 is *nīd qablim*
(should be 4^{f}-2^{b}) tighten string 4
the harp will be *nīd qablim*'

The set of *nīš tuḫrim* comes from the conjunction of pentads *išartum* and *kitmum* = b-a-g-f-e + e-d-c-b-a. The reconstructed text says that the 'unclear interval' is between strings '4 of the front' and '1 of the front'. Here again, it should be

an ascending system.

57 The Akkadian term *šumma* has been consistently translated by 'when' although it should be the conditional 'if'. It was argued that it meant the same thing. The Babylonians were keen on the usage of protasis (if) and apodosis (then): A protasis is the clause expressing the condition in a conditional sentence (e.g. if you asked me in I would agree). The apodosis is the main clause of a conditional sentence (e.g. I would agree in if you asked me).

58 The text does not say 'semi-tone'. It is an amount by which the 'unclear interval' is corrected. This quantity is unknown as the system does not rely on ratios and therefore is left to the appreciation of the musician's tonal memory. I shall replace the term by 'tighten'.

59 See also fn. 32, p. 97.

placed between strings '4 of the front' and string '2 of the back'.

'4. If the harp is *nīd qablim*
the 'unclear interval' between strings 7 and 4
is *pītum*
(should be 7-11)
tighten string 7
the harp will be *pītum*'

The set *nīd qablim* comes from the conjunction of pentads *kitmum* and *embūbum* = e-d-c-b-a + a-g-f-e-d. Here, the limitation of the span for the set places the 'unclear interval' *pītum* between strings '3 of the behind' and '4 of the front' and it is string '3 of the behind' which must be tuned-up.

'5. If the harp is *pītum*
the 'unclear interval' between strings 3 and 7
is *embūbum*
(should be 3^{f}-3^{b}) tighten string 3
the harp will be *embūbum*'

The set of *pītum* comes from the conjunction of pentads *embūbum* and *pītum* = a-g-f-e-d + d-c-b-a-g. The transliteration of the tablet, since this is where the text U7/80 starts, says that the 'unclear interval' *embūbum* is placed between strings '3 of the front' and string '3 of the behind' and that string '3 of the front' should be tuned-up.

'6. If the harp is *embūbum*
the 'unclear interval' between strings 6 and
3 is *kitmum*
(should be 6-10)
tighten string 6
then the harp will be *kitmum*'

The set of *embūbum* comes from the conjunction of pentads *pītum* and *nīd qablim* = d-c-b-a-g + g-f-e-d-c. The 'unclear interval' is *kitmum*. It should be placed on strings 9-10 of the Greater System.

'7. If the harp is *kitmum*
the 'unclear interval' between strings 2 and
6 is *išartum*
(should be 2^{f}-4^{b}) tighten strings 2 and 9
the harp will be *išartum*'

The set of *kitmum* comes from the conjunction of *nīd qablim* and *nīš tuḫrim* = g-f-e-d-c + c-b-a-g-f. The 'unclear interval' *išartum* is located between strings 2 and 6, while it should be string '2 of the front' and string '4 of the back'.

The second part is the reverse of the first part.

Musical Quantification of U7/80

Although incomplete, this fragmentary text holds a wealth of knowledge which coincides with the information extracted from the previous texts, nabnītu xxxii and CBS 10996.

The method given in U.7/80 places seven sets on a bi-pentadic span, or on an instrument with nine strings by simple tuning of one or two of its strings. This gives the following enneadic sets:

Part 1

išartum (1):	c-b-a-g-f-e-d-c-b
qablītum:	c-b-a-c-f♯-e-d-c-b
nīš tuḫrim:	c♯-b-a-g-f♯-e-d-c♯-b
nīd qablim:	c♯-b-a-g♯-f♯-e-d-c♯-b
pītum:	c♯-b-a-g♯-f♯-e-d♯-c♯-b
embūbum:	c♯-b-a♯-g♯-f♯-e-d♯-c♯-b
kitmum:	c♯-b-a♯-g♯-f♯-e♯-d♯-c♯-b
išartum (2):	c♯-b♯-a♯-g♯-f♯-e♯-d♯-c♯-b♯

nu-su-ḫ[u-um]

Part 2

išartum (2):	c♯-b♯-a♯-g♯-f♯-e♯-d♯-c♯-b♯
kitmum:	c♯-b-a♯-g♯-f♯-e♯-d♯-c♯-b
embūbum:	c♯-b-a♯-g♯-f♯-e-d♯-c♯-b
pītum:	c♯-b-a-g♯-f♯-e-d♯-c♯-b
nīd qablim:	c♯-b-a-g♯-f♯-e-d-c♯-b
nīš tuḫrim:	c♯-b-a-g-f♯-e-d-c♯-b
qablītum:	c-b-a-c-f♯-e-d-c-b
išartum (1):	c-b-a-g-f-e-d-c-b

The strings which are tuned up are printed red. Note that the last *išartum* is not at the octave of the first išartum. It is a 'semi-tone' (of an undetermined value) higher. Therefore išartum (1) is not equal to *išartum* (2).

The indications in the text are 'to tighten' and 'to loosen' the strings. The quantity by which it should be is not given. This means that while theoretically we should have a Just Intonation system, in practice it might have been quite different in function of mood and other factors, such as location, time of day, season, but also and most importantly on tonal memory. Had they insisted on precise pitches, they would have indicated them in ratios with which they were fully conversant.

However, ratios are meaningless on harps or lyres, and this is probably why they were not used. They are only effective when a string is divided with frets or fret-marks as guides, on lute types.

For the sake of demonstration, should we hypothesize that this structure was intended for Just Intonation, then figure 10 gives such quantifications.

The recital in U.7/80 is an exceptional narrative for the history and transmission of the earliest musical construction, from its pre-literate form onward. Then, with the advent of literacy, musicology over-flowed its banks to grow into the most sophisticated form ever achieved in any civilization – four thousand years ago. The reliability of the Sumero-Babylonian scribal discipline was such that even with the few tablets which have reached us by luck – and not by design – it was nevertheless possible to decipher some of the unique intricacies of Mesopotamian music making.

Text CBS 10996 describes the reduction to a heptachord of a triskaidecadic forerunner. It suggests a foundation pitch around which other pitches agglutinate in a manner not dissimilar to the development of language. They agglutinate as sets and subsets, pentadic and triadic, respectively.

Two conjunct triads (Fig. 11) make a pentad: *serdû+titur qablītu =išartu* (E-F-G/G-A-B rising); *titur išartu+isqu = niš tuḫri* (F-G-A/A-B-C rising); *titur qablītu+rebūtu = pītu* (G-A-B/B-C-D rising); *isqu+šalšatu=kitmu* (A-B-C/C-D-E rising); *rebūtu +šeru = qablītu* (B-C-D/D-E-F rising).

The reason for their position in my reconstruction of the *Greater Babylonian System* is not yet understood. However, their integration in my interpretation of the Hurrian song H6 corroborates their presence, complementing descending fifth, logically and aesthetically.

As they stand, these triads are an essential part of the Babylonian scape-sound. *The Greater Babylonian System* spreads onto 17 pitches with smaller spans of 15, 13, 11, 9, 7, 5 and 3, (Fig. 11) all based on the same principle of the sharing of a common axis of symmetry. This is supported by the iconography where the number of strings vary with periods coinciding with organological trends.

Pitch sets are composed of two conjunct pentads where the last pentad of a given ennead is also the first pentad of the ennead which follows (Figs 12, 13 and 14):

Pentads *nīš tuḫri* + *qablītu* = set of *išartu*
= c-b-a-g-f-e-d-c-b
Pentads *qablītu* + *išartu* = set of *qablītu*
= f-e-d-c-b-a-g-f-e
Pentads *išartu* + *kitmu*
= set of *nīš tuḫri*
= b-a-g-f-e-d-c-b-a
Pentads *kitmu* +*embūbu* = set of *nīd qabli*
= e-d-c-b-a-g-f-e-d
Pentads *embūbu* + *pītu* = set of *pītu*
= a-g-f-e-d-c-b-a-g
Pentads *pītu* + *nīd qabli* = set of *embūbu*
=d-c-b-a-g-f-e-d-c
Pentads *nīd qabli* + *nīš tuḫri* = set of *kitmu*
= g-f-e-d-c-b-a-g-f

or are composed of two conjunct pentads which also follow each other, conjunctly, where the last pitch of a set is the first pitch of the next and results in an order of descending contiguous pitches:

Pentads *nīš tuḫri* + *qablītu* = set of *išartu*
= c-b-a-g-f-e-d-c-b
Pentads *išartu* + *kitmu* = set of *nīš tuḫri*
= b-a-g-f-e-d-c-b-a
Pentads *embūbu* + *pītu* = set of *pītu*
= a-g-f-e-d-c-b-a-g
Pentads *nīd qabli* + *nīš tuḫri* = set of *kitmu*
= g-f-e-d-c-b-a-g-f
Pentads *qablītu* + *išartu* = set of *qablītu*
= f-e-d-c-b-a-g-f-e
Pentads *kitmu* +*embūbu* = set of *nīd qabli*
= e-d-c-b-a-g-f-e-d
Pentads *pītu* + *nīd qabli* = set of *embūbu*
= d-c-b-a-g-f-e-d-c

Unequivocally, this system, whether of 17, 15, 13, 11, 9, 7, 5 or 3 pitches, is built from pentads and triads and can also integrate a set of seven pitches.

The 'unclear intervals' are located at the following positions:

In *išartu*:
c-b-a-g-f-e-d-c-b = 1-2-3-4-5-4-3-2-1,
the 'unclear interval' is *qablītu* and is placed on
5-1b = f-b
In *qablītu*:
f-e-d-c-b-a-g-f-e = 1-2-3-4-5-4-3-2-1,
the 'unclear interval' is *nīš tuḫri* and is placed
on 1f-5 = f-b
In *nīš tuḫri*:
b-a-g-f-e-d-c-b-a = 1-2-3-4-5-4-3-2-1,
the 'unclear interval' is *nīd qabli* and is placed
on 4f-2b = f-b

In *nīd qabli*:
e-d-c-b-a-g-f-e-d = 1-2-3-4-5-4-3-2-1,
the 'unclear interval' is *pītu* and is placed
on 3b-4f (7-11) = f-b
In *pītu*:
a-g-f-e-d-c-b-a-g = 1-2-3-4-5-4-3-2-1,
the 'unclear interval' is *embūbu* and is placed
on 3f-3b = f-b
In *embūbu*:
d-c-b-a-g-f-e-d-c = 1-2-3-4-5-4-3-2-1,
the 'unclear interval' is *kitmu* and is placed
on 4b-3f (6-10) = f-b
In *kitmu*:
g-f-e-d-c-b-a-g-f = 1-2-3-4-5-4-3-2-1,
the 'unclear interval' is *išartu* and is placed on
2f-4b = f-b

The 'unclear interval' in each set gives its name to the set which follows: in *išartu*, the 'unclear interval' is *qablītu*. It gives its name to the second ennead: *qablītu*. In this set the 'unclear interval' is *nīš tuḫri*. It gives its name to the next ennead: *nīš tuḫri*, and so forth.

All 'unclear interval' are pentadic (f↓b) when the span has seventeen pitches (when the span is restricted, some 'unclear intervals' are tetradic, because of inversion) but have different names according to where they are placed. This shows that the enneadic set is a reduction of the *Greater Babylonian System* (*GBS*), as CBS 10996 is the reduction of the *GBS* for a pitch set of seven, or heptad.

Should we take 'unclear interval' location numbers as Gurney located them in 1992, where he follows the order of intervals in CBS 10996, they would be either pentads or tetrads. The location of the 'unclear interval' in the first part of the text is: 5-2; 1-5; 4-1; 7-4; 3-7; 6-3; 2-6, or (5-1-4-7-3-6). The second part is the inversion of the first part: 5-2; 2-6; 6-3; 3-7; 7-4; 4-1; 1-5, or (2-6-3-7-4-1).

This sequence is exactly the same that we find later with text CBS 1766, (Fig. 16) which, without any doubt is a heptatonic construction. This is how lack of meticulousness and hasty assumptions always lead to wrong conclusions.

If the enneadic sets, which constitute the basis for Babylonian music theory are composed of two conjunct pentadic intervals, then their description and purpose in CBS 10996 has been wrongly interpreted. Therefore, all postulations built from this assumption are consequently flawed.

Philology is only partially understood. My reconstruction of CBS 10996 has correctly positioned the 'unclear interval' interval *qablītu* perfectly in the middle of the grid. pītu means 'opening' and *kitmu* perhaps 'closing'; *išartu* means 'erect, straight'. All these terms would have had their meanings which at present remain obscure. Usually, various cultures use toponyms to name their scales. Greek theory has Ionian, Dorian, Phrygian, Lydian, Locrian, etc. Babylonian sets also use particular names though not toponymic.

For advocates of heptatonism, I must insist that to prove its existence, there must be evidence for its construction. Without it, the term may not be used. U7/80 has no evidence of it. The tuning of octaves does not prove that the system is octavial heptatonic. There is incompatibility between the heptatonic system and the octave[60]. A heptatonic set is made up of 6 just tones (5 just tones and 1 semitone). A Just tone equals 204 cents (9:8) and six of them amount to 1225 cents. The octave measures 1200 cents. In the context of U7/80, the octave exists as an interval shared between two conjunct pentads but it is not and interval contained within a pentad. Furthermore, a distinction must be made between the octave as an interval and the octave as a concept. These are two very different things. The octave as a concept is a sampling standard within which a certain number of intervals can be fitted for the purpose of measurement. It is a container of smaller intervals[61] as first coined by friend and scholar Amine Beyhom.

A problem remains. How enneadic sets where distinguished from pentadic sets since they have the same names? In 1977, Aaron Schaffer found a small fragment at the University Museum, Philadelphia which he thought might be part of the reverse of *nabnītu* xxxii. The word *siḫpu*, was found associated with each of the enneadic, or pentadic sets:

60 The term 'octave' has been borrowed from Mediaeval Western Christianity. It means a series of eight days following a festival. It is contended that by giving the same name to a series of eight notes, it would 'Christianize' it thus making of music a religious act. A more appropriate term should be 'octade'.

61 [Beyhom, 2003 ; 2010a ; 2013 ; 2017].

išartu /siḫip išartu
kitmu/siḫip kitmu
embūbu/siḫip embūbu
pītu/siḫip pītu
nīd qablim/siḫip nīd qablim
nīš tuḫrim/siḫip nīš tuḫrim
qablītum/siḫip qablītum

The order of the sets above corresponds to the second part of the text.

Line 11 of *nabnītu* xxxii is the header of a new list: [sa.]du.a! *pismu*. There, the word *siḫip* precedes sets. Would *pismu* or *siḫip* denominate pentadic or enneadic sets is not possible to say at present.

IV - YBC 11381: *9 Sets*?

A recently published Neo-Babylonian text[62] in the Yale Babylonian Collection, is one of the most significant additions to the corpus of music theory for the past fifty years.

The text lists nine strings. Each string number is followed by an incipit[63]. The nine strings are known from *nabnītu* xxxii and mentioned in U.7/80. Unlike their disposition in *nabnītu* xxxii where the nine strings are listed palindromically, symmetrically: 1-2-3-4-5-4-3-2-1, YBC 11381, has them listed continuously: 1-2-3-4-5-6-7-8-9, significantly.

Each line starts with the Sumerian sign 'sa', meaning 'string', followed by a number. I believe the nine 'sa' with their numbers are no longer used only for listing strings, as with *nabnītu* xxxii, but would also be used for naming nine enneatonic/bi-pentadic sets generated from the system described in text U.7/80.

Interestingly, this new text might be a precursor for Plato's quantification of his nine Muses. In a notoriously difficult passage of *Republic*, (545c-546d)[64] the Muses speak about two harmonies, two Pythagorean heptachords superimposed, Dorian and Phrygian, in such a way that their combination aggregates into an ennead/bi-pentad with pitch quantifications which would have come from the Babylonian model. Clio is 'a' 2400; Euterpe is 'g' 2700; Thaleia, is 'f', 3000; Melpomene is 'e' 3200; Terpsichore is 'd' 3600; Erato is 'c' 4050; Polyhymnia is 'b' 4320; Urania is 'a' 4800 and Caliope is 'g' 5400. Apart from Erato and Polyhymnia with typical Greek numbers, the other muses have Babylonian quantifications.

The way instructions are given in U.7/80 imply that they would have left room for local, regional, or, and national tone inflections in Old-Babylonian systems allowing for specific intervals to be tuned slightly wider, or slightly smaller than the Just paradigms. They were tuned by ear only, from a master's teachings through metaphors, and metonymy, and not with ratio theories.

Aristoxenos would have preferred speaking in terms of tension (ἐπίτασις) and relaxation (ἄνεσις), but how much of Aristoxenos' works are really his and not Western Mediaeval transpositions of Eastern theories, into Western ones, during and after the crusades.

Al-Fārābī, Latinised as Alfarabius, because of the complete disappearance of the Babylonian cuneiform script would have assumed that most he knew of the past would have mainly come from the Greek: He would have thus lost all knowledge of any Babylonian antecedence, and the Greeks were not eager at giving Babylon any credit.

Since U.7/80 relies only on tension and relaxation of strings, and not with ratios, it follows that the ratio of 2/1, the octave, is irrelevant. Although the 'Gurney/Wulstan theory', is interpreted as octavial, tense diatonic heptatonism, all seem to ignore that the first extrapolated scale of *išartum*, is not at the octave of the last *išartum* in both chapters.

It is either higher, or lower, by an unqualified semi-tone. It would appear logical that the Babylonians, after having adopted an enneatonic generative model, would have had nine subsets. These sets would have stemmed from their fundamental or generative model, as shown below, for the first chapter of U.7/80 with the second chapter relaxing the tension in each set as a reverse

62 [Payne, 2010]. As stated by Payne this tablet can be approximately placed as Neo-Babylonian based on its orthography.

63 In music, an incipit (from the Latin, meaning 'it begins') is an initial sequence of notes or words used in catalogues of musical texts.

64 [Cornford, 1997] omits the description of Socrates' 'sovereign number'. In [Plato and Waterfield, 1993] the latter notes 'scholars nowadays largely ignore the passage' – see [Crickmore, 2009]: 'Hesiod's "races" and "political degeneration" in Plato', p. 56-7.

process of chapter one.

Note that because we are certain, from the reading of U.7/80, that the last quatrain of the first part and the first of the second was išartum, it would be reasonable to assume that the series started with *kitmum*.

Tension (part one)

1 *kitmum*:	c-b♭-a-g-f-e-d-c-b♭
2 *išartum*:	c-b-a-g-f-e-d-c-b
3 *qablītum*:	c-b-a-g-f♯-e-d-c-b
4 *nīš tuḫrim*:	c♯-b-a-g-f♯-e-d-c♯-b
5 *nīd qablim*:	c♯-b-a-g♯-f♯-e-d-c♯-b
6 *pītum*:	c♯-b-a-g♯-f♯-e-d♯-c♯-b
7 *embūbum*:	c♯-b-a♯-g♯-f♯-e-d♯-c♯-b
8 *kitmum*":	c♯-b-a♯-g♯-f♯-e♯-d♯-c♯-b
9 *išartum*":	c♯-b♯-a♯-g♯-f♯-e♯-d♯-c♯-b♯

Relaxation (part 2)

9 *išartum*":	c♯-b♯-a♯-g♯-f♯-e♯-d♯-c♯-b♯
8 *kitmum*":	c♯-b-a♯-g♯-f♯-e♯-d♯-c♯-b
7 *embūbum*:	c♯-b-a♯-g♯-f♯-e-d♯-c♯-b
6 *pītum*:	c♯-b-a-g♯-f♯-e-d♯-c♯-b
5 *nīd qablim*:	c♯-b-a-g♯-f♯-e-d-c♯-b
4 *nīš tuḫrim*:	c♯-b-a-g-f♯-e-d-c♯-b
3 *qablītum*:	c-b-a-g-f♯-e-d-c-b
2 *išartum*:	c-b-a-g-f-e-d-c-b
1 *kitmum*:	c-b♭-a-g-f-e-d-c-b♭

It will be noted that these sets proceed in fifths: c-g-d-a-e-b-f♯-c♯-g♯, therefore in the thetic disposition. However, the sets are here in the dynamic disposition (Fig. 15) and that therefore, the eighth set is not the repetition of the first one and the ninth is not the repetition of the second one, one octave higher.

Had the system been octavial, then the first and eighth sets would have been identical and so would have been the second and the ninth.

It is possible that the nine chants listed in the text would have been sung to the following scales:

sa 1: May Aššur, the king of the gods, improve your dominion for you.
c-b♭-a-g-f-e-d-c-b♭ = g-f-e-d-c-b-a-g-f
= *kitmum*
sa 2: May Ištar, who created mankind, grant you well-being and longevity.
c-b-a-g-f-e-d-c-b
= *išartum*
sa 3: May Daragal make you rival the fierce weapon (s and) the raging storm.
c-b-a-g-f♯-e-d-c-b = f-e-d-c-b-a-g-f-e
= *qablītum*
sa 4: Enkidu, treat kindly the Lady, the protective spirit who created good things, the lamassu.
c♯-b-a-g-f♯-e-d-c♯-b = b-a-g-f-e-d-c-b-a
= *nīš tuḫrim*
sa 5: May Damkianna make your appeal, your prayers, and the stroke of your nose always pleasing to the lord of lords. c♯-b-a-g♯-f♯-e-d-c♯-b= e-d-c-b-a-g-f-e-d
= *nīd qablīm*
sa 6: May Endašurimma present your artful advice and your precious words daily.
c♯-b-a-g♯-f♯-e-d♯-c♯-b= a-g-f-e-d-c-b-a-g
= *pītum*
sa 7: May Endukuga always let your footstep fall on a prosperous road and a smooth path.
c♯-b-a♯-g♯-f♯-e-d♯-c♯-b= d-c-b-a-g-f-e-d-c
= *embūbum*
sa 8: May Enudtila constantly establish abundance, plenty, and prosperity for the pastures of your people.
c♯-b-a♯-g♯-f♯-e♯-d♯-c♯-b=g-f-e-d-c-b-a-g-f
= *kitmum*
sa 9: May Enmešarra crush the forces of those who wrong you and of your enemies. May he scatter the weapons of your adversaries.
c♯-b♯-a♯-g♯-f♯-e♯-d♯-c♯-b♯=c-b-a-g-f-e-d-c-b
=*išartum*+ undefined quantity

Thus, it is possible that the nine sets were known, at some point during the late Neo-Babylonian period, no longer by their names but by their numbers. There is a parallel in the text which follows (CBS 1766) where the names of sets are also substituted by numbers. This also applies, much later, to Ecclesiastical Modes such as 'mode of the first tone', 'mode of the second tone',[65]etc., and no longer by their original Greek names.

V - CBS 1766:[66]or first evidence of heptatonism

This unusual rectangular tablet dates from the late Neo-Babylonian period, early last half of the first millennium. It has the drawing of an irregular heptagram[67] etched within two concentric circles,

65 The seven ecclesiastical modes: mode of the first tone (Ionian) (mode of c) c-d-e-f-g-a-b-c = 1 1 ½ 1 1 1 ½; mode of the second tone (Dorian) (mode of d) d-e-f-g-a-b-c-d = 1 ½ 1 1 1 ½ 1; mode of the third tone (Phrygian) (mode of e) e-f-g-a-b-c-d-e =½ 1 1 1 ½ 1 1; mode of the fourth tone (Lydian) (mode of f) f-g-a-b-c-d-e-f = 1 1 1 ½ 1 1 ½; mode of the fifth tone (Mixoly-dian) (mode of g) g-a-b-c-d-e-f-g = 1 1 ½ 1 1 ½ 1; mode of the sixth tone (Eolian) (mode of a) a-b-c-d-e-f-g-a = 1 1/2 1 1 1/2 1 1; mode of the seventh tone (Locrian) (mode of b) b-c-d-e-f-g-a-b = 1/2 1 1 1/2 1 1 1.

66 [Waerzeggers and Siebes, 2007].

67 In general, a heptagram is any self-intersecting heptagon, a seven-sided polygon. It is the 7/3 heptagram which is depicted in CBS 1766. This is the smallest star

at the top left corner, with annotations both lexical and numeral. Under the heptagram, there are eleven columns spreading onto the whole width of the tablet. Columns two and three have seven numbers each. Column four is empty. Columns five, six and seven are inscribed with only one line of numbers. A header spreads along the entire length of the columns but at present resists interpretation.

The heptagram and the column two represented in figure 16 constitute the first evidence of a heptatonic construction, and therefore of conceptual, if not practical heptatonism. This is based on the names of seven strings inscribed on each point of the heptagram. The names of the strings are given in the same order as they were in *nabnītu* xxxii, without the two last strings (second behind and behind string).

Another significant matter is that the strings are also described with numbers from one to seven. The intersecting lines link the points of the heptagram in a pattern generated by the numbers in column 2: 2-6 = b-e; 6-3 = e-a; 3-7 = a-d; 7-4 = d-g; 4-1 = g-c; 1-5 = c-f; 5-2 = f-b. This heptatonic construction consists in the alternation of descending fifths and ascending fourths until the last ascending fourth reaches 5-2 = which should be f-b , a just fourth. However, the b would be conflicting with the initial 'natural'[68] 'b'. Therefore, to suit the heptagram, the is made 'natural' so that it equates to the initial 'b'.

This system is radically different in its construction from all previous systems. It indicates, if not a change, but at least an addition to Babylonian theory. While the older system is of linear construction, CBS 1766 is of cyclical expression, two very different concepts. The names of the strings/pitches described in earlier texts are replaced by numbers from 1 to 7.

The system relies only on just fourths and fifths for its construction, and no longer on fifths, fourths and, or thirds. This was a remarkable innovation, typical of Babylonian scholarship that the Greeks would have adopted.

However, it could also be surmised that this innovation was originally Greek and adopted by Babylonian theory during the Orientalizing period, fertile in exchanges between both cultures, but this is highly conjectural as there is no reliable chronological (or other) evidence from the Greek side of the coin. Furthermore, there is no evidence of any heptatonic representation with circumscribed heptagrams in the history of Ancient Greek theory. This theory has always been linear, conceptually and transposed as a tangible monochord on which ratios of string-lengths were applied.

This *modus faciendi* will remain with Boethius, and later theoreticians. It is highly probable that Near-Eastern scholarship adopted the cyclical model which perdured into Islamic theory but was never adopted in the West until much later. Therefore, CBS 1766 might be Babylonian, or perhaps a Babylonian interpretation of a Greek linear model, hypothesis which I would advance with little conviction.

The Near-East never willfully adopted the octavial concept and remained attached to smaller intervals known as *ajnās* which agglutinated to one another forming scales. Later, probably with the indoctrination from the crusades, tetrads were added to pentads and triads. There would have been further influence from crusaders who remained in the Levant and probably Westernised local trends. There was further contamination during the French Mandate, in Syria and Lebanon where Maqām musicians and teachers were instilled with Western heptatonism.

The belief persists and has greatly contributed to the degeneration of Oriental intonation. But I would advance with conviction that the Maqām and other musical forms practiced in the Near-East are direct inheritors of the Babylonian models.

polygon which can be drawn in two forms, 7/2 and 7/3, as irreducible fractions.

68 I use the term 'natural' reluctantly, as it conveys an impression of normality for the Western tense diatonic scale.

VI - H6: The 'Proof in the pudding?'

The Hurrian songs[69] are well documented[70]. Tablet H6 comes from a collection of about thirty which could be restored from three fragments: (RS13.30 + 15.49 + 17.387). The other texts were mostly broken in small fragments which could not be joined. They date from about 1400BC and were excavated at the site of Ugarit, modern-day Ras-al-Shamrah in North West Syria. The tablets were written in the agglutinative Hurrian language. Fortuitously, musical terms were written in 'Hurrianized' Babylonian making it reasonably easy to read. There were hitherto unknown terms the meaning of which remaining obscure. The writing runs parallel to the longest side. It is divided in three. The first part generally spreads onto the obverse. A double line with two winkelhaken[71] is drawn at about the middle of the tablet. Musical notation is written under the double lines.

The musical notation can be segmented in sets and numbers associated with them:

According to Kilmer, the colophon says: [*an-n*]*û za- am-ma-rum ša nid-qib-li za-l*[*u*]*-z*[*i ša* DINGIR.MEŠ TA ^{m}*Urḫiya*] ŠU ^{m}*Am-mu-ra-bi*. This roughly translates as: This is a song in the set of *nid qibli*, a *zaluzi* for the gods, composed by *Urḫiya* and written by the scribe *Ammurabi*.

The Chicago Assyrian dictionary says that a *zamāru* is 'a song with or without instrumental accompaniment'. However, I think that the word 'song' is inappropriate because there are instances where the word is used for an *adapu*-instrument. Since an instrument does not sing, I prefer the use of the term melody: 'a sung melody', or an 'instrumental melody', to avoid confusions. For instance, there is a *za-ma-rum šá pít-ni* which the CAD translates as 'to sing to the accompaniment of the *pitnu*-instrument'. However, this could also translate as 'a melody played on the *pitnu*-instrument'. In most cases, it is the *ḫalḫallatu*-drum which is mentioned as accompaniment to the voice, but percussion is rhythmical rather than melodic. However, we have instances where '*šušqqûssu ina sammî li-iz-za-mir-ma*' translates as: 'let her (Babylon's) exaltation be sung to the accompaniment of the harp'. The only instruments of which we are certain that they accompanied the voice were the *ḫalḫallatu* and the *alû* which are drums; the *balaggu* and the *sammû* are string instruments. My view is that popular instruments such as the pastoral inu, a type of primitive lute, would have provided some basic accompaniment to a song. On the other hand, I would think that ritual singing, mostly, would have been a cappella. The more solemn cultic chants would have had percussion accompaniment.

It took centuries for the Christian Church to adopt the organ. Plain-Chant should have remained unaccompanied because a well-tempered tuned instrument is anathema to ecclesiastic modality. The Babylonian clergy might have had similar views, although not for the same reasons, but it is usually for reasons of spirituality that religious singing mostly remains unaccompanied, in most cultures.

With regard H6, I believe the song did not have any instrumental accompaniment. Had there been, it would have been written down. They certainly could do so. Kilmer's hypothetical accompaniment has been forced-fitted as justification for her belief in simultaneous dyads. The notion of instrumental accompaniment is not a simple matter as it introduces the concept of 'absolute' tuning in a world where tuning was 'relative'.

In the absence of a standard pitch, instrumental accompaniment would have been problematic unless one specific instrument accompanied one specific voice, exclusively. On the other hand, a street or a folk musician could have accompanied him- or herself should the tuning fit with their own 'tonal' register, but this certainly would have been exceptional rather than habitual. This problem is of no concern to us, in the West, or at least since the seventeenth century AD, as equal temperament tuning allows for transposition which would certainly not have been possible at Ur or Babylon.

Additionally, the principle of accompaniment,

69 I write 'songs' in reaction to the sempiternal denomination of any ancient music as 'hymn', with a religious connotation. The term 'song' means that the melody can be either secular or religious.

70 [Nougayrol, 1955], [Schaeffer, 1962], [Nougayrol et al., 1968].

71 The Winkelhaken (from German 'angular hook'), also simply called a hook, is one of five basic wedge elements appearing in the composition of signs in Akkadian cuneiform.

thousands of years ago, was one which would have involved conceptual understandings for which there is certainly no evidence at that period.

Some of the terms and numbers in figure 17 are difficult to read and therefore, the number of beats in the last column to the right reflect these problems. My experience, shouldered by logic, tells me that there must have been a regular infrastructure in this melody. It is written on six lines; it has six bars per line.

I would confidently guess that the four lines at the centre (2, 3, 4 and 5) amount to thirty-six beats because six times six intervals are thirty-six.

I can only but assume that the first and the sixth lines being introductive and conclusive would have twice thirty-six beats, therefore seventy-two beats. However, the rhythmical values of the sets are irregular. This is probably because music had to fit the text and not the contrary.

This suggests that a single melody, whether a song or a hymn, might have different sets of lyrics to accompany and that inevitable metrical variations of the lyrics would be echoed in the time signature for each segment.

The colophon in the text says that this song is in the set of natqabli, Babylonian *nīd qablim*, which is: e-d-c-b-a-g-f-e-d.

My methodology in this interpretation is as follows (see figure 17):

Each set is followed by a number.

Each line has sets and subsets amounting to six.

The numbers are rhythmical notation.

The numbers following the intervals prolong the last beat.

One number beat equals to two interval beats.

This is the process:

The first cell of the first line is qablite. (qablītu). qablite in the set of nīd qablītu equals five beats: ...qablite followed by 3 equals 5+(6-1) = 10 beats (5 = beat in the interval. (6-1 6 is double 3 and -1 is subtracting the last beat of the interval:

My interpretation of the song (Fig. 18) lends itself to analysis. This is a critical point as there is no music without structure.

Here, it is built on the A B C formula. There is an introduction at the first line (A), and a coda at the last (C).

The refrain of the song (B) is composed of four lines of six bars each with a total of 36(?) beats each amounting to six (irregular bars) amounting to a 36/8 time signature, per line split in six bars.

The coda leads back to the introduction to repeat the whole song, as indicated on the tablet with a double winkelhaken on the double bar separating lyrics from music.

However, I am not suggesting that my interpretation is how the piece sounded in 1400 BC. The subsets which I have interpreted, in tense diatonic scale, would have been played with intonations similar to Maqamian ajnās.

The Babylonian or Hurrian musicians were unable to write down particular intonations for each of their pentads and triads the inflexions of which being as refined as they were complex, and therefore impossible to notate. They still are. However, they were and are inscribed in the memory of the genetic unconscious. Therefore, they had different names. Their recalling, as conditioned reflexes, would immediately suggest how they sounded. Similarly, the accordion evokes Paris; the *ādhān* evokes Cairo; the *shofar* evokes Jerusalem; pipes evoke Edinburgh, etc. Maqamian *ajnās*, like Babylonian pentads and triads, are called *ajam*, *jiharkah*, *mustaar*, *bayati*, *busalik*, *hijaz*, *kurd*, etc., as Babylonian pentads and triads are called *išartum*, *qablītum*, *šaššate*, *isqu*, etc.

Coranic declamation uses *ajnās* but it must be reminded that these intonations are not specifically Islamic. They were shared by most if not all cultures in the Ancient-East, and continuously throughout history to our days. I am inclined to think that Babylonian music would not have been very different. Hebraic cantillation in Synagogues of Morocco, and Christian music in the Levant, also share these intonations, certainly not as a conscious adoption, or association with Islamic declamation, but as the reminiscence of an unconscious knowledge.

In Damascus, during the 2011 Oriental Landscapes Conference, I submitted my interpretation of H6 to leading Maqām musicians at the Dar al-Assad Opera House. They hummed along my interpretation as it was played. After my

presentation, they corrected the melody which I was playing electronically (in the tense diatonic scale), to its proper intonation, and suggested how to play it as it should. These musicians, after over 3000 years, recognized H6 as part of their heritage.

This anecdote is certainly not an academic proof for the authenticity of my interpretation, but it is, certainly, as far as I am concerned, a proof much more significant than any other[72].

I have titivated the title of this last chapter with the addendum 'proof in the pudding'. The reason is that my view of Babylonian theory, which clearly diverges from the established version, is consistent throughout, contrarily to Kilmer's. There are no points in this little work which are not fully tested, no more than there are points which do not fit in with the general description of the theory in all texts available to us. The intervals of pentads and triads are the most obvious origins for the Maqamian ajnās, and suggest a continuous usage of Babylonian theory from its origins to our days. It is Babylonian music which shaped Early Greek music which in time slowly evolved away from its original model; it is Babylonian music, probably, which shaped some part of Byzantine music[73]; It is Babylonian music which gave the early Mediterranean world musicological tools with which it could, in turn, develop its own concepts.

Music theory was born in Mesopotamia, it was the earliest theory ever developed and is at the source of all other Mediterranean systems and perhaps others.

All music theories of the Ancient Western and Oriental Worlds carry the Babylonian gene and it is therefore not surprising that Plain-Chant modality is so close to it.

Musicological Conclusions

Babylonian music rests on a series of descending pentads and ascending triads with infixes. Two conjunct triads make a pentad and two conjunct pentads make an ennead. The system is essentially descending enneadic, or preferably descending bi-pentadic. Triads, pentads and enneads make up the elements of music similarly to the *ajnās* of the Maqām form, of which they are likely to be the source. The sets are organised in systems of enneads, either conjunct when the last pitch of an ennead is the first pitch of the next one, or in organised pentadic conjunction where the last pentad of an ennead is the first pentad of the next one. This is the dynamic arrangement of the system. There is evidence that as early as the Old-Babylonian period the thetic system was also used. It allowed for all sets of a system being contained within a fundamental enneatonic set.

Numbers following pentads and triads indicate the time by which the last pitch of a set should be prolonged. There were other forms which might have been embryonic, and others complementing the system, such as pentatonism and heptatonism, respectively. The ambitus or span of the Greater Babylonian System could expand to 11, 13, 15 and 17 pitches, always arranged in symmetry from the central common pitch, or axis of symmetry. There were nine enneatonic sets as we know from a Neo-Babylonian text, and also from an Old Babylonian tablet which suggests seven enneatonic, but also possibly nine sets. It is probable as with Maqamian *ajnās*, that infixes of pentads were played in any order to suit a composition. Infixes in pentads, while initially diatonic in construction, would have been modified to express mood as with ajnās, and like ajnās, their names would reflect these variations in intonation.

My exposition of Babylonian music theory radically differs from the established interpretation. However, it is so closely related to Oriental forms, such as the Maqām, that it is difficult to ignore this relationship. Ἀπόδοτε οὖν τὰ Καίσαρος Καίσαρι καὶ τὰ τοῦ Θεοῦ τῷ Θεῷ[74].

Ultimate Remarks

The following gems are lifted from Sach's

72 A video of this text translated by the author with Oriental adaptation by Rosy Azar Beyhom and Amine Beyhom is available at https://www.youtube.com/watch?v=gynhfxQ1IO4. It must be noted that the singer because of her Western operatic training was unable to give an appropriate Oriental intonation to the piece.

73 For a rational and comprehensive introspection into Byzantine Music, see [Beyhom, 2015].

74 'Render unto Caesar' is the beginning of a phrase attributed to Jesus in the synoptic gospels, which reads in full, 'Render unto Caesar the things that are Caesar's, and unto God the things that are God's'. [Matthew 22:21]

The Wellsprings of Music: In his *General History of the Science and Practice of Music*[75], Sir John Hawkins wrote[76] that 'the music of the Barbarians [Orientals] was said to be hideous'. Although he studied Greek music he did not realize that the *chroai* were Oriental. He scorned Oriental music not because it was hideous, but because it was said to be hideous. As for his chapters on the Greeks and the Hebrews, on the contrary, there was no danger of unfavourable reports: conceivably, there were no ear-witnesses.

The two sections of the book were easily filled with learned quotations from literary sources well-known to all the erudite contemporaries. The music itself was absent, to be sure; but being Biblical or Greek, it must have been perfect by definition.

In K.C.F. Krause's unremarkable *Darstellungen aus der Geschichte der Musik*[77], we read with astonishment that

> '...in Antiquity, which was the childhood of music, only simple unadorned melody was known, as is the case today with such peoples as the Hindus, Chinese, Persians, and Arabs, who have not yet progressed beyond the childhood age'. This is true Hegelian progressivism: how far have we come in our mature age (or is it senility, if not worse?). Not to mention the profound ignorance behind the notions of Hindus, Persians, and Arabs singing in 'simple unadorned melody' – they who are unrivalled masters in the art of highly adorned singing, and leave simplicity to the lower forms of children's songs – and to the West' [78],

while

> The reader who reaches for the monumental Geschichte der Musik by August Wilhelm Ambros[79]finds a whole Buch on the Kulturvölker des Orients, indeed on the Primitives. But on these pages, he also finds the most bewildering pronunciamientos such as: 'Assyrian music seems never to have risen above the level of a mere sensual stimulus', or: the music of Babylon 'was quite certainly voluptuous, noisy, and far from simple beauty and noble form'; and Phoenician music was mainly meant to drown 'the cries of the victims who burned in the glowing arms of Moloch[80]'.

More recently, Stefan Hagel, in his 'Is *nīd qabli* Dorian? – Tuning and modality in Greek and Hurrian music[81]', says:

'A comparison with Ancient Greek music suggests a largely independent development of musical form at least as early as the first half of the second millennium on[82]'.

How could Greeks having borrowed mathematics, astrology, medicine, mythology, religion, divination, literature, law, etc., from the Orient, would have forgotten by some extraordinary phenomenon all about Babylonian music on their way home?

Further:

> 'It is significant that this system was not orientated towards melody, as was Ancient Greek notation and music theory, but to instrumental practice'.

What an amazing statement!

Later:

> 'For that reason it [the Babylonian system] will survive for a considerable period of time only in a mainly traditional if not backward-orientated musical culture. But in Greek music history the melodic possibilities had soon become too rich to be contained within such a reduced harmonic framework'.

What an extraordinary feat of Hellenic supremacism, well in keeping with Krause and Ambros.

Plus ça change, plus c'est la même chose.

75 [Hawkins, 1776].
76 For this paragraph: [Sachs, 1962, p. 6].
77 [Krause, 1827].
78 Quote and commentary from [Sachs, 1962, p. 7].
79 [Ambros, 1862].
80 [Sachs, 1962, p. 7–8].
81 [Hagel, 2005]: the title is already biased as should it not be: 'Is Dorian *nīd qabli*' rather than the contrary?
82 For this quote and the following: [Hagel, 2005], Abstract, p. 287.

ILLUSTRATIONS

Line	Sumerian	Akkadian	Translation
1	sa.di	*qud-mu-u[m*	front string
2	sa.us	*ša-mu-šu-um*	next string
3	sa.3.sa.sig	*šal-šu qa-a[t-nu*	3rd thin string
4	sa.4.tur	*a-ba-nu-[ú*	4thsmall, Ea-created
5	sa.di.5!	*ḫa-am-[šu*	5th string
6	sa.4.a.ga.gul	*ri-bi úḫ-ri-im*	4th behind string
7	sa.3.a.ga.gul	*šal-ši úḫ-ri-im*	3rd behind string
8	sa.2.a.ga.gul	*ši-ni úḫ-ri-im*	2nd behind string
9	[sa.1]a.ga.gul	*úḫ-ru-um*	behind string

Table 1 Transliteration and translation of Sumerian and Babylonian terms in *nabnītu* xxxii.

1		2		3		4		5		4		3		2		1
a		g		f		e		d		c		b		a		g
g		a		b		c		d		e		f		g		a
	tone		tone		1/2		tone		tone		1/2		tone		tone	

Fig. 1 Position of tones and semi-tones in the enneadic/bi- pentadic system of *nabnītu* xxxii.

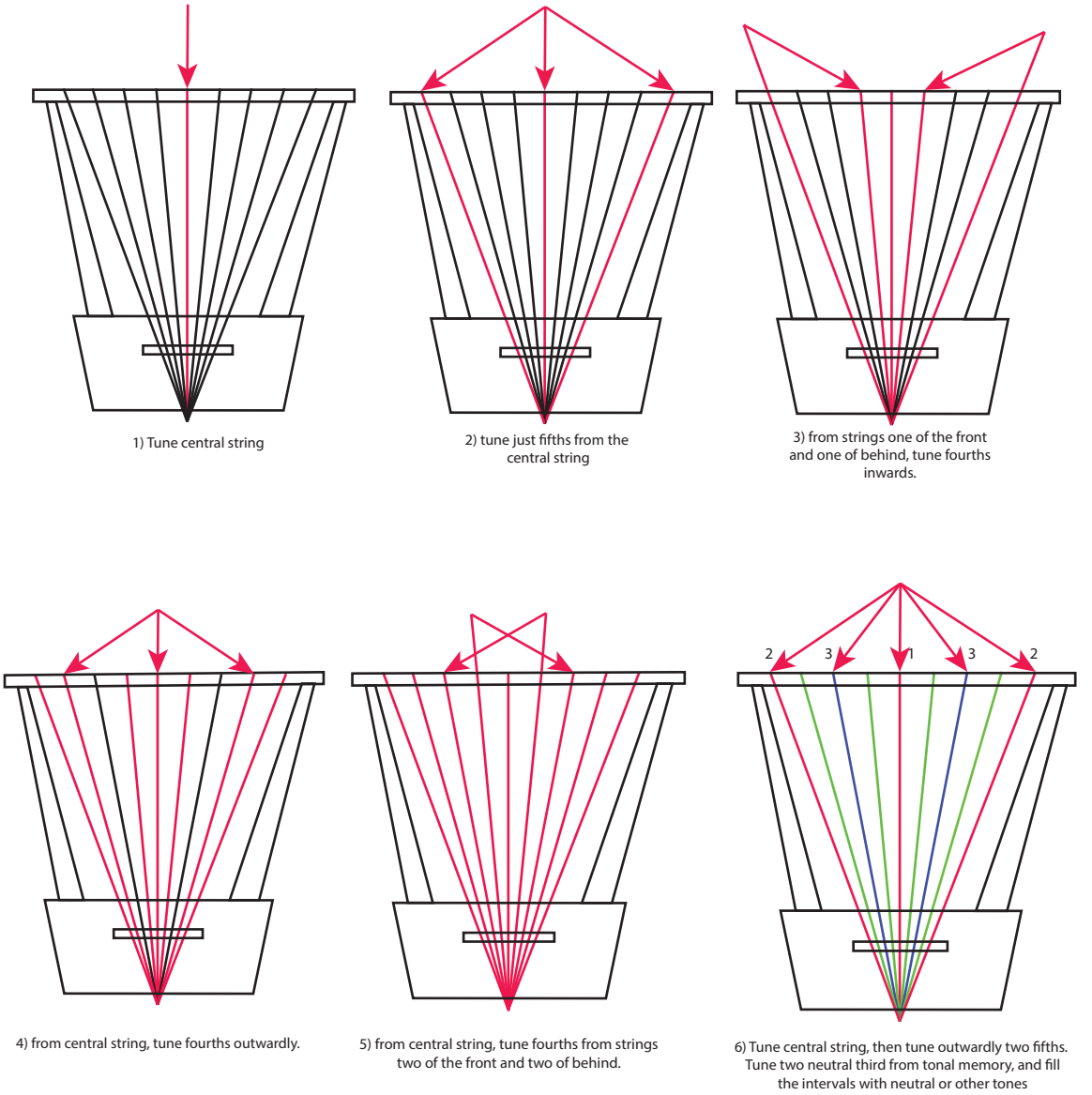

Figure 2. (1-2-3-4-5-6). Illustrated tuning procedure derived from *nabnītu* xxxii. But it is also possible that they tuned with fifths and thirds only. In this case, after the second process when the central and two outer strings are tuned fifths apart, Just Major thirds are tuned from the outer strings. Then infixes are tuned from pitch memory. I would be in favour for this second type of tuning because it would be better suited to the interval list in the next tablet.

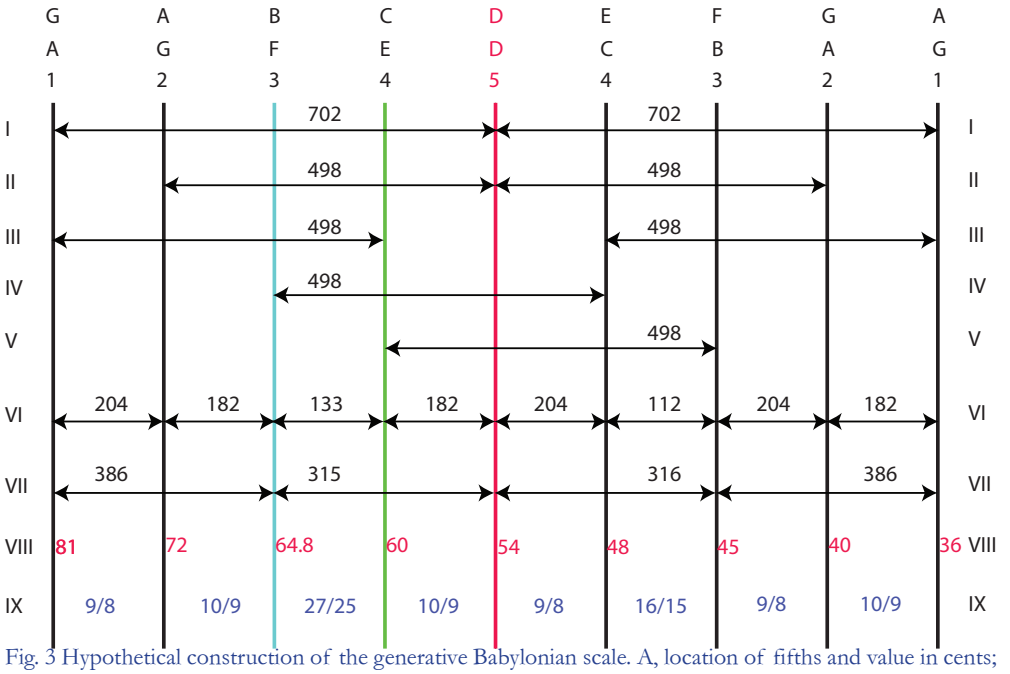

Fig. 3 Hypothetical construction of the generative Babylonian scale. A, location of fifths and value in cents; B, C, D and E, location of fourths and size; F, interval values of string-pitches in cents between each pitch and location; G, interval values of major and minor fifths and location; H, location and pitch quantification of each string-pitch; I, location and ratio values of each interval. Ratios are in blue; quantifications are in red.

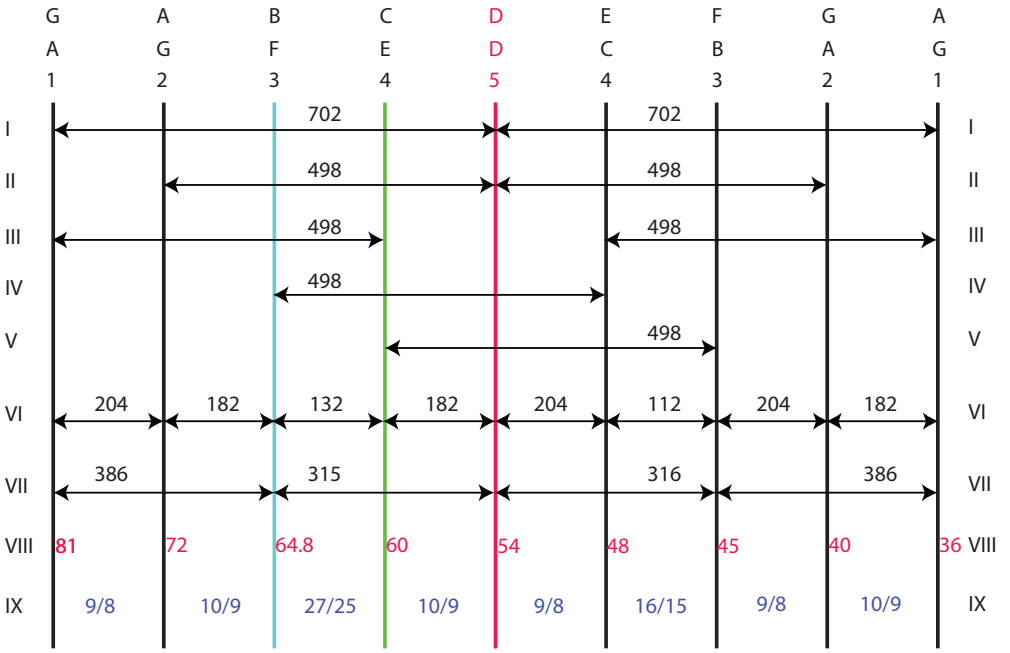

Figure 4. Quantifications, ratios and cents in bold and underlined indicate changes due to the reformation when 64.8 was corrected to 64. This brought new quantifications in the first pentad (encircled in red). There, the tritone is 610, almost equal to 612, the tritone in Just Intonation.

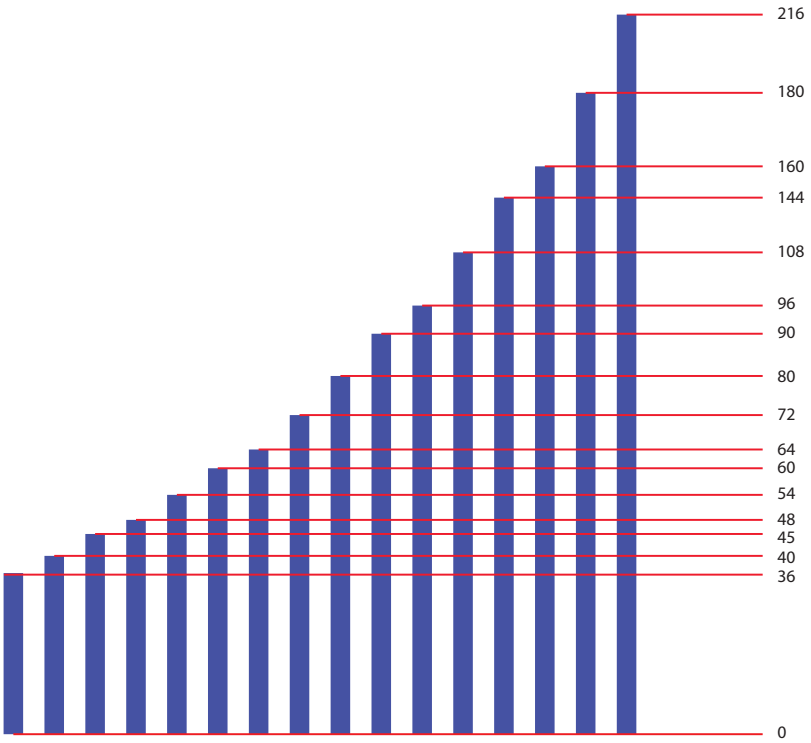

Figure 5. Babylonian quantification as pattern for string lengths, (with identical sections and mass) or air column lengths, for instrument makers.

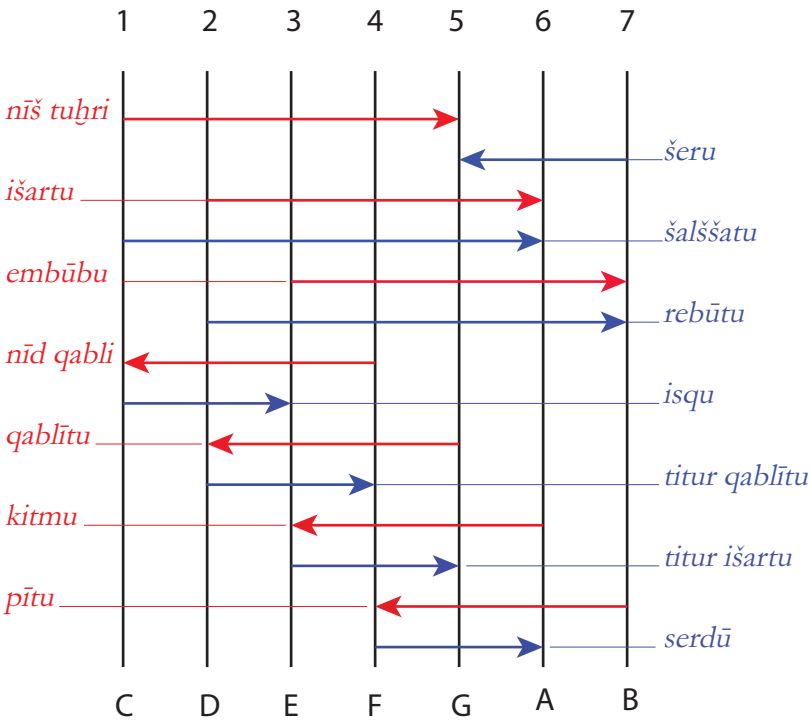

Figure 6 Graphic representation of CBS 10996, Kilmer's version. Names in red indicate principal pentadic sets. The tetrads in this graphic are obviously inverted pentads. Names in blue are secondary (triadic) subsets.

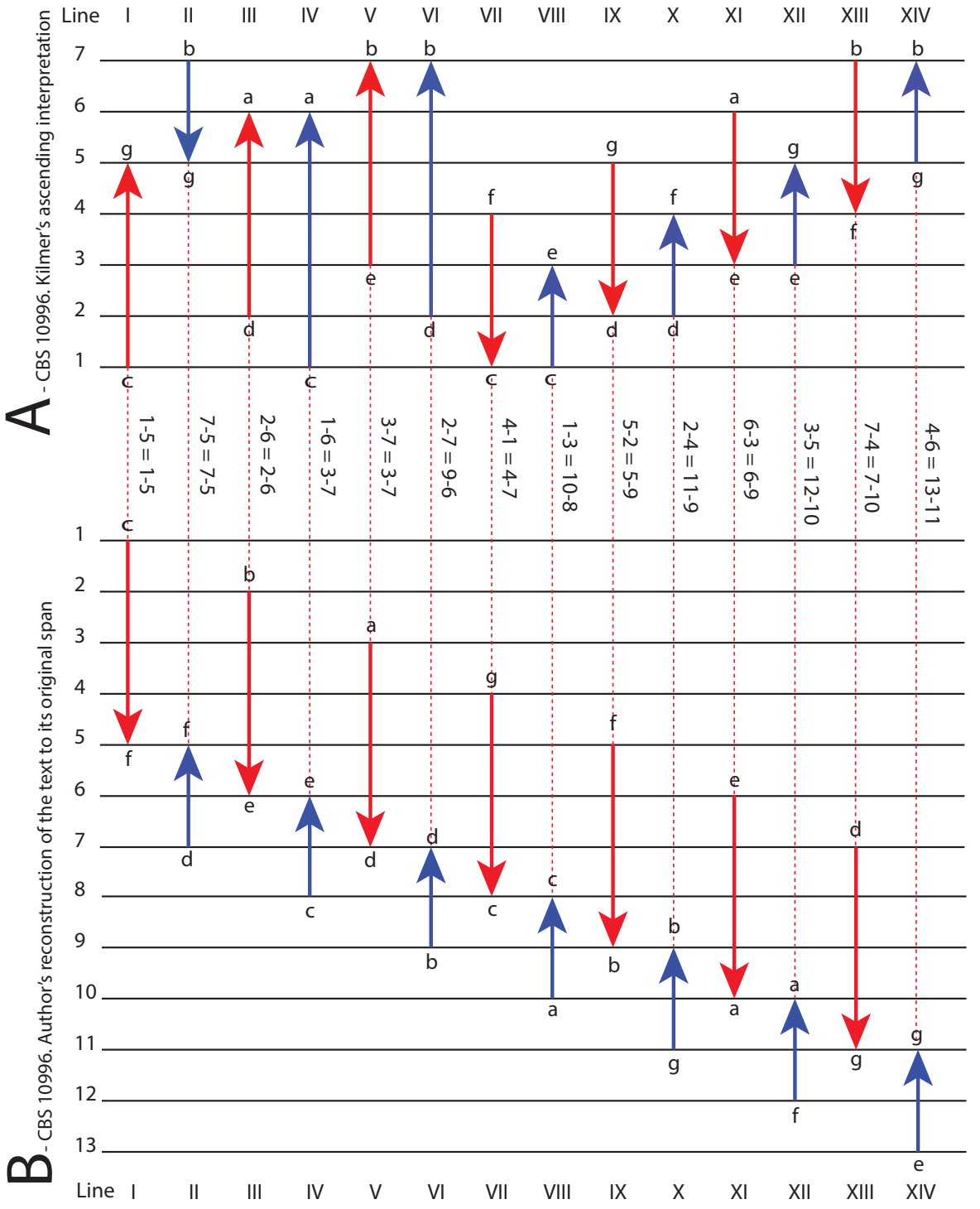

Figure 7. The seven lines at the top are the transliteration of the original text in its descending polarity. Lines 1 to 13 of the lower graphic shows the reconstruction of what would have been the original span, in a descending order. Numbers between lines 1 of A and 1 of B give interval inversions when they appear.

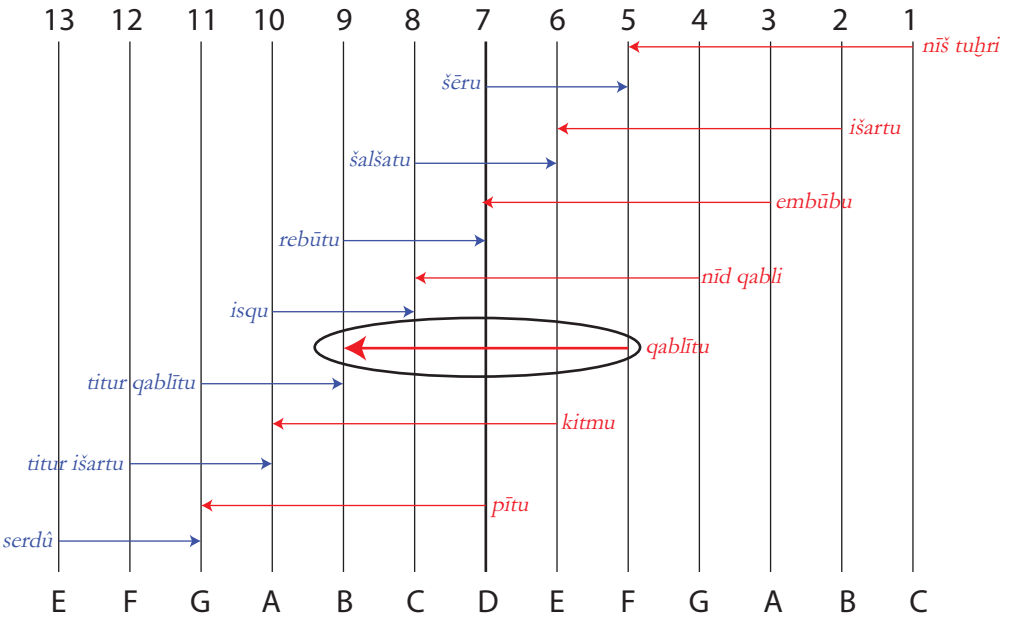

Figure 8. Author's reconstruction of the Greater Babylonian System of descending pentads and ascending triads. The position of Babylonian pentads is extrapolated from text CBS 10996. Therefore, this is how they would have been located prior to their contraction into a heptatonic framework. I have inscribed qablītu in an ellipse because in Babylonian the term means 'middle' which in this case is perfectly suited. qablītu sits exactly in the middle of the grid, it is the only tritonic pentad in the system and is symmetrical with D as its axis: B(semitone)C(Tone)D(Tone)E(semitone)F.

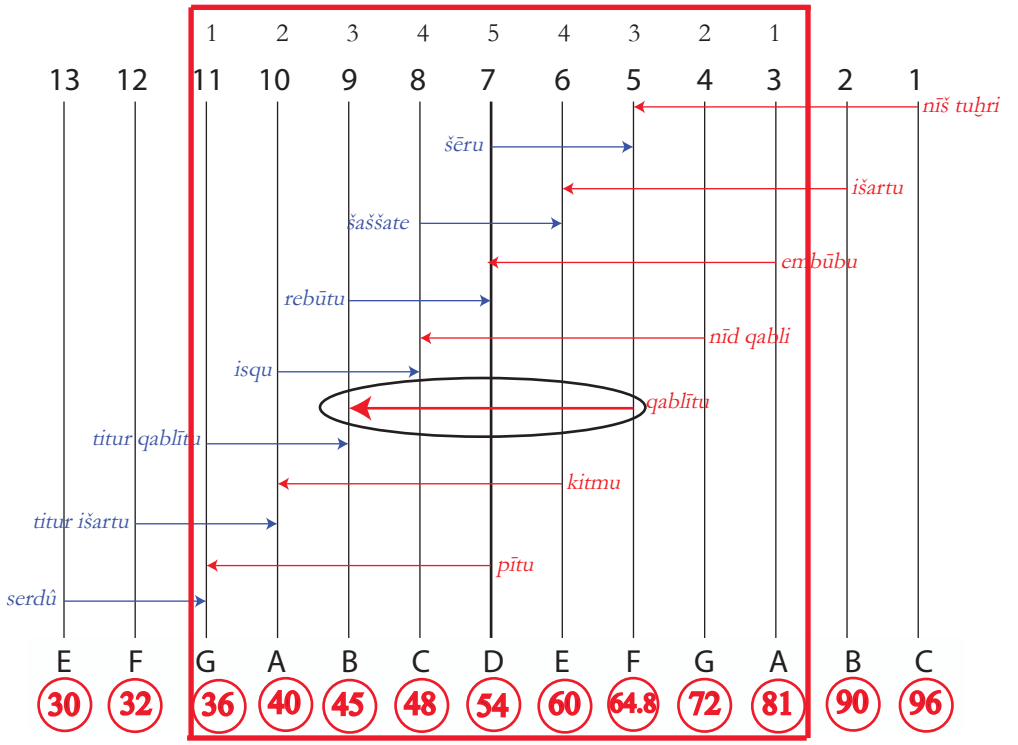

Figure 9. Superimposition of quantifications to the original interval series. Quantification 64.8 has been reduced to 64.

Tune up									
	I	II	III	IV	V	IV	III	II	I
išartum	1290	1200	996	792	588	498	294	90	0
qablītum	1290	1200	996	792	680	498	294	90	0
nīš tuḫrim	1382	1200	996	792	680	498	294	182	0
nīd qablim	1382	1200	996	884	680	498	294	182	0
pītum	1382	1200	996	884	680	498	386	182	0
embūbum	1382	1200	1088	884	680	498	386	182	0
kitmum	1382	1200	1088	884	680	590	386	182	0
išartum	1382	1292	1088	884	680	590	386	182	92
Tune down									
išartum	1382	1292	1088	884	680	590	386	182	92
kitmum	1382	1200	1088	884	680	590	386	182	0
embūbum	1382	1200	1088	884	680	498	386	182	0
pītum	1382	1200	996	884	680	498	386	182	0
nīd qablim	1382	1200	996	884	680	498	294	182	0
nīš tuḫrim	1382	1200	996	792	680	498	294	182	0
qablītum	1290	1200	996	792	680	498	294	90	0
išartum	1290	1200	996	792	588	498	294	90	0

Figure 10. Analysis of values of sets in cents showing that the last išartum of the first chapter is not at the octave of the first set. The same applies to the second chapter. The two sets differ by 92 cents throughout. 92 cents is the larger limma which is the defect of a fourth, 498 cents, increased by an apotome, 112 cents (total 610 cents) from a fifth, 702 cents, and hence the interval by which the fourth must be sharpened to be an apotome below (i.e. the 'leading note' to) the fifth and hence the interval by which the fourth is sharpened on modulating into the dominant.

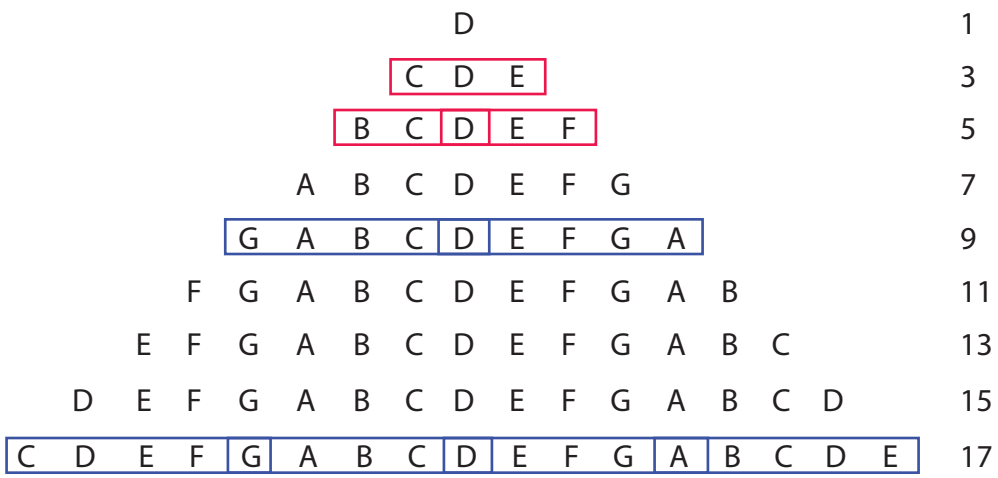

Figure 11. Pyramid of systems. This pyramid shows the gradual structure from an initial pitch, to a triad, then the conjunction of two triads amounting to a pentad, then of two conjunct pentads amounting to an ennead, then of four conjunct pentads amounting to the Greater Babylonian System or heptadecade.

Nomenclature	Approximative pitch	Quantification
1. *nīš tuḫrim*	E^4 - D^4 - C^4 - B^3 - A^3	$24^{(9:8)}$ $27^{(10:9)}$ $30^{(16:15)}$ $32^{(9:8)}$ 36
2. *išartum*	D^4 - C^4 - B^3 - A^3 - G^3	$27^{(10:9)}$ $30^{(16:15)}$ $32^{(9:8)}$ $36^{(10:9)}$ 40
3. *embūbum*	C^4 - B^3 - A^3 - G^3 - F^3	$30^{(16:15)}$ $32^{(9:8)}$ $36^{(10:9)}40^{(9:8)}$ 45
4. *nīd qablim*	B^3 - A^3 - G^3 - F^3 - E^3	$32^{(9:8)}$ $36^{(10:9)}$ $40^{(9:8)}$ $45^{(16:15)}$ 48
5. *qablītum*	A^3 - G^3 - F^3 - E^3 - D^3	$36^{(10:9)}$ $40^{(9:8)}$ $45^{(16:15)}$ $48^{(9:8)}$ 54
6. *kitmum*	G^3 - F^3 - E^3 - D^3 - C^3	$40^{(9:8)}$ $45^{(16:15)}$ $48^{(9:8)}$ $54^{(10:9)}$ 60
7. *pītum*	F^3 - E^3 - D^3 - C^3 - $B^{3(b?)}$	$45^{(16:15)}$ $48^{(9:8)}$ $54^{(10:9)}$ $60^{(16:15)}$ 64

Figure 12. Quantification of pentads showing that they all differ in content. Quantifications are given in regular numbers and in ratios.

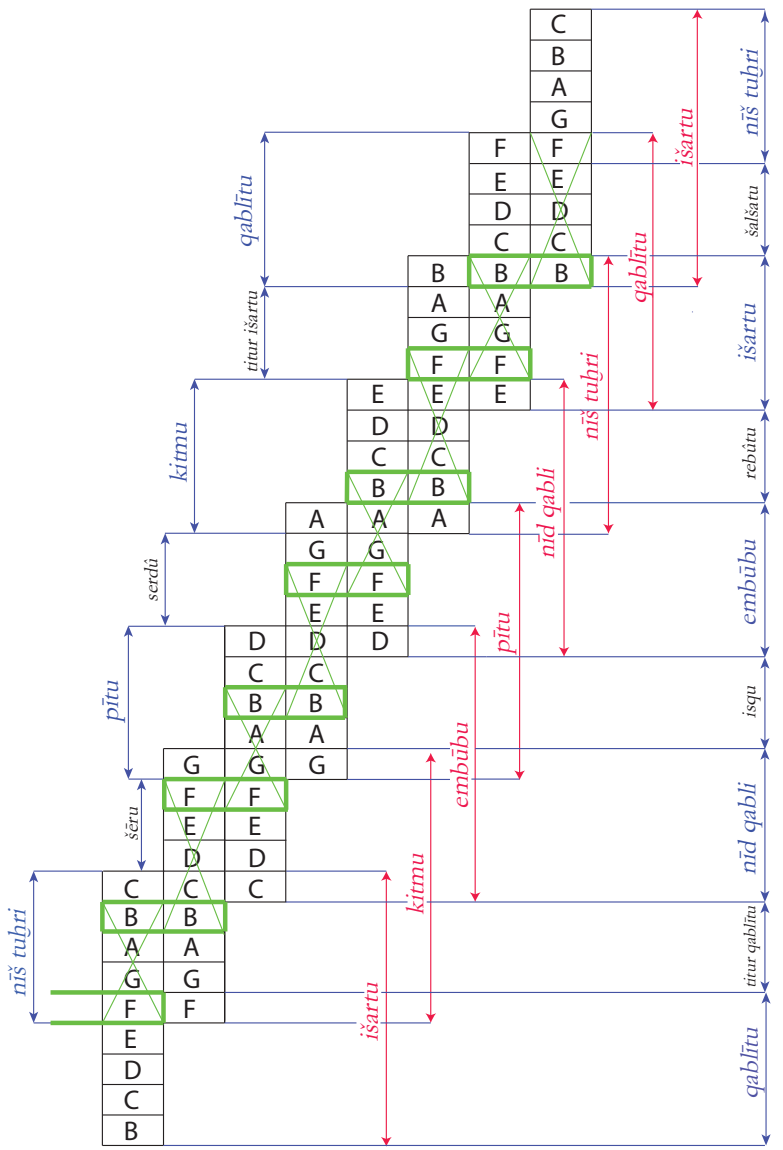

Figure 13. Set construction with superimposition of pentads (last pentad of a set being the first pentad of the following set). Pentads (principal intervals) are in blue; triads (secondary intervals) are in black; sets are in red. Tritones are crossed with green lines with their conjunction pitch framed in thick green lines.

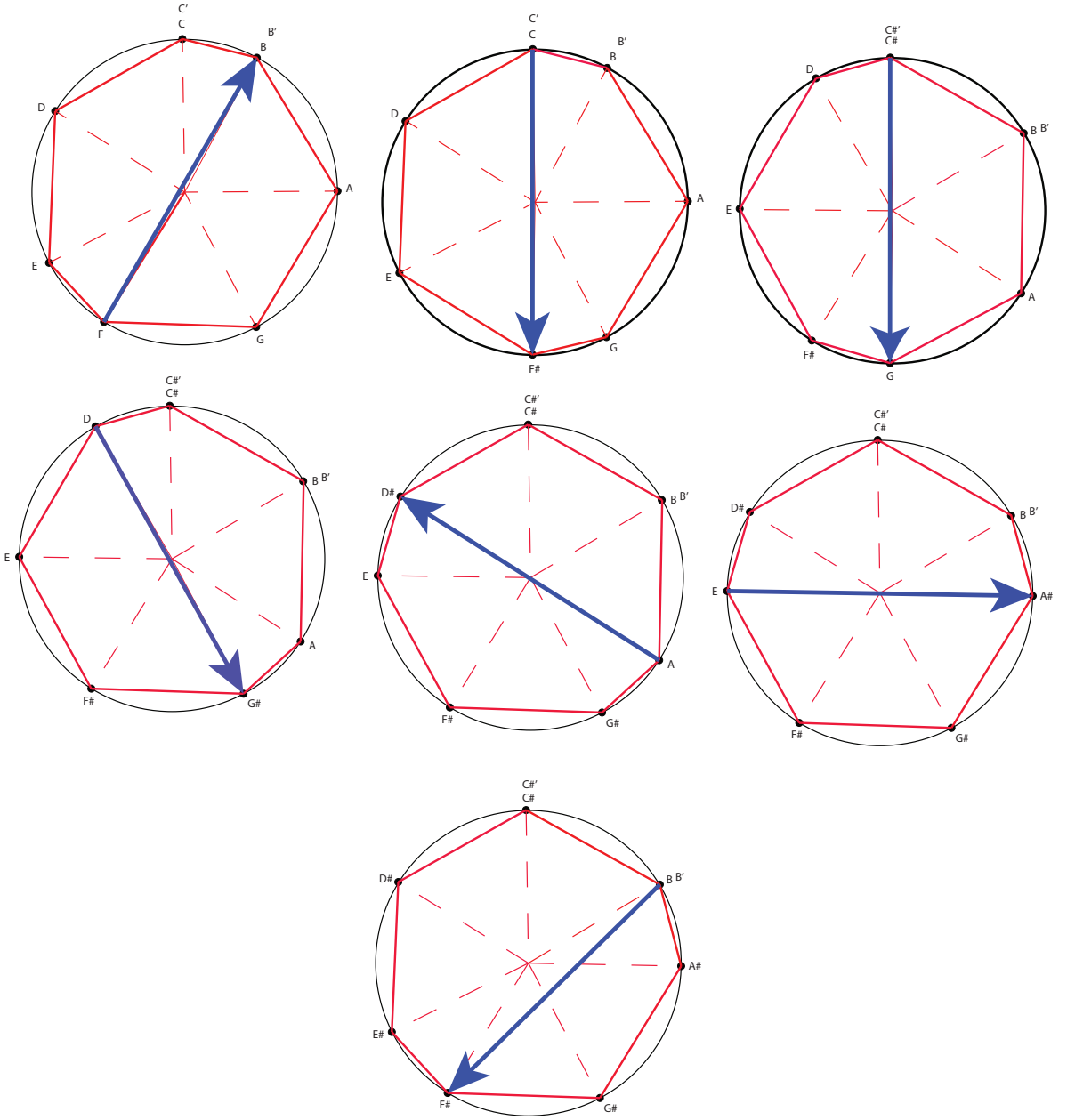

Figure 14. Circular representation of the sets generated in U.7/80. There is no evidence that Babylonians used this form of representation of their sets mainly because their system was enneatonic. Only heptatonism is suited for its representation in an encircled heptagram. (In my representation, note that I have doubled Cs and Bs to fit enneatonism within a heptagram): 1. *išartum*; 2. *qablītu*; 3. *nīš tuḫrim*; 4. *nīd qablim*; 5. *pītum*; 6. *embūbum*; 7. *kitmum*. Blue arrows indicate the tritones and their polarity.

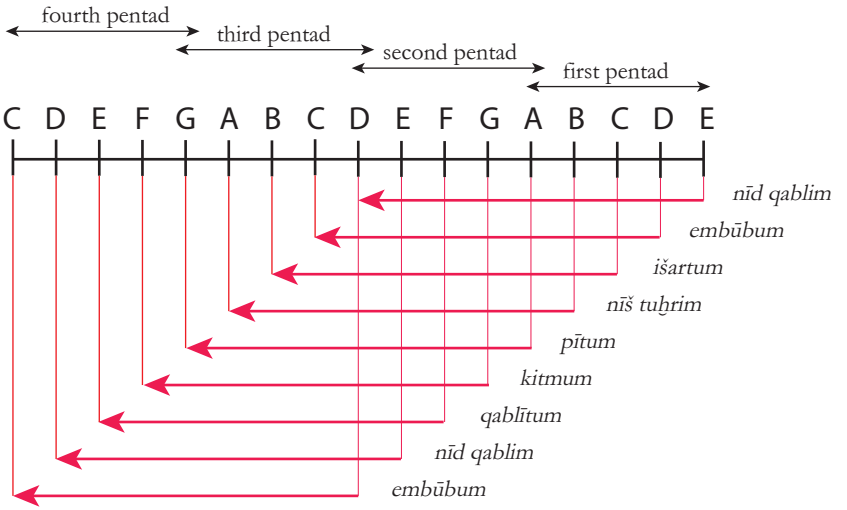

Figure 15. Placement of the nine sets on the greater Babylonian System of 17 pitches in a dynamic disposition.

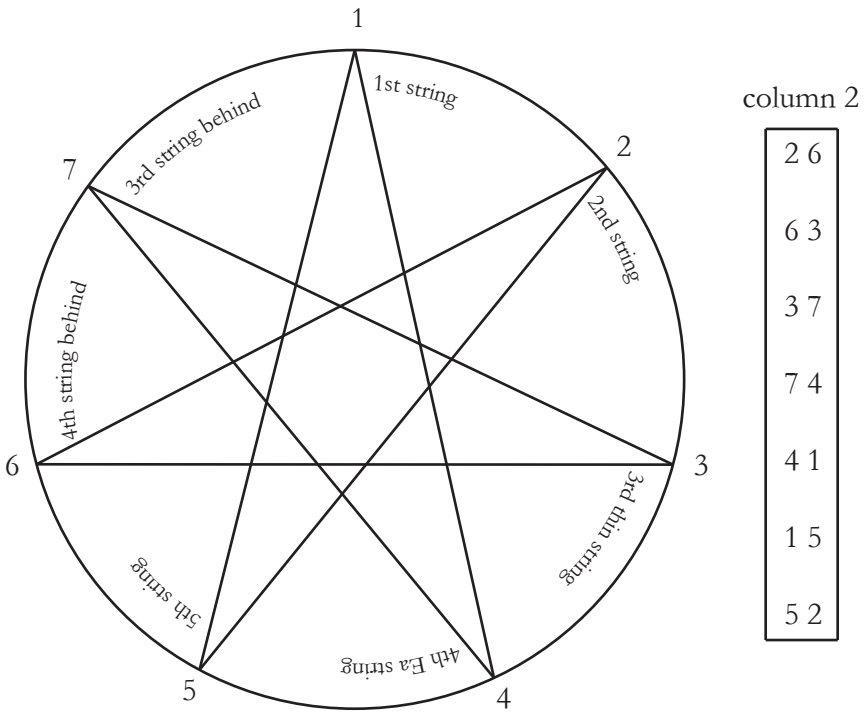

Figure 16. CBS 1766, selective lexical and numeral translation.

No	I	II	III	IV	V	VI	Beats
1	*qablite 3*	*irbute 1*	*qablite 3*	*šaḫri (?)1(?)*	*titimišarte 10*	*uštamari (?)*	70 (?)
2	*titimišarte 2*	*zirte 1*	*šaḫri 2 (?)*	*šaššate (?) 2*	*irbute 3 (?)*	*šaššate 2 (?)*	38 (?)
3	*umbube 1*	*šaššate 2*	*irbute 3(?)*	*natqabli (?) 1*	*titarqabli 1*	*titimišarte 2 (?)*	38 (?)
4	*zirte 1*	*šaḫri 2*	*šaššate 4*	*irbute 1*	*natqabli 1*	*šaḫri 2*	38 (?)
5	*šaššate 2(?)*	*šaḫri 1*	*šaššate 2*	*šaḫri 1*	*šaššate 2*	*irbute 4(?)*	38 (?)
6	*kitme 2*	*qablite 3*	*kitme 1*	*qablite 4*	*kitme 4 (?)*	*qablite (?) 4(?)*	60 (?)

Figure 17. H6 notation reconstructed.

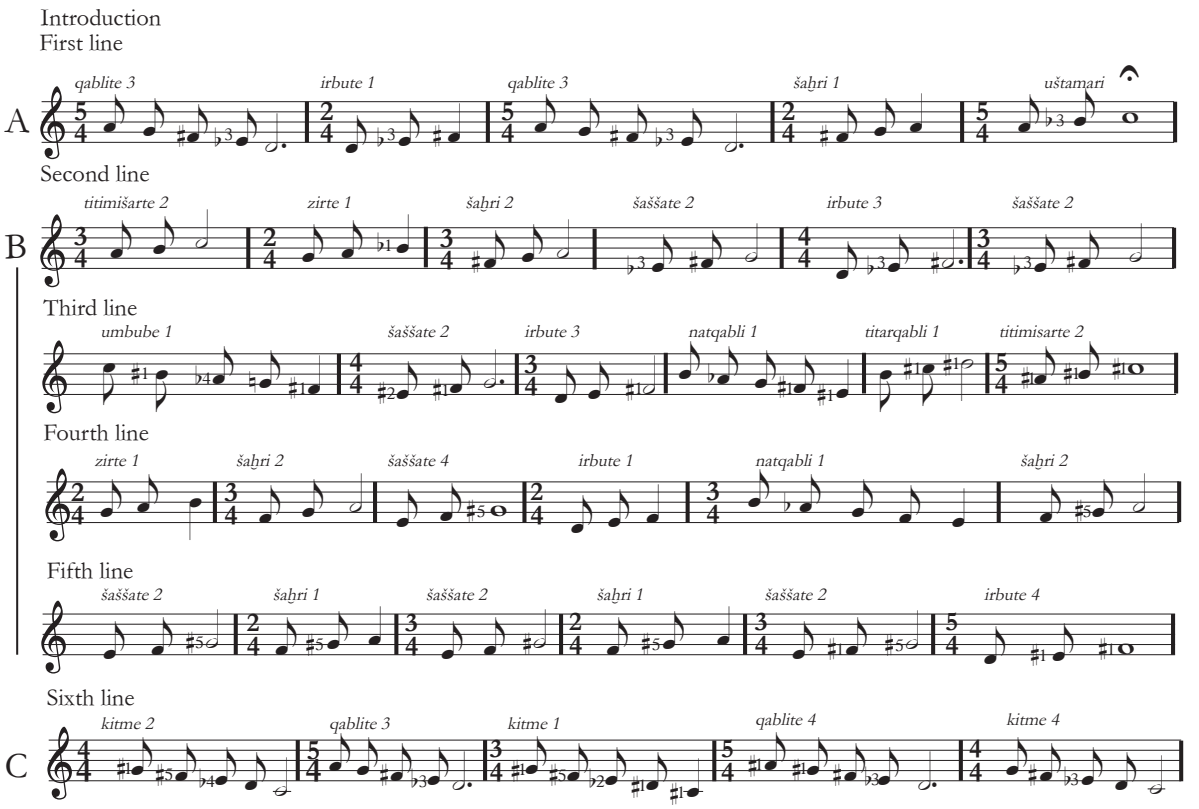

Figure 18. Author's interpretation of Hurrian song H6 in Near-Eastern intonation according to advice from Damascus and implementation with the collaboration of Rosy Azar Beyhom and Amine Beyhom from Beirut. The first bar of the introduction is the fourth bar of the conclusion. It is the musical version of well-known catch-lines often used in Mesopotamian texts. Numbers after accidentals indicate: #1= 1 comma sharper = 22.64 cents; #2= 2 commas sharper = 45.28 cents; #3= 3 commas sharper = 67.92 cents; #5= 5 commas sharper = 113.2 cents; b1= 1 comma flat = - 22.64 cents; b2= 2 c mmas flat = -45.28 cents; b3= 3 comma flat = - 67.92 cents; b4= 4 comma flat = - 90.57 cents.

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THROUGH THE EYES OF PLATO

Howard Barry SCHATZ

Introduction

Science and religion are often thought of as mutually exclusive. With science the arbiter of truth, and religion the arbiter of belief, could there possibly be a definitive body of knowledge where truth meets belief? This would be a precious knowledge indeed, and the quest for that knowledge is well articulated by the following passage, describing the Faylasufs, a 9th century Muslim sect in search of religious truth: They wanted to find the kernel of truth that lay at the heart of all the various historical religions, which since the dawn of history, had been trying to find the reality of the same God[1].

Whatever we think we might know about this "kernel of truth" is subject to rigorous scrutiny from scientists and clerics. Karl Popper's critical rationalism suggests that any claim to knowledge can be empirically falsified, but never proven empirically true. From Popper's perspective, any person who claims to know something can never justifiably lay claim to the truth, because truth is independent of our cognitive activity. It exists, or it doesn't exist, and our role in the matter is immaterial. From the classical scientific perspective of inductive reasoning, individuals are justified to say that they know something when generalizations are made from observations derived from a step-by-step accumulation of data. Traditionally, the study of metaphysics or religion evades epistemological discussions because reason is usually replaced by faith, and generalizations typically can't be derived from observed data, nor can metaphysical theories be evaluated sufficiently with universal criteria to be falsified.

Inductivists and falsificationists both fall into the category of rationalists, but the conclusions of rationalists might only be true within a particular context, which introduces the concept of relativism. For anthropologists, cultural relativism denies the existence of universal truth, framing truth within the limitations of language and cultural context. But, perhaps the scientific community should also be considered a distinct culture. Scientific truth at any given point in time must, of course, be based on the application of objective, universal criteria, but any determinations are also subject to a consensus within the scientific community. Physicist and philosopher Thomas Kuhn claimed that scientific progress was not always a linear and gradual process, so he introduced the locution 'paradigm shift' to describe an exponential shift of the baseline thinking within the scientific community. Kuhn writes that 'when the paradigm is successful, the profession will have solved problems that its members could scarcely have imagined and would never have undertaken without commitment to the paradigm.'

As we pay homage to the life and work of Ernest G. McClain, we find him grappling with ancient texts that lay claim to truth and wisdom, including the great philosophical writings of Plato, the writings of Scripture, and the oldest Hindu text, the Rig Veda. Ernest focused his efforts on a common mathematical/musical tradition that occurs within a matrix of cross-cultural contexts. We might wonder if this tradition is indicative of some "kernel of truth," since it seems to transcend cultures. If the same mathematics crops up in different cultures, could it suggest that science is not far behind; or, must the mathematical deep structure of metaphysics and religion necessarily be categorized as arithmetic subjectivism?

Joseph Campbell suggests that the origins of this common mathematics can be traced back to the discoveries of 4th millennium Mesopotamian astronomers and writes that '...the time and place of the date for the invention of the arts of writing, mathematics, and astronomical reckoning there:

1 Karen Armstrong, *A History of God* (New York: Random House, 1993), 170-173.

the time and place of the initiation of this whole remarkable effort to translate celestial mathematics into the ordering principle of life on earth[2].

Astronomy reached its highpoint of development during the Old Babylonian period[3].Campbell references a text by Alfred Jeremias, which suggests that the wisdom of celestial mathematics was then adapted to a variety of cultural contexts. Babylon was, moreover, according to a constant tradition, the home of astronomy ('Chaldean wisdom'), and there the science of the stars formed the basis of all intellectual culture[4].

Ernest's window into the cross-cultural matrix of ancient wisdom writings was Plato's Pythagorean mathematical model based on the theory of vibrating strings known in ancient times. Ernest went about his business, exploring the archaeomusicology of different cultures from his Pythagorean perspective, completely unaware that a paradigm shift would take place during his lifetime that would enable science to bring Plato's metaphysics and the God of Abraham into the laboratory for the first time. In an attempt to justify this dramatic claim, allow me to provide additional background that should help make my case.

Any discussion of objective versus subjective reality should also address the notion of absolute versus relative truth. If we were to use Einstein's relativity as the authoritative model to determine if an absolute truth exists, we would discover that an underlying absolute framework and unity of nature does indeed exist. Einstein's special theory of relativity required that all relative contexts be made manifestly covariant against an absolute framework in order to match 'apples to apples.' However, Einstein spent the last 30 years of his life trying to further define that absolute framework as a Unified Field Theory. Unfortunately, he was unsuccessful, but Einstein's efforts beg the question of whether today's scientists share Einstein's vision for the so-called end of physics, i.e., formulating the theory of the universe according to a finite number of principles.

In 1984, physicists John Schwarz and Michael Green worked together to discover the comprehensive mathematical solution that Einstein was never able to find. The Schwarz-Green mechanism gave birth to superstring theory. Superstring theory is based on the harmonic series that results from a classical Newtonian vibrating string. But, additional mathematical constraints were imposed by the requirements of 20th century physics to account for all the forces of nature. Schwarz and Green had to make their superstring manifestly-covariant, to account for the principles of relativity; and they had to quantize it, to account for the rigors of quantum mechanics. There were several other mathematical constraints, but their breakthrough formula described a tiny harmonic superstring vibrating in ten abstract dimensions. All elements on the Periodic Table of Elements are structured by spherical harmonics, and superstring theory states that each of the sub-atomic wave-particles (protons, electrons, gravitons, gluons, etc.) may be closer to standing wave harmonics than to discrete particles. These wave-particles vibrate into ten abstract dimensions rather than the usual four dimensions (three spatial dimensions plus time) that we currently know exist. If the foundation of the universe turns out to be the harmonic series, as superstring theory suggests, then we must ask, what role, if any, does the harmonic series play within Plato's metaphysics, or within the original conception of monotheism's God?

Ernest is part of an important lineage of archeomusicologists who have charted the deep structure of the ancient wisdom writings. His pick and shovel are the integer ratios of ancient string theory. Ernest has taught us that ancient string theory defines the foundation of Plato's metaphysics, the Rig Veda and Scripture. Like Einstein, Plato believed in objective reality and in the unity of nature. Determining the truth of Plato's metaphysics was limited to the exact sciences of antiquity, as summarized by the Pythagorean tradition, better known as *The Harmony of the Spheres*[5]. Central to Plato's metaphysics was his

2 Alfred Jeremias, *Ages of the World*, cited by Joseph Campbell, The Mythic Age (MJF Books/Princeton University Press, 1974) 149.

3 O. Neugebauer, *History of Ancient Astronomy, Astronomy and History: Selected Essays* (New York: Springer-Verlag, 1983), p. 50.

4 Campbell, Op.Cit. *The Mythic Age*, p. 149.

5 Joscelyn Godwin, *The Harmony of the Spheres*

theory of Forms. Since it is only possible to bring an instantiated, perceptible form into the laboratory, the transcendent nature of Plato's idealized form seems, once again, to relegate Plato's mathematical model to the realm of arithmetic subjectivism. After all, how could the cosmology, cosmogony, and ontology of Pythagoras and Plato be subjected to the empirical scrutiny of a laboratory? It seems absurd to even suggest such a thing. Certainly, Ernest never concerned himself with scientific proofs.

Popper's philosophy of science differs from the usual scientific approach insofar as it prefers the least likely, unfalsifiable theory. It prefers the least likely candidate, because it is that candidate which typically has the highest information content and appears to be the most vulnerable to falsification. The modern scientific method, however, prefers the most likely candidate to scrutinize. It is safe to say that Plato's metaphysics would easily qualify as a least likely candidate since it has the highest information content. But, can we even compare Plato's mathematics to the rigors of superstring theory and m-theory, the most successful 21st century paradigm? Superstring theory is solving problems that physicists 'could scarcely have imagined and would never have undertaken without commitment to the paradigm.' Imagine that by providing a 'theory of everything' for modern physics, that same theory of everything also defines the deep structure of Plato's metaphysics, the God of Abraham, and the spanda (Sanskrit: sacred vibration) of the Rig Veda.

Long before Ernest ever realized that Pythagorean string theory might be the predecessor of modern string theory, he knew that it held the key to understanding allegory within antiquity's greatest religious and philosophical texts. The rest of us have the luxury to ponder the epistemological, religious, and scientific implications of Ernest's work, but without Ernest's window on antiquity, there would be little to discuss. Now that string theory is in vogue, the spotlight is shifting to focus on Ernest's work, because his books uniquely reestablish the tight coupling between science, philosophy, and religion. Consider the possibility that science and religion were born as fraternal twins within the same ancient cradle.

For Plato, ontological truth was eternal and unchanging, but he considered man's perceptions of that truth to be subjective, changing, and limited. Plato famously made this point in his 'Allegory of the Cave.' Plato believed that whatever our senses recognized in the material world is subject to change, but for him, reality went deeper, instantiating abstract, non-material (yet still substantial) form from an unchanging eternal source of number and vibration.

One important by-product of number and vibration was Plato's belief in the Greek concept of ethos, which described how a culture's music directly impacted the soul, the character of its citizens, and the nature of its government. He believed that audible music either disturbed or enhanced the inner vibrational essence of everyone and everything. Reality may not have been perceptible to the senses, but the hidden, inaudible vibrations of the 'World Soul' was the basis of each human soul, and the essence of the entire manifest cosmos. In short, ancient string theory was at the heart of what it meant to be a Pythagorean.

Ernest demonstrated that the 'Rosetta Stone' to decipher Plato's mathematical allegories was the so-called Pythagorean Table, or Lambdoma diagram, which expressed the dual sequence of harmonics and sub-harmonics[6]. Thanks to what I learned from Ernest, my own texts demonstrate that this very same mathematical table was encrypted in what is arguably the oldest monotheistic text, the *Sefer Yetzirah* (Book of Creation), the only text attributed to Abraham by the Orthodox Jewish community. My books attempt to demonstrate how the harmonic series, at the core of superstring theory, originally defined the God of Abraham[7]. My examples attempt to demonstrate that the *Sefer Yetzirah*'s Lambdoma Table shapes a great deal of Biblical allegory, which should sufficiently demonstrate that the *Sefer Yetzirah* is authentically Abrahamic[8].

(Rochester, Vermont: Inner Traditions International, Ltd., 1993) xii.

6 Ibid., p. 371.

7 Howard Schatz, *The Lost Word of God* (New York: Tone Circle Pub.: 2007) pp. 30-40.

8 Ibid., 113-255; Howard Schatz, *The Science of*

The paradigm of superstring and m-theory continues to gain acceptance within the scientific community as the most viable candidate for a theory of everything. Proving a theory of everything based on the harmonic series would also create a de facto proof that God is a harmonic series, if it can be substantiated that the God of Abraham was originally defined as a harmonic series. Once Biblical scholars acknowledge the Lambdoma Table as the original definition of monotheism's God, then even agnostics and atheists would find it difficult to deny that God exists. If the 'Image of God' was originally defined in terms of the transcendent mathematical formula for a harmonic series, then God's instantiation into matter describes the substance of God (Greek: ousia; Hebrew: *pnimiyut*) as the vibrational essence of all matter and energy in the universe. This 'substance of God', for example, is the 'Image of God' shared among the three 'co-equals in eternity' better known as the Holy Trinity (*homoousian*).

This paper will attempt to reconstruct Ernest's Platonic window into antiquity, and to articulate his observations about the unique role of Equal Temperament as the 'Embodied Vision' of Plato, Scripture, and the Rig Veda. This paper will examine ancient string theory and compare it to recent developments in modern string theory. Finally, it will be demonstrated that the intersection of science and religion not only unlocks the secrets of civilization's most sacred texts, but also describes a powerful 21st century framework for peace.

I. Mathematics, Music, Philosophy & God

My years of independent study at Brooklyn College with Ernest McClain were the happy result of a rather arcane paper that I wrote for an undergraduate course at Brooklyn College in the early 1970's. My music history professor, as well as the Chairman and Deputy Chairman of the department asked me to explain my paper to them, but since Pythagorean mathematics wasn't their strong suit, they decided to give the paper to Dr. Ernest McClain. What Ernest taught me over the next few years would change my life. He also became something of a surrogate father, who guided me through some challenging times. I loved him as a mentor and as a friend.

Unfortunately, after receiving my degree in music composition, we lost contact. Years later, I tracked him down in Washington D.C., where I paid him a visit. He mentioned to me that I had been his only formally assigned student in Pythagorean studies during all his years of teaching. Knowing all that he had to share with the world, I felt privileged but saddened that his breakthrough thesis deciphering the mathematical riddles within a host of ancient philosophical and religious writings was not trumpeted by an army of disciples. I confided in him that his teaching gave my life direction and meaning, and I gave him a recently completed draft of my first book. He was pleasantly surprised that I 'stayed with it.' After reading my manuscript, he was gracious enough to write a wonderful foreword. He then introduced me to the Berkeley Institute of Biblical Archeology and Literature (BIBAL), where I was warmly received, despite being humbled by the accomplishments and credentials of its members. BIBAL folks provided a rigorous sounding board for Ernest's work during his later years. It offered him the opportunity to expound on his writings, and he won a growing number of supporters.

Ernest always enjoyed relationships with noted scholars, but perhaps none were more important than his collaboration with philosopher Antonio de Nicolas. A week before Ernest died, he asked me to complete a Festshrift chapter (living homage) dedicated to his much loved collaborator, Antonio. I was deeply honored by Ernest's trust in me, but the publisher was anxious to move forward, and it was published before I was able to fulfill Ernest's request. I'm delighted that I have the opportunity to submit this chapter for inclusion in a Gedenkschrift dedicated to Ernest.

Ernest's thesis can be summarized by the somewhat preposterous sounding notion that the first few prime numbers structured and shaped history's greatest philosophical and religious texts. Ernest inherited a methodology initially rediscovered by a German 19th century judge and music theorist named Albert von Thimus.

Religion: A Framework for Peace (New York: Tone Circle Pub., 2013).

The significance of his writings would have been lost if not for the work of a Swiss-German scientist named Hans Kayser. Kayser's work then inspired the writings of Swiss pianist, composer, and musicologist Ernst Levy. When Ernst Levy emigrated to the US in 1941, he taught at several colleges, including MIT. In 1959, he joined the staff at Brooklyn College as a colleague of Siegmund Levarie and Ernest McClain.

Ernest became heir to a Pythagorean methodology that enabled him to illuminate the deep structure of Plato's Dialogues, the Bible, and the Rig Veda. Ernest's stepping stone to even earlier civilizations was his knowledge of the mathematical/musical details describing the Pythagorean tradition. Ernest's thesis is based on the integer ratios defined by the mathematical structure of sound, known today as acoustics. A more recent name for this discipline is "string theory," since its physics derives from the theory of vibrating strings known in ancient times. Still, it is natural to wonder how comparing string lengths could possibly shape myth and allegory within the literature of the Ancient Near and Middle East.

Ernest provides us with the following table as a summary of Plato's musical constructs. (See table 1[9]) Ernest dedicates a chapter to each of the musical tuning systems listed in the table below, and he describes how Plato modeled four different cities, each based on one of these four tuning systems. Each city demonstrates a different type of political regime based on the inner reality of its leaders and inhabitants.

To quote Ernest: 'Platonic ratio theory, music theory, political theory, and astronomy are equivalent representations of an abstract cosmological "systems" theory[10].' Plato's first principles are defined by number and music, and these principles must be understood if one hopes to unlock the meaning of Plato's writings. In the following passage from *Laws* 967e, Plato justifies his musical approach to philosophy and knowledge: '...Moreover as I have now stated several times, he who has not contemplated the mind of nature which is said to exist in the stars, and gone through the previous training, and seen the connection of music with these things, and harmonized them all with laws and institutions, is not able to give a reason of such things as have a reason[11].

Through the eyes of Plato, Ernest was able to explore even older musical traditions, surveying the ancient literature of Sumer, Babylon, India, Egypt, and the religions of the Levant. His analysis of the common mathematical elements in Sumerian, Hebrew, and Hindu mythology supports the thesis of a leading scholar on ancient Sumer, Samuel Noah Kramer, who believed that Sumerian cosmology, theology, ethics, and system of education permeated to a greater or lesser extent the thoughts and writings of all the peoples of the ancient Near East[12]. Ernest is our Sherpa guide through the hidden passes of Scripture's Holy Mountain, Babylonian Ziggurats, and Egyptian Pyramids. His legacy includes a suite of mathematical/musical tools that need to be mastered by anthropologists, archaeologists, historians, musicians, and theologians, if they hope to penetrate the ancient writings. The *Torah*, for example, is rich with musical metaphor. One has to understand how integer ratios create Plato's four tuning systems, because they also shape Biblical literature. From this perspective we might best think of Plato as the last true Kabbalist. Ernest lived and breathed musical metaphor, and freely moved from one cultural context to another at a dizzying pace, often surveying a cross-section of cultures in a single paragraph.

My own particular obsession has been an effort to expand on the religious and scientific implications of Ernest's work with respect to the harmonic series, i.e., 'the chord of nature[13].' The harmonic series is a natural, empirical phenomenon

9 Ernest G. McClain, *The Pythagorean Plato* (York Beach, Maine: Nicolas - Hays, Inc., 1978), p. 14.

10 Ibid., p. 58.

11 Plato, *The Laws*, trans. by Benjamin Jowett (New York: Dover Pub., 2006), 967e

12 Ernest G, McClain, *The Myth of Invariance: The Origins of the Gods, Mathematics and Music from the Rg Veda to Plato* (New York: Nicolas Hays Ltd., 1976), p. 129; Samuel Noah Kramer, The *Sumerians* (Chicago: University of Chicago Press, 1963) 291

13 Howard Schatz, *The Chord of Nature and the Evolution of Music Theory in Music in Human Adaptation,* eds. Dr. Daniel J. Schneck and Judith K. Schneck (Blacksburg: Virginia Tech, 1997), pp. 423- 36.

with a simple yet profound scientific formula: integer multiples of a fundamental vibration, in other words, simple counting. The simple counting that defines the harmonic series helps us to better understand Ernest's analysis of Plato's cities and associated tuning systems. Each of the sections in this paper have been subtitled according to the sets of integers that define them:

- Pythagorean Tuning: The First Four integers
- Just Tuning: The First Six integers
- Archytas Tuning The First Seven Integers
- Equal Temperament: The Rejection of Integers

A 'Primer' has been included in the Appendix to provide the reader with some basic knowledge about the integer ratios that define ancient string theory. Mastering these basics should put us in a better position to understand the various tuning systems, and how Ernest applied them to Plato's Dialogs. I have taken the liberty to include several of my own diagrams, derived from Scripture, when I find them to be consistent with Ernest's thesis.

II. Pythagorean Tuning: The First Four Integers

The first four integers (1:2:3:4) extend into matter through the geometry of point, line, plane, and solid. As the transcendent Monad extends into matter as a Dyad, a geometric perspective suggests that 'The One' becomes one something, and that 'point' extends into matter as a one dimensional length with two endpoints. Three points determine a two dimensional plane by adding width to length. Adding a fourth point on a different plane adds depth and creates space, thus cubed numbers have three dimensional volume. The simplest Platonic solid is a tetrahedron, created from four points.

When we use these first four numbers to divide the string on a monochord, the generation of tones has historically suggested a procreation metaphor, beginning with the ratio of 2:1. This ratio creates a womb-like octave container of all the subsequent divisions of the string that 'gives birth' to all subsequent tones. God's number = one, and the number two, creating the octave container, is symbolic of woman. The number three symbolizes 'man' and the musical perfect fifth generated by the number three 'impregnates' the female octave generated by the number two with new and different notes. Additional powers of three generate all the different named pitches within each female octave container. The function of the female number two transposes each named note (D, E, F, G, etc.) to higher or lower octaves. Octave transposition results from multiplying or dividing string length by two without changing its note value. When constructing the city of Ancient Athens from Pythagorean Tuning based on powers of two and three, the acropolis of Athens had a single tone generating fountain (male powers of three) bringing forth divine musical 'waters.'

For Plato, musical procreation proceeded from the octave 'womb' (1:2) that contained or gave birth to musical twins: the arithmetic and harmonic mean. The metaphor of a musical twin derives from the dual function of the number three as the tone generator of both musical fifths (2:3) and musical fourths (3:4) which become musical complements within the octave (2:4). We must transpose the proportion 1:2:3:4 to 6:8:9:12 in order to fit both the arithmetic and harmonic means into a single octave. As we can see in Figure 1, the interval separating these arithmetic and harmonic twins is the interval of a Tonic (8:9).

This diagram depicts the Pythagorean Perfection of perfect octaves (1:2), perfect fifths (2:3), and perfect fourths (3:4) using older Greek names. It describes the foundation of music theory within ancient Greece since the time of Pythagoras. Ernest tells us that these ratios define the only fixed tones in Pythagorean tuning theory and appear as the invariant element in every Platonic mathematical allegory[14].

The *organum* became the basis of Church music for about one thousand years. It typically started with a unison plainchant melody harmonized by the same melody a perfect octave, perfect fifth, or perfect fourth away. As we can see in the diagram, these three intervals all derive from the first four harmonics.

The Tyrant's Allegory (Pythagorean Tuning)

If the reader has any doubt that Plato's mathematical riddles married number to philo-

14 McClain, Op.Cit., *The Pythagorean Plato*, p. 9.

sophy, they merely have to encounter the many passages that have baffled translators for centuries. Here is an excerpt from "The Tyrant's Allegory" within Book IX of the *Republic*. It should provide some insight into how Plato linked number and geometry to his ethics, politics, and philosophy:

> The tyrant is the third removed from the oligarch, and has therefore, not a shadow of his pleasure, but the shadow of a shadow only. The oligarch, again, is thrice removed from the king, and thus we get the formula 3 x 3, which is the number of a surface, representing the shadow which is the tyrant's pleasure, and if you like to cube this 'number of the beast,' you will find that the measure of the difference amounts to 729; the king is 729 times more happy than the tyrant. And this extraordinary number is nearly equal to the number of days and nights in a year (365 x 2 = 730); and is therefore concerned with human life. This is the interval between a good and bad man in happiness only: what must be the difference between them in comeliness of life and virtue!

There is nothing vague about Plato's use of number, but the correspondence that Plato draws between number and a state's political system is baffling to the uninitiated. A musical context solves all the mathematical riddles. Plato believed that music affected the soul, the character, and the body politic. He believed that imperfections could only exist within the external world of matter, and not within the inner spiritual realm of the soul. Within Pythagorean Tuning, new tones could only be generated by powers of three, since powers of two create redundant tones in either higher or lower octaves. Powers of two might therefore be omitted from certain musical constructions.

In the bit of prose quoted above, Plato explains that the King (=1) compares to the Tyrant by a difference of 729, which he then derives for us by taking the cube of the number of the beast (9^3 = 729). If we recall Figure 1, the number 9 creates a whole tone or Tonic (8:9) when measured against the third octave boundary (2^3 = 8). We should take note that the number 9 is not a prime number. The first principle of music theory states that it takes a prime number to generate a new sound. With this in mind, we factor 9^3 = 729 into 3^6 = 729. This factoring exercise enables us to recognize that Plato's number of the beast also appears within the New Testament's *Book of Revelation*, as Satan, the ultimate Tyrant. Within Revelation this number is interpreted as three sixes (666), but Ernest informs us that it is not three 6's but six threes (3^6 = 729) that defines the *Diabolus in Musica*[15]. Figure 2a arrives at the Tyrant's number using 3^6 = 729 exponing the sounds in a single row. In Figure 2b each tone spins around the tone circle coming to rest in its appropriate tonal location. Figure 2c describes a series of three whole steps (9:8) to finally arrive at the location of the Tyrant (9^3 = 729). If the divine wisdom of Plato's philosopher-king corresponds to the D at the middle of the exponed row, and at the top of the tone circle, then the Tyrant's location would be diametrically opposed at the bottom of the tone circle. The gap or musical comma that exists between the A♭ and G♯ at the bottom of the circle in Figure 2b is generated between the two farthest extremities from the divine: 729/1 and 1/729. We should not be too surprised to learn that this musical comma also corresponds to the 'abyss of the Beast' within the *Book of Revelation*.

In Figure 2c[16] we are playing a musical scale one step at a time, with each step being a major second (9:8). When we add musical intervals to one another we multiply them as fractions. In this case, three whole tone steps of ratio 9:8 creates the cumulative distance of three tones (tritone) which bisects the tone circle (1:√2:2). Pythagoras' irrational constant of √2 may be the first known irrational number, causing great unrest for Pythagoreans. There is a popular story that the Pythagorean philosopher Hippasus divulged the discovery of this irrational number revealing the universe's imperfection, and was therefore drowned at sea as punishment from the gods.

Since each power of three became increasingly removed from the divine wisdom of the D at the top of the tone circle, the Tyrant's number (9^3 = 3^6 = 729), located at the bottom of the tone circle, is as far away as one could get from Divinity at the top. The essence of the Tyrant, therefore,

15 McClain, Op.Cit., *The Myth of Invariance*, p. 117.
16 McClain, Op.Cit., *The Pythagorean Plato*, p. 37.

embodied the antithesis of the philosopher-king's divine wisdom. The World Soul (explained in the next chapter) is created from the same triple progression as the Tyrant. But, because the Soul is divine it was necessarily limited, and could not extend closer to the Evil of the Tyrant ($3^6 = 729$) by any more than three powers of three, limiting the endpoints of the soul's construction to $3^3 = 27$ and $1/3^3 = 1/27$. This manifests in the soul of Plato's aristocratic philosopher-king as a man who knows his limitations, and is thus fit to rule with wisdom and within reason. The continued increase in powers of three within the soul of each successive 'generation of guardians' becomes the root cause (although not the only cause) of a gradual degeneration through Plato's five political regimes: Aristocracy, Timocracy, Oligarchy, Democracy and Tyranny.

The philosopher-king was considered an aristocrat who ruled selflessly, neither tempted nor distracted by personal gain. Rule by a philosopher-king (Aristocracy) was most desirable, but often limited to the mythological founders of the city. Each successive generation of the city's guardians created an exponential increase of the various prime numbers (Plato's genetic components), and if the vibrational essence of a guardian's being was not properly limited, they would tend to pursue personal interests. The final phase of this political devolution, from Democracy to a Tyrannical government, is symptomatically based on the conflict between rich and poor.

Most cities in ancient Greece were either Oligarchies or Democracies. Moderation and temperance within an Oligarchy was channeled toward not wasting wealth. Athens, as a city limited to essentials, became Plato's model for Democracy. Athenians did not demand more than what was owed to them, for the greater good, and this principle was the crux of what it meant to be a Democracy. The degeneration of that democracy begins when freedom is cherished above governance, and the idea spreads that a citizen is free to do whatever they want, while happiness is generally associated with material wealth. Democracy takes the final step toward Tyranny when there is no discipline whatsoever, and chaos ensues. A Tyrant typically seizes power, embodying injustice and Evil. The Tyrant is generally hated and might be killed by the masses who seek democracy and freedom from Tyranny. Here is Plato's musical procreation metaphor describing how political systems degenerate:

> This whole geometrical number is sovereign of better and worse begettings. And when your guardians from ignorance of them cause grooms to live with brides out of season, the children will have neither good natures nor good luck. Their predecessors will choose the best of these children; but, nevertheless, since they are unworthy, when they, in turn, come to the powers of their fathers, they will as guardians first begin to neglect us by having less consideration than is required, first, for music, and, second, for gymnastic; and from there your young will become more unmusical. (Republic 546a-d)

The city of Atlantis began well, but it gradually became corrupt. Plato modeled the city of Magnesia on his most complex tuning system, but, he considered it his second best city after ancient Athens. It was considered second best because rigorous musical laws were in place to temper the city's musical complexity. Strict and well-enforced musical laws in a complex tuning model kept citizens true to their inner being. Plato considered moderation and temperance the most important virtue. Within musical tuning systems, Equal Temperament embodies equality among all tones, as if each tone were the essence of a democratic citizen, willing and able to sacrifice for the greater good. There could be no injustice or inequality in an equally tempered system. From a mathematical perspective, Equal Temperament was not possible in Plato's time, and, as his ideal tuning system, Plato believed that it could only exist in Heaven.

The World Soul (Pythagorean Tuning)

The Demiurge was the Greek artisan-like creator of the universe. As Pythagorean Tuning's powers of three extends from the divinity of the Monad toward the evil of the Tyrant, all 12 notes of the chromatic scale would be created along the way. However, Plato's World Soul only extends

through three powers of three: $3^3 = 27$ and $1/3^3 = 1/27$. In order to stay in close proximity to the divine wisdom, the construction of the World Soul was limited to the seven tones described in Plato's Timaeus. Pythagorean Tuning[17] (2p 3q) uses powers of two and three with 'means inserted' as depicted in Figure 3. Ancient Athens was modeled on Plato's World Soul because it stayed within the constraints of Pythagorean Perfection, limited to perfect octaves, fifths, and fourths. Moderation in all things, and tempering one's demands was Plato's path to Heaven.

Ernest's musical constructs enabled me to recover the *Timaeus* mathematics within the oldest Kabbalistic Hebrew text, known as the *Sefer Yetzirah*[18]. It is not coincidental that exactly the same mathematical construction generated what Jewish tradition would call Supernal Man, a functional equivalent to Plato's World Soul. In Figure 4, we can see powers of 3 as the serpent's path -- manifesting as man's alter-ego -- slithering up and down the Tree of Knowledge of Good and Evil. In Figure 5, we see Adam's powers of 3 and Eve's powers of 2 extending their hermaphroditic soul into the cursed Earthly realm where they would require Knowledge of Good and Evil[19].

More recent Hebrew writings have been reduced to a verbal reflection of a lost mathematics. For example, the *Likutei Amarim Tanya*[20], the Bible of Chasidus for Judaism's Lubavitch sect, describes Supernal Man as "two extensions — right and left..." When I brought the *Tanya* to Ernest in the early 1970's he managed discover another important verbal reference to a different underlying mathematical construct. He described its mathematical details in a postscript to his 1976 publication *The Myth of Invariance*. From the *Tanya*, Ernest was able to diagram Isaac Luria's 72:144:288 double octave[21] describing Luria's Kabbalistic doctrine of *Hishtalshelut*, containing the divine sparks that 'descended degree by degree through myriads of levels, in the chain-like descent of the worlds, in the manner of cause and effect[22]. Although most of the mathematics had been long lost by the time Isaac Luria was born (ca. 1534), the mathematical skeleton was clear to Ernest[23], [24].

III. Just Tuning: The First Six Integers

The Church clung to Pythagorean Perfection for about a thousand years, until applied music theory evolved to its next phase during the early Renaissance, when major thirds (ratio 4:5) and minor thirds (ratio 5:6) appeared in the music of the period, within a method of harmonization called fauxbourdon. Its most well-known exponents were composers Guillaume Dufay and John Dunstable. The term Just Tuning generally refers to any intonation based on the integers of a harmonic series. Within this context, however, it refers to the musical ratios created by the first six integers (1:2:3:4:5:6). Ernest's mentors and colleagues, Siegmund Levarie and Ernst Levy, taught Ernest about the power of the *senarius* as defined by the first six integers. The *senarius* might be best described as the most powerful subset of nature's harmonic series.

Here is a passage from Ernst Levy and Siegmund Levarie's classic work on acoustics Tone: *A Study in Musical Acoustics:*

> The first six numbers are known as the senarius. There is a special formative power inherent in the senarius – a force that sets limits and thereby shapes the given elements. The pulling-apart initiated by the different forces of two and three is reconciled at six. The quality of the senarius – not the quantity now – has left an imprint on the cosmos as much as on our thinking. Crystallography operates primarily with ratios based on the senarius. Snowflakes that deviate from the norm of the hexagon are rare exceptions. The senarius becomes manifest again and again in affinity calculations of

17 Ernest G. McClain, A New Look At Plato's *Timaeus, Music & Man*, Vol. I, Number 4, 1975, p. 353.

18 Howard Schatz, *The Lost Word of God* (New York: Tone Circle Pub., 2007), p. 156-61.

19 Howard Schatz, *The Science of Religion: A Framework for Peace* (New York: Tone Circle Pub., 2013).

20 Schneer Zalman of Liadi, *Likutei-Amarim-Tanya* (New York-London:Kehot Publication Society, 1973), Chapter 52.

21 Ernest G. McClain, The Star of David as Jewish Harmonic Metaphor, *International Journal of Musicology*, 1998, Vol. 6.

22 Zalman, Op.Cit. , Chapter 6.

23 Schatz, Op.Cit., *The Science of Religion.*

24 Schatz, Op.Cit., *The Lost Word of God*, p. 175.

> chemical elements, in chromosome numbers, in plant structure, et cetera [25].

Imagine the first four integers of Figure 1 extended to include the integers five and six. The prime number five enables us to add major thirds (4:5) and minor thirds (5:6) to the perfect octaves, fifths, and fourths generated by the first four integers. Ernest describes Plato's legislation of 'wedding feasts' in a fragment of the *Republic* that describes four pentatonic marriage patterns where neither family invites more than five friends of both sexes, and the number of relatives and kinsmen from either side is similarly limited. (see figure 6a[26])

Plato then combines these four additional patterns into a seven note diatonic scale (along with its reciprocal). Taken together, they generate 11 different tones, that now includes major and minor thirds as "friends of the family" of octaves, fifths, and fourths. (See figure 6b[27])

If we transpose all 11 notes into the 360:720 octave, they can be exponed in a single row. We then get a single chromatic scale with the same tones both waxing and waning. This perfect symmetry reflects the chromatic scale of a widely used tonal calendar. The Hebrew calendar, for example, included 360 regular days plus five festival days. Egyptian mythology and astronomy used it as well in order to reconcile the cycles of the sun and moon. (See figure 6c[28])

Likutei *Amarim Tanya*: Chapter 52, also references the senarius and Just Tuning:

> Thus, it has been explained above in the name of the Tikkunim, that six Sefirot nest in Yetzirah. They the six Sefirot or middot comprise, in general, two extensions, right and left, right representing kindness, and left, severity.

In the passage above, the word Sefirot translates as counting, thus we encounter a familiar reference to the six integers of the senarius configured as the polar aspects of the soul.

Within Scripture, the six *Sefirot* generate these two trees in the Garden of Eden using smallest integers[29].

Once these smallest integers are transposed and purged of impurities on the 'four-horned altar' (see Figure 9) the sanctified remnants produce an identical 360:720 chromatic scale. These musical constructs derive from (and provide insight into) the derivation of Mesopotamian pyramids and ziggurats. Within the context of Biblical allegory, these constructs were called tents and holy mountains.

As a result of this musical construct, Genesis allegory takes shape as follows: Jacob was renamed Israel ('He who struggles with God') after he wrestled with an angel. The 12 sons of Israel produced 11 tonal sons each of whom was to be given a share in the land of Israel according to their birthright. Levi forfeited the 12th share as leader of the priesthood in order to compensate for the sins of the beast. As first-born, Reuben's double share was denied because he committed adultery with his father's wife. In Figure 8b, take note of Reuben and Levi falling outside of the purity of the altar construction. Jacob gave Reuben's double share to Joseph sons Ephraim and Manasseh, and thus this important Biblical allegory is structured in all its subtleties by the acoustical Four-Horned altar construct.

When the Greek gods divided the world among themselves, Poseidon received Atlantis. In this passage, Ernest describes the virtues of Poseidon and his five pairs of twin sons, shadowed by the seed of gradually degenerating births:

> ...his first born twins, Atlas and Gadirus, are now properly represented by A and G [the harmonic and arithmetic means within the octave]...[30] In its early years Atlantis was a veritable paradise; its citizens were men of great virtue while the god's strain in them was still vigorous, but in later generations the human temper (i.e. the products of 5) began to

25 Levarie, Sigmund and Ernst Levy, Tone: *A Study in Musical Acoustics* (Kent, Ohio: Kent State University Press, 1968) p. 30.

26 Ernest G. McClain, Musical Marriages in Plato's Republic, *Journal of Music Theory,* Fall 1974, Vol. 18.2.

27 McClain, *The Pythagorean Plato, p.* 25.

28 Ibid., p. 25.

29 Schatz, *The Lost Word of God*, p. 234.

30 Ernest G. McClain, *The Pythagorean Plato* (New York: Nicolas-Hays, 1978), p. 82.

predominate (over the factors of 3)[31].

Plato introduced Just Tuning within legislated constraints, but once these constraints were abandoned during the degeneration of Atlantis, Plato documented its demise in his musical model as the "worst" possible city, and held it in contrast to Ancient Athens, a city "limited to essentials." In Book II of Plato's *Republic*, Socrates described Atlantis as 'luxurious … feverish … and gorged with a bulky mass of things...'. Atlantis was no longer governed by Necessity but by unbridled possibility[32]. Successive generations of guardians begin to neglect the populace by having less consideration than is required, first, for music, and, second, for gymnastic; and from there your youth become more unmusical. Ernest's faithful reconstruction of Plato's tunings reflect its citizenry's chaotic feverishness and corrupt government. In *Critias*, we read:

> Atlantis became mighty, and sent its fleet across the ocean and into the Mediterranean Ocean through the strait of Gibraltar. And a little later Atlantis dominated the whole basin, but for the area where we find ancient Greece. Wealth amassed in Atlantis, where they took to building palaces that were paneled with ivory inlaid with gold and silver. Luxury living spread. But so long as the people obeyed their wise kings and the good laws, things went well. But the might and splendor of Atlantis bred corruption, alas. Then laws were not as much esteemed and lived up to as earlier. Envy and injustice won an entrance. Bad circles of influences started and didn't stop... Now Zeus and his brother Poseidon intervened: The unjust inhabitants had to be punished by the natural catastrophe Solon retells from, and others. A horrible earthquake, volcano eruption and a tidal wave left ruin and death behind. Atlantis sank into the ocean and was gone. - Critias 114-115

IV. Archytas Tuning: The First Seven Integers

This tuning system included the prime number seven which is not part of the senarius nor part of sexigesimal Base-60 numeral system that was the earliest numeral system in use. The Bible also utilized the prime number seven but, since it was not a Base-60 number, gave it special treatment since we know that God did not create objects or beings on the Sabbath. Plato's friend Archytas divided a string into seven equal sections, and fully utilized seven-smooth numbers, which included prime numbers two, three, and five, as well as the prime number seven, as one of the building blocks comprising the deep structure of the city Magnesia. The upper limit of this tuning system was 7! = 7 x 6 x 5 x 4 x 3 x 2 x 1 = 5040 which became the population limit in Magnesia of 5040 'landholders' (Figure 9). We should take note that the Republic's Marriage Allegory describing Just Tuning had a tonal index of 6! = 720 (see Figure 6c).

Since this tuning system is more complex due to an additional prime number tone generator, it required a higher numerosity octave to fit all the 'new arrivals.' These 11 tones would be seen again as Poseidon and his five pairs of twin sons that founded ancient Atlantis, and later as the boundary markers of the city of Magnesia. The number of landholders within Magnesia's boundary markers was strictly held to 5040. The tones generated from: $2^p\ 3^q\ 5^r\ 7^s <= 5040$ included members of the highest property class from powers of three; members of the next property class from powers of five; and members from the third property class from powers of 7[33].

The next 19 new guardians would be of even lesser birth, and Magnesian society would suffer if not for the strict musical laws that kept it functioning. These new guardians were generated symetrically around their parent guardian, derived from the musical interval on either side of seven, namely the septimal third (6:7) and the septimal tone (7:8). The parent generation of 18 guardians plus this new generation of 19 guardians made for a total of 37 guardians. Strict laws were listed in his dialogue Laws that regulated all aspects of music in Magnesian society. Ernest lists twelve such regulatory laws. Here are just a few of them:

31 Ibid., p. 95.

32 Necessity was the goddess who personified Destiny within Greek mythology. Only Necessity could control the decisions of her daughters, the Three Fates: Lachesis, Clotho, and Atropos.

33 Ernest G. McClain, Op.Cit., *The Pythagorean Plato*, p. 102-107.

•The three kinds of choruses proper for boys, young men, and old men

•The music for drinking parties

•A catalogue of musical forms: hymns, laments, paeans, dithyrambs, nomes

•Prescribing musical style, due to the low level of public taste

•The appointment of music supervisors: Supervisors of choral contests must be over 40, and supervisors of instrumental contests over 30, and choral members who fail to attend the election of a Chief Organizer are liable to fines.

These laws were so extensive and strict, that Plato was accused of being a fascist. Magnesia was a widely diverse population, and the city's vibrational essence needed to be strictly regulated for its inhabitants to live amidst this great diversity in an ordered and peaceful way. Since Magnesia was more populated and diverse than Ancient Athens, it required stronger laws to protect its citizenry. It compared well to the diversity and burgeoning needs of Atlantis, which grew at a rate that far exceeded what its laws could control, thus anarchy and chaos ensued. Atlantis suffered natural catastrophe as punishment, at the hands of Zeus and Poseidon. Atlantis was Plato's worst city, modeled to contrast with Athens and Magnesia as his best and second best cities. But, not even the virtues of Ancient Athens, Plato's best city, could compare to Socrates' description of an ideal city limited to essentials. His Utopian celestial city was Callipolis (Greek: beautiful city).

Callipolis was most unified and moderately governed because the Three Fates would temper the Siren's tones generated from its central fountain.

V. Equal Temperament: The Rejection of Integers

Since each harmonic produces entirely new divisions of a vibrating string, it appears that the evolution of man's musical vocabulary has incorporated the first few prime number tones into the music of the time in the exact order of their appearance within the harmonic series[34]. We might think of this process as tuning in to the chord of nature. Perhaps the evolution of human hearing plays a role as we perceive and absorb the hidden structure of sound one harmonic at a time.

As we review the history of music, Organum was generated from the first four harmonic integers: 1,2,3 and 4, describing music that was prevalent during the Middle Ages. The next evolutionary step added the next sequential harmonic integer -- the number 5 – which was used in ratios 5:4 (major thirds) and 6:5 (minor thirds), and incorporated into the musical vocabulary of the Renaissance. According to this theory, the number seven would be the next harmonic to be perceived and incorporated. During the Enlightenment, Johann Sebastian Bach introduced the interval of a seventh in his music, but he was not happy with Archytas Tuning, since the natural interval created by seven divisions of a string was not aesthetically pleasing. The number seven generated ratios of 6:7 and 7:8, creating the interval of a septimal third and a septimal (supermajor) second, respectively, which locates a B♭ at 231 logarithmic cents. In an Equally Tempered scale, that note would be exactly 200 cents. Thus, it is called a supermajor second because it is larger than two 100 cent semi-tones.

Bach's work *The Well Tempered Clavier* would cover all 12 major and 12 minor keys within Well Temperament, and Well Temperament would become standardized as Equal Temperament about 150 years later, once music theorist Simon Stevin's 17th century manuscript *Van De Spiegheling der singconst* was finally published. Bach was not the first composer to attempt this, but his influence had a major impact on all composed music ever since. We might say that Bach finally realized Plato's idealized vision of Equal Temperament.

Today, we have learned to measure Equal Temperament on a logarithmic scale. Each half-step consists of 100 cents for each of the 12 chromatic notes = 1200 logarithmic cents. Today, the art of piano tuning is the art of mistuning a piano in order to split the Diabolus in Musica into 12 equal slices, hiding it from human perception. Simon Stevin's 17th century text is said to be the first, but somewhat flawed attempt to split the $\sqrt{2}$ evenly among the 12 chromatic tones, so that each semi-tone received a tiny portion ($12_{\sqrt{2}}$ or $_2$

34 Howard Schatz, *The Chord of Nature and the Evolution of Music Theory*, Op.Cit. 423- 436.

1/12). It's as if we were dissecting the Devil into 12 impotent little pieces. Metaphorically speaking, we might say that Bach, a religious composer, attempted to destroy the Devil with his music.

If we were to continue our analysis of how prime numbers within the harmonic series influenced history's musical vocabulary, our next consideration might be the chord extensions of Claude Debussy's tone paintings, when he added the next prime number harmonics, the 11th and 13th, albeit within equal temperament. The human ear has only evolved to hear about 16 harmonics. We might also want to take into account the words of Jean-Phillipe Rameau, a contemporary of Bach. Rameau warned that the acceptance of Equal Temperament would one day result in a musical practice that did not even recognize distinct musical modes. Rameau's prophetic words became reality on the day that Arnold Schoenberg proposed 12-tone music. Schoenberg exaggerated the concept of Equal Temperament by completely ignoring the harmonic series as a formative musical principle. Schoenberg's 12-tone system ignored all 12 harmonic modes by melodically structuring his system according to the arbitrary rule that all 12 tones had to be played before any tone could be repeated. This new chromaticism created a new, unnatural type of musical scale. From a historical perspective, 12-tone music might seem to be a logical extension of the late Romantic chromatic tradition of Richard Wagner and Gustav Mahler. But, neither Mahler nor Wagner ignored harmony, they just took much longer for their harmonies to resolve, due to their extended melodic chromaticism. By definition, 12-tone music rejected the harmonic series outright, and thus it rejected nature's guidelines.

After Schoenberg, the next generation of musicians, not unlike Plato's unmusical generations, introduced aleatoric music, in which random chance become the main determinant of which notes would be played. John Cage wrote a piece called *4 minutes and 33 seconds*, in which there was no sound except random sounds made by the audience. Cage's approach was said to draw inspiration from the randomness of throwing I Ching yarrow sticks. This turned the mirror of performance on the sounds made by the audience. This might be an interesting philosophical idea, but it is not music from an aesthetic perspective, nor from the perspective of Greek ethos. As Plato's writings suggest, randomness was analogous to political anarchy, and only those composers who remained harmonic in their approach would be worthy of a chapter in music history. Despite the best efforts of Arnold Schoenberg to formalize the end of harmonic music, nature has a way of winning out.

The common harmonic practice period enabled composers to transpose from one key to another with impunity, unquestionably producing the world's most beautiful music, and its most renowned composers. The beauty of this music created over the past 300 years presents a strong case to prove Plato's theory that Equal Temperament defines celestial harmony. Greek moderation and temperance for the greater good was the most compelling virtue within Plato's model cities as well as his philosophy. Ernest argued that Plato, Scripture, and Hinduism's Embodied Vision idealized Equal Temperament as the celestial Music of the Spheres.

Let's take a closer look at Plato's ideas about celestial music, and see what modern day string theory has to say about it. For example, do the exact sciences of antiquity that spelled out the deep structure of Plato's philosophy need to be brought up-to-date in any way? And, what does modern string theory say, if anything, about the state of 21st century music, and where it is headed?

VI. The Celestial Harmonies

In his work, *De Architectura*, the first Roman architect, Vitruvius expressed his belief that the ideal proportions of the human body reflected the cosmography of the universe. Leonardo's well-known Vitruvian Man quotes this text while attempting to visually demonstrate this Vitruvian principle. Da Vinci attempts to reconcile the symbolism of a circle's spirituality with a square's materiality. With this in mind, there has been an ancient effort of geometers to square the circle by using only a finite number of steps with compass and straightedge. Kabbalah's most sacred riddle,

the circle and the square within was symbolically related to Leonardo's depiction, but the actual sacred riddle demonstrates knowledge of the Pythagorean Theorem some 1200 years before the birth of Pythagoras, engraved on a cuneiform tablet called YBC 7289 (a Yale catalogue number) as discussed in Ernest's first book.

Within the Kabbalistic tradition, the circle and the square within defines the Sabbath, as well as the ancient priestly practice of seven circuits around the sacred cube. It relates geometry and human proportion. Within the Book of Exodus, it was known to Bezalel, the chief architect of the Tabernacle in the desert. According to an important Talmudic teaching, Bezalel knew how to permute the letters with which Heaven and Earth were created[35]. A three-term continuous proportion can be established between the circle, the square, and the diagonal of a square, as the mean term, that also functions as the diameter of the circle. All this has been carved into stone using sexagesimal numbers on the cuneiform tablet that Ernest brings to our attention. Thus, the Pythagorean Theorem may have first been used to solve for the length of the diameter/diagonal, the *Axis Mundi* of man's being, suggesting man's difficulties reconciling the Pythagorean comma that exists within the macrocosm of the Heavens and is reflected in the microcosm of man's spiritual and material being.

The irrational mean between Heaven and Earth is the irrational square root of two, correct to five places. We derive √2 as the irrational hypotenuse of an isosceles right triangle when each of the congruent sides = one. The term 'irrational,' when used in this sense, does not mean that it was without reason, but that it was without ratio. Pythagoreans espoused that the world was built on integer ratios, and, their reported dismay can be attributed to what this mathematics suggests about the relationship between Heaven and Earth. An irrational number defining the *Axis Mundi* implied imperfection in the manifest world. This particular three term proportion was the model for an observer reconciling materiality and spirituality within the *Axis Mundi* of one's own being, and it would have a profound impact on religious and philosophical thinking. Morality and repentance from sin were the only known solutions to the Evil that was believed to be embodied by √2. Plato's Tyrant's allegory and the Bible's Satan present the clearest evidence of how the Quadrivium's arithmetic, music, geometry, and astronomy shaped religious and philosophical thought. The musical realization that Equal Temperament would powerfully mitigate the original theological schema for Evil, is no longer appreciated by a world that has all but lost any knowledge of the religious and philosophical implications embodied by √2.

For Plato, the dichotomy between matter and spirit implied that the celestial harmonies of Equal Temperament were only possible in Heaven. However, Plato did his best to strictly regulate the standards for music here on Earth in order to have a positive impact on man's soul, man's character, and the body politic, as already discussed. Once the early Church fathers learned about Pythagorean Perfection, it would become the basis of Church music for over 1000 years. Music was not composed for pleasure, but it did have a ceremonial function that was intended to enhance the parishioner's spiritual experience. Plato's strict musical regulations were introduced in Laws because the Celestial harmonies of Equal Temperament were not practically possible. Equal Temperament was Plato's theoretical and idealized construct. But, Plato may not have been the only one to envision the celestial harmonies of Equal Temperament.

Equal Temperament also became the focus of Ernest's work with Antonio de Nicolas as the key to the language of 'Embodied Vision' and Enlightenment within de Nicolas' schematization of Hindu thought as four-dimensional man. (see table 2)

Ernest relates the Hindu ideal described by de Nicolas' Embodied Vision to Plato's Vision of the celestial harmonies as Equal Temperament. Plato formalized his models of ancient cities based upon different tuning systems, with each providing an Image of the city's inhabitants, its laws, and its form of government. In his work with de Nicolas, Ernest described how each of these Images change

35 Aryeh Kaplan, *Sefer Yetzirah* (York Beach, ME: Samuel Weiser, 1990) 26.

their musical and societal context, and must be sacrificed' to make way for the next musical and societal context. Plato's four model cities and tunings include the following Images. (See table 3)

Perhaps we should not blindly accept the premise of Original Sin, and the apparent imperfection of the manifest universe, without a more precise scientific perspective on the nature of that imperfection. After 300 years of a musical practice based on Equal Temperament, one might argue that the celestial music should have taken hold by now, embraced civilization, and brought peace to the world. Clearly, that is not the case. If we have been enveloped by celestial harmonies for 300 years while the world slips deeper into chaos, then we are compelled to take a closer look.

The Three Fates tempered the sounds of the Siren's song within Plato's Republic, because Plato believed that Equal Temperament compensated for the perceived imperfections that manifested within the physical realm. Plato described whorls of sound and color that spun on the Spindle of the goddess Necessity. Necessity's three daughters were the Three Fates who tempered the sounds of the Sirens by putting out their hand to interrupt and slightly temper their spin (frequency) in order to produce the celestial harmonies. Plato therefore viewed moderation and temperance as the most important divine virtues, but for Plato the Three Fates might be considered the gatekeepers of Heaven, because they purge the imperfection of number as one enters the Gates of Heaven.

For Pythagoras, integer ratios were the final arbiter of absolute truth that reflects the deep structure and internal order of the universe. For Plato, numbers don't lie to us in Heaven, but one's perceptions could lie to us on Earth. His theory of Forms would suggest uninstantiated ghostlike forms of the sun and the moon that are spiritual reflections of the physical plane. But, these idealized Forms would remain perfectly in-sync with one another. Within the transcendent heavenly realm of number and sound, powers of two, powers of three, and powers of five, are all musically in-sync. In this transcendent realm of uninstantiated Form, there would be twelve 30 day lunar months that would fit precisely into a 360 day solar year. In a perfect world, the sun would travel around the earth at a rate of 1° per day, without the five extra festival days. There would be exactly two six-hour lunar tides in each 12 hour day, and the length of each day wouldn't fluctuate. There would also be two 6-hour tides in a 12 hour night. But, since these are all approximate amounts, and nothing in the manifest universe seems to fit precisely, Plato reasoned that Equal Temperament's perfection was only possible in Heaven, where number and vibration comprising the inner vibrational essence was tempered by the Three Fates. This perfect ethereal substance then instantiates into the imperfections of a physically manifest form.

Within the manifest physical universe things are perceived as imperfect and out-of-sync; the sun, moon and planets are out-of-sync within their perceived orbits around the earth[36]; the duple, triple, quintuple progressions are all musically out of sync; just as the inhabitants of Plato's different cities, their laws, and their governments, were out-of-sync within themselves and with one another (with the exception of Callipolis). But, it was widely assumed that monotheism's God or polytheism's gods could not have possibly created an imperfect universe. In Scripture, Original Sin became the root cause of all that is wrong with the world. This burden of guilt still weighs heavily on man's conscience in day-to-day life. The celestial harmonies of Equal Temperament were thought to be a way to heal mankind, helping to free mankind from sin, and empowering him to return to the purity of Eden.

Let's return for a moment to Greek mythology, and compare the beauty of equally tempered music composed over the last 300 years to the song of the Sirens in Homer's Odyssey. Plato's Republic suggests that the music of the Sirens was seductive, but imperfect, and it could only be made perfect by the Three Fates (Clotho: spinner, Lachesis: allotter, and Atropos: unturnable) who tempered the spinning bowls strung along the "Spindle of Necessity[37].

36 The heliocentric perspective of Copernicus put the Sun at the center of known universe, however, what is being referenced here is the earlier geocentric astronomy of Ptolemy, which put the Earth at the center.

37 Musically, the 8th bowl was an octave duplication of

Over the last 300 years, civilization has been immersed in the celestial harmonies with little apparent affect. One possible alternative is to wait longer for the celestial music to take hold. A second alternative suggests that Plato, Scripture, and Hinduism may have gotten it exactly backwards. What if civilization is being lured onto the allegorical rocks, not by the Siren's song, but by the Fate's tempered song? Could Ernest possibly be reporting a Platonic mis-calculation? Could it be that Equal Temperament fools the ear but not the soul? To play Devil's Advocate, could Equal Temperament be a kind of Trojan Horse that eats away at our sense of inner peace and well-being? Could it possibly be that Equal Temperament itself contributes to the root-cause of the world's current state of chaos, as the cause of further degeneration within each successive generation? A third possibility exists that would render the Greek principle of ethos completely or partially wrong, implying that equally tempered music would have little or no effect on the human condition whatsoever. Within the context of modern science, there is a fourth, most likely alternative, that forms the basis of this paper's closing thoughts.

After seeing the highly questionable results from our 300 year experiment with celestial Tuning, we might wonder whether Plato might have considered outlawing all equally tempered music. If we continued this line of reasoning, perhaps the music of the future should not contain any melody whatsoever. Rameau explained that any melody implies shifting harmonic roots, which then implies that tuning adjustments must be made to the tree's melodic branches. In other words, an E that occurs in a melody sounding as the 10th harmonic of a C harmonic series, has a different tuning than the E that sounds as the ninth harmonic of a D harmonic series. Further, Rameau suggests that nature has no template for melody. The history of Western music has standardized on an arbitrary 9:8 harmonic scaler quantity chosen by the Greeks to mark off steps for a melodic scale. Could Plato's search for perfection paradoxically be bastardizing nature's integer ratios? One observation suggests that both equal temperament and the 'chord of nature' are preferable to the various anarchic musical developments within 20th century music, such as 12-tone music, which rejects any embrace of harmonic principles.

We must keep in mind that Equal temperament embraces harmonic principles, even if it is only an approximation of nature's harmony. From an emotional perspective, a world without melody, and without the diverse harmonies that become possible within Equal Temperament, would seem rather bleak. When the chorus enters in the third movement of Maurice Ravel's masterpiece *Daphnis and Chloe*, the listener feels like they are entering the Gates of Paradise itself.

Consider that Plato might have characterized the impossible diversity and unmanageable chaos of our current world as reminiscent of the lawless anarchy gripping Atlantis before it sank into the sea. The words moderation and compromise are no longer in the lexicon of American politics. If today's lawmakers were philosopher-kings, we would be imposing rigorous laws to mitigate the bombardment of our senses by the current technology explosion. In addition to musical laws, Plato would stress gymnastics and a rigorous liberal arts education that would teach society about the power of harmony to heal or corrupt our soul and our civilization. If we consider the possibility that there is some truth to the Greek principles of ethos, then the modern age of science may or may not corroborate the impact of tone on the human physiology and psyche. Once Ernest demonstrated that acoustics provides the deep structure of the ancient wisdom writings, the framework for a new Renaissance has emerged that promises to transform ancient metaphysics into modern physics.

Conclusion

Ernest McClain has spent his life deciphering the ancient wisdom texts, however he has never attempted to espouse or dispense this so-called ancient wisdom. He never defended the veracity of the Greek principle of ethos, nor can one call him a mystic, a religious believer, or even a philosopher. Ernest was a scholar and explorer of ancient worlds who provided the missing mathematical details

the tonic note, thus sounding 7 different tones.

of the The Harmony of the Spheres that Joseph Campbell took notice of but could never explain. Ernest's writings shed light on the deep structure of myth and allegory within a broad syncretistic matrix of ancient cultures and literature. If the Renaissance defined history's cultural bridge between the Middle Ages and modern history, Ernest rediscovered a methodology that is able to reveal the lost meaning of ancient literature, begging the modern world for a new Renaissance.

I am convinced that Ernest's musical approach is the only correct way to parse the literature of the earliest civilizations. Long hidden layers of meaning become accessible because Ernest has applied his knowledge of Plato and Pythagoras to other cultures. He has taken us on a survey of cuneiform tablets in Sumer and Babylon, hieroglyphic stelae and artifacts in Egypt, the earliest Vedic text that ultimately made its way to India, the epics of Homer, and the Scriptures of Israel, Rome, and Mecca. The deepest meaning of these ancient and sacred texts have finally been brought into focus, because Ernest inherited and mastered the legacy of the mathematical/musical suite of tools initially rediscovered by Albert von Thimus. Once mastered, Ernest viewed earlier history through the erudite lens of Platonic philosophy and Pythagorean number. His contribution has made it possible to access civilization's most precious literary lights in what is, unfortunately, a very dark world.

I would like to thank Ernest for bringing meaning to my life, and for the legacy of his work. Modern culture suffers from a case of tunnel vision and ignorance. Ernest's life challenges us all to do better. I suspect that his methods reconstruct knowledge lost in the Library of Alexandria. After seeing the ancient world through Ernest's eyes, it is my hope that musicologists, philosophers, theologians, anthropologists, archaeologists, etc,... are able to make further progress on the path that our beloved friend and mentor has set before us.

Appendix: A McClain Primer

Arithmetic is an old term for number theory. Number theory studies integers and their properties, such as its divisibility. A number's divisibility determines whether it is a prime or composite number, and it is from the study of prime numbers that the ancients discovered how arithmetic integrates with music. They discovered that it takes a prime number to generate a new (as yet unheard) sound. The nature of this special relationship has been captured within modern number theory by the terms: five-smooth and seven-smooth, terms which were coined by an American computer theorist and mathematician named Leonard Adelman. In number theory, a smooth number would simply be an integer that factors completely into small prime numbers. However, five-smooth numbers are also called regular numbers, because they evenly divide into 60 or into any power of 60 with prime factors of 2, 3 and 5. Since seven-smooth numbers do not divide into 60, they are not considered regular numbers. Therefore, seven-smooth numbers were not terribly useful in sexagesimal, Base-60 arithmetic, which was the earliest positional method of numeric notation used in Sumer and Babylon dating back as far as the 4th millennium BCE. The glyphs used to symbolize the earliest numbers were not the Hindu-Arabic positional numerals that we use today, nor were they using symbols resembling non-positional Roman numerals. The earliest civilizations in Mesopotamia used positional cuneiform glyphs, while Egypt used positional hieroglyphs.

Today, the world has standardized on a decimal Base-10 number system, also a positional notation system like Base-60, but instead of 59 unique digits with a positional space holder, Base-10 utilizes nine digits and zero. Within these first nine digits the prime numbers are two, three, five and seven. The fact that seven is not a regular number explains why the number seven was given such special treatment within Scripture's Creation allegory, within Plato's city of Magnesia, and even within the history of music composition and performance. Perhaps we switched from Base-60 to Base-10 because we had 10 fingers and 10 toes, thus making our count of small herds of sheep and goats much easier. What

is more likely, however, is that Base-10 developed from the way Mesopotamians organized Base-60 sets into helpful Base-10 subsets. From this perspective, the Mesopotamian system might be considered mixed-base, and Base-10 seems to have evolved from there. Ancient Egypt was probably the first to standardize on Base-10.

In order to understand different numerical bases, perhaps its best to start with Base-2, which is also called a binary system. Base-2 is best for modeling modern-day computers. It consists of only 0's and 1's, and since an electrical switch is either on (=1) or off (=0), a computer can be modeled as an organized bunch of on-off switches. Base-60 became the first important historical system because it modeled nature. Regular numbers were used to integrate astronomy and time, which helps explain why the clock is our oldest Base-60 artifact.

As a metric of time, there are 60 seconds in a minute, and 60 minutes in an hour. In geometry and astronomy, there are 360 degrees in a circle, and 60 seconds in a minute of latitude or longitude, and 60 minutes in a degree of latitude and longitude. The fact that there are twelve factors of 60: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30 & 60, also made Base-60 very flexible to apply to different contexts. For example, there are approximately 6 hour lunar tides, 12 hour days, and 12 hour nights. One cycle of day and night was never described as 24 hours because 24 is not a factor, multiple, or power of 60, so the term 12 double-hours was used to indicate a complete cycle. There are also 30 day lunar months, and 360 day solar years (360 is a multiple of 60). Base-60 was useful to Pythagoreans and Kabbalists who sought to define both the spiritual and material nature of reality through number. It accomplished this by integrating arithmetic, music, geometry and astronomy into a holistic macrocosm of the Heavens as a mirror reflection of the microcosm of the soul. For example, all objects and beings were thought to be created from the natural sounds generated by the three prime number factors of 60: two, three, and five. Within Scripture, those sounds were thought to have emanated from the Voice of God. The use of Base 60 also explains why Plato's upper numerical limit, his so-called sovereign number, was 60^4 = 12,960,000 (a power of 60), as well as why God's holiest name YHVH is equivalent to 60^5 = 777,600,000 (a power of 60).

Composite numbers generated an additional harmonic with each prime number factor. Prime numbers higher than 7, like 11, 13, etc., were not thought to be part of the deep structure of reality because they were not so readily apparent in nature. The notion that Creation was based on regular numbers two, three, and five explains why God rested on the seventh Day. The number seven did not generate objects and beings within Creation, but it was given transcendent status as the Sabbath. The number seven also played a significant role in Plato's musical allegories. Even though seven was not a prime number factor of 60, it was among the prime numbers less than 10, and therefore, in any attempt to switch from Base-60 to Base-10, it's unique role had to be defined and reconciled with primes two, three and five.

Within the Jewish Kabbalistic tradition, the Ten Utterances of God were the ten sequential integer vibrations of God's Voice that created the world in six Days. Within Hebrew Scripture's 1 Samuel 16:23, the ten strings on David's lyre soothed King Saul's madness by channeling what we might imagine to be divine celestial harmonies. Lesser mortals might begin to replicate the notes on David's lyre by simply filling up glasses with specified amounts of water to create a very crude musical instrument, and then plunking out a tune with a piece of silverware. The two questions we might want to answer are: how much water do we put into each of the different glasses in order to recreate the tuning of David's lyre. And, if we could imitate the sound of his lyre, would that give us access to the patriarch's celestial harmonies?

My first lesson with Ernest was not spent filling up water glasses, but by learning about the sound of vibrating strings on an ancient single stringed instrument known as a monochord. The 'how much water' question requires knowledge of the same integer comparisons as the 'how long a string' question.

A monochord (See figure 11) might be constructed from a single 36 inch string fastened between ends of a block of wood, with a yardstick placed directly beneath the string so we could

visually divide the string length in halves, thirds, quarters, etc.. To divide the string in half we would stop our 36 inch string with a movable bridge at the 18 inch mark. This is comparable to a violinist dividing one of the violin's strings in half by stopping it with their finger. To divide the string into thirds we would stop the string every 12 inches. However, since there is only one movable bridge, we would move it to 12 inches, leaving a remaining section of 24 inches. If we then compare these different lengths of string using fractions we would have 1/3 the length of string versus a 2/3 length of string. As a musician, we would write these two fractions as the integer ratios 1:3 or 3:1 and 2:3 or 3:2. We would then follow a similar procedure if we divided the string length into quarters, fifths, sixths, etc.

A musical ratio is a comparison of two numbers. If we are comparing three or more number-sounds we call it a proportion. In the fundamental vibration defined by the integer ratio 1:2, the wavelength is twice the string length. The first number (=1) would be the numerator of the fraction ½, corresponding to the sound made by plucking or striking the entire length of string. The second number (denominator of the fraction ½) would correspond to divisions of the string length, and the sound made by plucking half of the string's length. The listener might compare these two notes by playing them on the white notes of a piano. We would see that these two tones were eight notes away from one another on the piano. The musical interval produced by these first two tones is called a perfect octave.

Let's move on to describing the proportion 1:2:3, which compares three number-sounds as a sequence or progression beginning with the unison or undivided string, which we then compare to the sound of half the string length. The third number would divide the string into thirds (denominator 3) producing the musical interval of a perfect fifth. Don't get thrown off by the fact that the number 3 generates the musical interval of a perfect fifth. If we measured the distance from the previous notes we played, we would see that the proportion 1:2:3 stacks the musical interval of a fifth on top of the previous octave, both preceded by the tonic note.

The prime numbers 2, 3, 5, 7, 11, 13, 17, 19,... each produce entirely new divisions of a vibrating string, but Western music tends to ignore the higher primes, and the harmonics those higher primes generate. Human hearing appears to be limited to 16 harmonics, so the last audible prime number would be 13. Although it might sound strange to a Western ear, Indian Hindustani music makes more use of the smaller musical intervals created by higher prime numbers.

Prime number integers have been slowly incorporated into our musical vocabulary one harmonic at a time. Adding harmonics one at a time begins with a musical "unison"; the number 2 defines an octave with ratio 1:2 (or 2:1); the number 3 is associated with a musical fifth of ratio 2:3, whereas 1:3 really defines the interval of a twelfth because a 1:2 octave can be thought of as a hidden component of the interval within a ratio of 1:3. The proportion 1:2:3 therefore stacks the intervals of a perfect octave + perfect fifth above the fundamental root sound, generated by the number 1 -- the open string. Similarly, when we count to four we are stacking up all the component intervals along the way: an octave (1:2) + a fifth (2:3) + a fourth (3:4), and so on... This principle of a larger number containing the smaller previous number defines what the history of music theory calls a super-particular or epimoric ratio, which can be written in the form we see here, with the larger number in the numerator: $n+1/n = 1+ 1/n$.

What follows (See table 4) is a table of super-particular ratios for numbers ranging from 1 to 10: If we look at the inner workings of a piano, we see that longer strings sound lower than shorter strings. Pitch (highness or lowness of sound) is based on its frequency of vibration. Whenever we divide the string-length by ½ we double its frequency. Whenever we divide the string length by 1/3 we multiply the frequency by its reciprocal 3/1 , etc.. Since frequency is measured in cycles per second (cps), if a cellist plays a concert A vibrating at 440 cps, and then plays a 1:2:3 progression, the next note would be another A at twice the frequency (440cps x 2/1 = 880cps) and half the string-length (36 inches x ½ = 18 inches). The next note would sound an E a perfect fifth higher in frequency than

the last note played (880 * 3/2 = 1320cps), while the length of the string would be shorter by 2/3. Plato understood the reciprocity of frequency and wavelength in terms of the reciprocal harmonic and arithmetic ratios produced by a single vibrating string. This concept of reciprocity is important to the ancient writings because it models the dichotomy between the spiritual and material world, which then became the prototype for Heaven and Hell within Scripture.

Perhaps it is best to think of these integer ratio intervals like a child's building blocks that stack from the bottom up. The bottom of this stack always begins with the number 1. As we continue to count, the number 4 is a compound number with prime factors of 2 x 2. Powers of 2 simply stack octaves on top of one another. For example, 2^2 generates two successive octaves (1:2:4), while 2^3 = 8 would stack three octaves (1:2:4:8). The number 5 is associated with both major and minor thirds (ratio 4:5 and 5:6). The number 7 generates septimal minor third (ratio 6:7) and septimal major second (7:8). As we just pointed out, the number 8 = 2^3 converts to three successive octaves; the number 9 is a composite number (3^2= 9) that would stack two perfect fifths one on top of the other (C to G then G to D), musically called a secondary dominant. Since the D appears in the next octave, octave reduction would require us to divide the string length by 2, which allows us to transpose the D into the same lower octave as the initial C. Multiplying or dividing any tone number by 2 enables us to control the sounding octave.

Plato and Scripture focused on the first 10 integers to shape their respective metaphysics. For Plato, and modern physicists, the essence of matter was defined by the 'chord of nature, now called the harmonic series. Dividing a string into measurable segments enabled Pythagoras to explore the structure of this natural harmonic phenomenon using integers and a measuring stick to accurately divide the string. Of course, Pythagoras had no idea that the harmonic series would one day become established as a naturally occurring empirical phenomenon. The mathematics describing a harmonic series can be traced back at least as far as Babylonian cuneiform tablets, almost 1500 years before Pythagoras[38]. With the discovery of a harmonic series as a natural scientific phenomenon, metaphysics was put on empirical ground for the first time.

There is a sticking point for even the most respected of academics attempting to read Ernest's books. When we measure string length against our monochord's yardstick, we are visually measuring quantities in a linear fashion. If musical sounds were perceived in a linear way, then each octave could be thought of as a whole apple pie. We might cut 8 equal slices of pie to represent the 8 notes contained within an octave (there are actually 7 different tones in an octave, since the eighth tone repeats the first). If we visually divide our apple pie into thirds, then each slice would be 1/3 of the whole pie (ratio 1:3). Unfortunately, we don't hear in the same way that we see. We see linearly, but we hear logarithmically[39]. For example, in the figure below, each ascending octave sounds exactly the same to our ear, but in order to perceive a series of rising octaves, the string length must get exponentially smaller with each octave. All octaves sound the same, whether rising or falling in pitch, but the string lengths must shorten exponentially in order to produce each new rising octave:

C ..... C 1 .....C2 ..... C3 ... C4 frequency
1/1 .....1/2 .....1/4 .....1/8....1/16 string length

The disparity between linear visualization and logarithmic hearing also explains why the interval of a third does not derive from 1/3 of a string's length. Similarly, the musical interval of a fifth is not 1/5 of the string's length. In fact, what nature presents to us is the unfortunate coincidence that the musical interval of a fifth is created by

38 Jöran Friberg (2007) Amazing Traces of a Babylonian Origin in Greek Mathematics, *World Scientific*, Singapore 2007.

39 The logarithm of a number is the exponent to which another fixed value, the base, must be raised to produce that number. For example, the logarithm of 1000 to base 10 is 3, because 10 to the 3rd power = 1000 or 10^3= 1000. Logarithmic exponents enable us to stack musical intervals on top of one another. For example, if we select the interval of a perfect fifth generated from a numerical base of 3, then we can raise that base to some exponent, stacking each fifth one on top of the other as we go along, for example: 3^1 = one musical fifth, 3^2 = two musical fifths, 3^3 = three fifths, etc. In other words: 31 generates the interval from C to G; 3^2 = C to G plus G to D; 33 = C to G plus G to D plus D to A.

a numerical ratio of 1:3. Conversely, the musical interval of a third is created by a numerical 1:5 linear ratio. In my experience, this basic acoustical fact confounds more people than one might imagine, as they struggle to grasp how number converts to sound. The fact that a 1:7 ratio generates an interval that we tend to associate with a tempered minor seventh on the piano, adds to further confusion. The number three creates a musical fifth, while the number five creates a musical third; and, the number seven creates a musical seventh -- yikes! This is all too confusing for most people to process. During the transition from number to music, it is important to keep in mind when you are in the realm of number and when you are in the realm of music. Please disregard the coincidental transitions of 3 -> 5, 5 -> 3, and 7 -> 7, or you might soon confuse up and down.

Now that we have a feel for the basics of Ernest's Pythagorean vocabulary, we are in a better position to study how these concepts shaped the writings of Plato, as well as the earlier writings of Scripture, the Rig Veda, and a host of other ancient wisdom writings.

City:	Callipolis	Athens	Atlantis	Magnesia
Character:	Celestial (Ideal)	Moderate (best)	Luxurious (worst)	Practicable (2nd best)
Tuning System:	Tempered	Pythagorean	Just	Archytas
Tone Generators:	$2^p\ 3^q$	$2^p\ 3^q$	$2^p\ 3^q\ 5^r$	$2^p\ 3^q\ 5^r\ 7^s$

Table 1

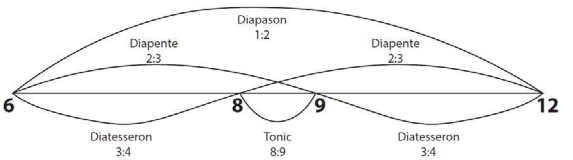


Figure 1. Transposing the First Four Integers to a 6:12 Octave.

Ab	Eb	Bb	F	C	G	D	A	E	B	F#	C#	G#
729	243	81	27	9	3	1	1/3	1/9	1/27	1/81	1/243	1/729

Figure 2a: Triple Progression: Exponed in a Row

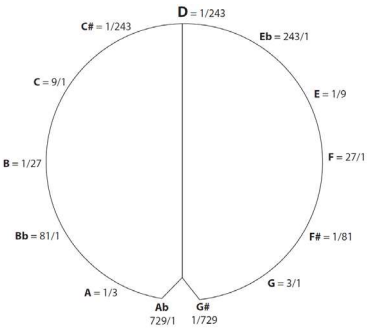


Figure 2b: Triple Progression: Wrapped Around in a Tone Circle.

Notes:	D	e	f#	G#<> Ab
Steps:	1	9/8	81/64	729/512
Intervals:	[9/8]	[9/8]	[9/8]	

Figure 2c: Cube the Number of the Beast

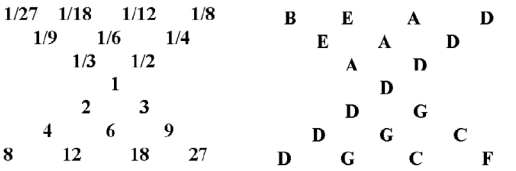
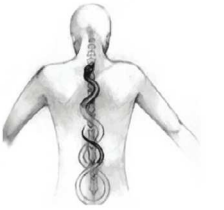


Figure 3: The World Soul (Pythagorean Tuning)



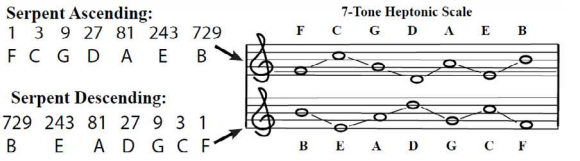


Figure 4: Ascending and Descending the Tree of Knowledge.

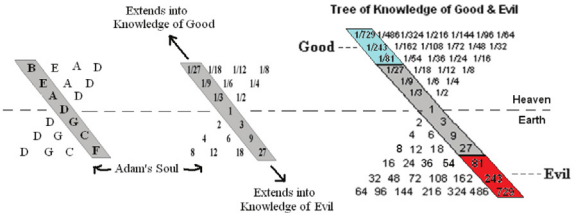


Figure 5: Adam & Eve's Fall from Grace Extends Pythagorean Tuning.

"the root 4:3"	6	:	8	::	9	:	12		
	12	:	9	::	8	:	6		
"friends" 4:5	24	: 30 :	32	: 36 :	45	: 48		"waxing"	
	60	: 48 :	45	: 40 :	32	: 30		"waning"	
"friends" 5:6	30	: 36 :	40	: 45 :	54	: 60		"waxing"	
	36	: 30 :	27	: 24 :	20	: 18		"waning"	

Figure 6a

"Guardians"	30	32	36	40	45	48	54	60
Greek Dorian	D	eb	f	G	A	bb	c	D
Reciprocal Dorian	D	c#	b	A	G	f#	e	D

Figure 6b

Tones:	D	eb	e	f	f#	G	A	bb	b	c	c#	D
"Waxing"	360	384	400	432	450	480	540	576	600	648	675	720
"Waning"	720	675	648	600	576	540	480	450	432	400	384	360

Figure 6c

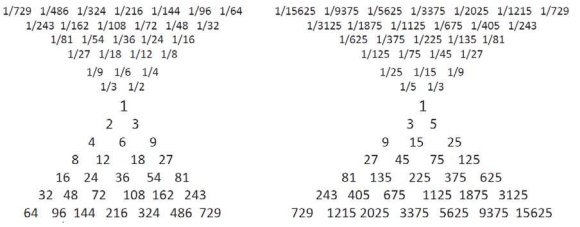


Figure 7

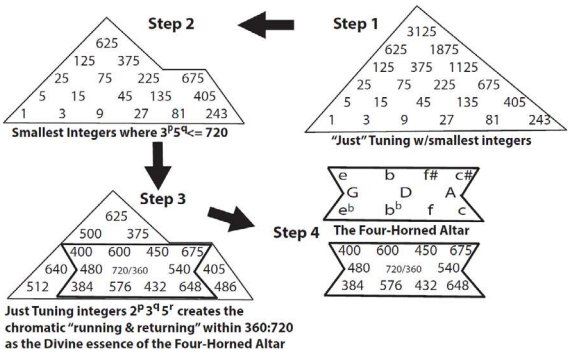


Figure 8a: Derivation of the Four Horned Altar.

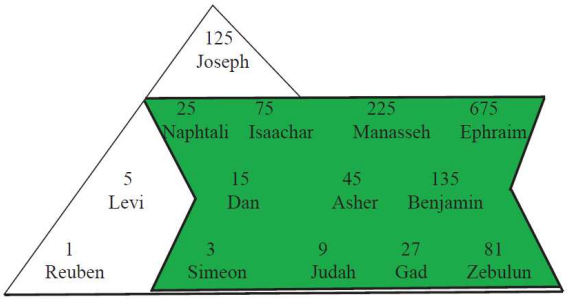


Figure 8b: The Tent of Jacob.

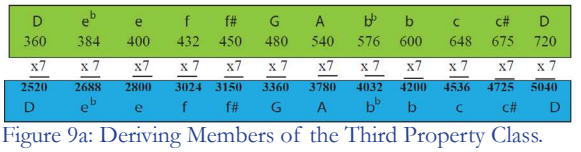


Figure 9a: Deriving Members of the Third Property Class.

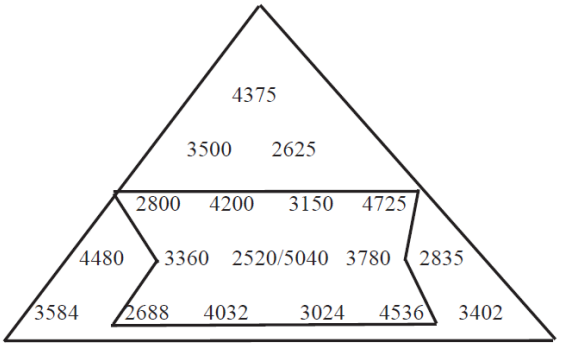


Figure 9b: Magnesia's 18 Parent Guardians.

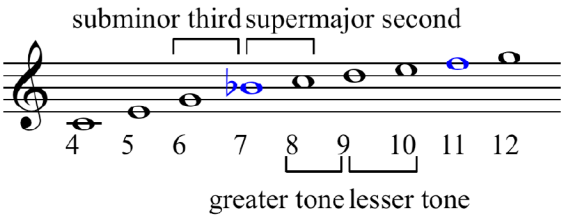


Figure 10

De Nicholas	McClain
Non-Existence	Non-being: undifferentiated tonal spectrum
Existence	Being: World Soul contained by the material Body of the cosmos
Images & Sacrifice	Context and relinquished context of each tuning system
Embodied Vision	Equal Temperament: the celestial Image that embodies Enlightenment

Table 2

City	Tuning System	Description
Athens	Pythagorean	Tuning to integer powers of 2, 3 $= (2^p 3^q)$
Atlantis	Just	Tuning to integer powers of 2, 3, 5 $= (2^p 3^q 5^r)$
Magnesia	Archytas	Tuning to integer powers of 2, 3, 5, 7 $= (2^p 3^q 5^r 7^s)$
Callipolis	Equal Temperament	Eliminating imperfection through temperament

Table 3

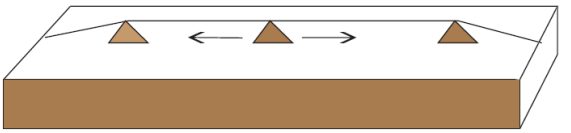


Figure 11

2:!	perfect octave
3:2	perfect fifth
4:3	perfect fourth
5:4	major third
6:5	minor third
7:6	septimal minor third
8:7	septimal major second
9:8	major second
10:9	minor second

Table 4

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MEMORIES OF ERNEST G. McCLAIN

Gerald M. TURCHETTO

I first encountered Ernest G. McClain through a typewritten manuscript draft of his first book, *The Myth of Invariance*. I had recently fallen off of the back of a turnip truck and landed at the State University of New York at Stony Brook campus, where in my sophomore year my interests began to concretize around philosophy. At that time I was taking a course in Buddhism with a philosophy professor named Antonio T. de Nicolas. All the students in the class were required to give a presentation on a topic of their choice. Antonio and I had met in prior classes and were developing a philosophical relationship, and so he felt comfortable sharing with me this typewritten manuscript of Ernest's (still awaiting publication), with the recommendation that I should give a presentation on one of its chapters. (He and Ernest already were in a productive working relationship at that time.) As I'd never heard either of "Ernest G. McClain" or of "tuning theory" (although like everyone at that time, I played the guitar), this was all new territory to me.

Ernest could be rather unforgiving in his presentation, and so the naïve reader of *Myth* had a bit of work to do to get any traction with the book. I nonetheless found that, in all, the basic set-up was not terribly difficult. The mathematics involved was grammar school level, albeit a bit slippery due to the constant complementary perspectives required. But even an equal-tempered guitarist could, without too much trouble, decipher what was going on.

Ernest's representation of harmonical tuning theory by way of the circular mandala was a nice and easy way to visualize the complementary possibilities of various tunings. His "yantras," on the other hand, visualizing the same information as his mandalas, but allowing a finer level of detail and showing tonal and arithmetic relationships in a more sophisticated and revealing manner, nonetheless were destined to try the reader's patience (and graphical ability). However, as a coding scheme, they were easy to interpret and construct, if one had the time and patience.

All this prolegomenous material took a while to get through, and wasn't to be the subject of my presentation in my Buddhism class—it only would've been a mathematics lesson without import. After I had become comfortable with the material, I chose a section from *Myth* on the *Revelation* of St. John and gave my presentation on that. It was well-received although I doubt anyone found it a revelation of any kind.

That was my first encounter with Ernest—St. John of Patmos and Antonio T. de Nicolas brought us together.

But still, Ernest was only a name to me. My memory is faulty, but I think it was another year or so before I actually met him, at his apartment in Manhattan. (We may have been corresponding by post in that interval.) This was a fruitful meeting: short on amenities (although I remember him serving a lovely tea) but long on learning. Among his many interesting artifacts was his monochord, and finally I got to hear what "natural" thirds sounded like—off-key, to my ear. But following his various placements of the triangular "fret" along the monochord and listening as he did so, the mathematics and the music finally merged in my experience. All the mathematical decisions that drive the music, and all the musical decisions that drive the mathematics, were brought home to me, along with the intense frustration of wanting to make them work together "harmoniously" without compromising the integrity of either. Good luck! That dilemma is at the core of it all. (If you follow Ernest's work, you might say that it's the "square root" of all evil.)

I continued to follow Ernest's work, in his articles and in his subsequent *Pythagorean Plato*,

with great interest, and I was convinced that he was truly on to something of extraordinary significance. I was familiar with all the ad hoc numerology that "scholars" and amateurs have indulged in when discussing ancient cultures, but Ernest's work had nothing arbitrary about it: it was based on the science of acoustics and tuning theory. It had a solid empirical foundation. The question for me became: how did it work to help us understand the texts of ancient cultures?

At the time the understanding of other cultures, including ancient ones, was an area of philosophical interest for me ("cross-cultural hermeneutics," as I called it). I ultimately joined the graduate school at Stony Brook as a doctoral student in philosophy, with Antonio de Nicolas as my advisor. In casting around for a thesis topic, I settled upon Plato's *Republic* as the test text on which to utilize Ernest's harmonic hermeneutic—a "harmoneutic," if you will.

I noted that this troublesome work of Plato's—his most famous by far—contained a lot of material that simply could not be accommodated by interpretive schemes that had been generated over the past 2,000 years. In fact, so frustrating were some of Plato's passages that some translators (who, rightfully, shall remain nameless) simply excised them from their translations. Ernest's *Pythagorean Plato* went a long way towards remedying this on a case-study basis, but I wanted to use Ernest's harmoneutic to show how *every* detail about the city—not just the patently mathematical ones—is generated by Socrates using the same decision-making process that a musician would use to investigate and develop a scale or tuning system. I wanted to see if the entirety of the *Republic* could be covered in this way, leaving out nothing and leaving nothing unexplained. I wanted the possibility of cross-cultural hermeneutics propounded by de Nicolas to be instantiated by the arch text of western philosophy. (An interesting side note to this was the conclusion that Plato was the last of the eastern philosophers, rather than the first of the western philosophers.) This was my doctoral thesis, which rests in likely well-deserved obscurity in the Music Library (!) at the university. Nonetheless, for me it was the clear demonstration—clearer than even Ernest himself imagined—of the fecundity of Ernest's work for understanding other cultures. (Ironically, I'm unsure whether or not Ernest ever had access to a copy of it to read.)

At some point—I can't recall when—Ernest himself tried to attenuate the numerological presentation of his work and produced *Meditations Through the Qu'ran*. Without a doubt, this is his most approachable work, relatively free (at least in its first edition) of the mathematical (seeming-) congeries of his first two books, and displaying what a beautiful stylist he was. Its timeliness for today goes without saying.

It would have been interesting for me to remain in academia and militate for Ernest's position in increasingly convincing and sophisticated ways, but life intervened and in the early 1980s, knowledge of Ernest's work and 50 cents could get you a ride on the subway in New York City. So off to the "real world" I went, leaving the ivory tower to the latest batch of turnips.

Somehow, I re-established contact with Ernest when I returned to academia to teach in the year 2000. I was surprised that he still was active—and was he ever. He had fearlessly taken on the battle with the Bible, which would largely occupy him for the remainder of his life. We began an internet correspondence of the type that many of his current admirers have been part of throughout these years. Justifiably or not, Ernest always regarded me as someone who understood him well—albeit I was a "philosopher," a species for which he had little use. An "essential tension" in our relationship was my desire for his work to be more systematic, whereas Ernest, as he always was quite, most, and incessantly fond of saying, had to do things "his way." My main concern, which I expressed many times to him, was that he avoid expressing himself in such a way as to attract numerologists and other hermetics, or allow his work to be thusly interpreted. I thought that a systematic presentation—one driven by the scientifically-based necessity behind the harmoneutical moves he was making—would help ensure that his work was taken seriously, and the new-ageists and ancient-ageists would shudder at having to do real mathematics and flee. However,

he didn't seem to think of this as a problem; again, he had to do things "his way." I respected that, although I continued to worry about the scholarly reception of his work.

I wanted to be of some help to him, but myself quickly flamed out at the university. Things therein had changed too much over twenty years, and too much within myself as well, for me to be comfortable there, and I left after a year. Nonetheless, Ernest and I continued corresponding by e-mail. At first I began to collect these e-mails as a type of record of his work, but, after time, as their number quickly approached the boundary between countable and uncountable infinities, it became impossible to do so.

Although his work always was interesting, in later years I often found it frustrating. His numerical and textual connections often appeared to me to be arbitrary, not necessary—or, alternatively, his work appeared to be fractions of a larger synoptic whole which he could not seem to articulate *in toto*. One of the challenges he faced was that the extraordinary range of his reading seemed to cause an overwhelming "flood" of possible connections, all of which he wanted to explore. Further, it seemed that his faith in the musical use of number was wavering, as he explored other mathematical rubrics, such as gematria, as a way of gaining insight into ancient texts. Although I knew how fruitless were my admonitions to him to remain focused on music and on the necessary relationships obtaining between tuning theory and text, I occasionally made them anyway. But, of course, Ernest had to do things "his way," and, chastened, I understood. I myself was too long out of the game to be an active contributor (plus, my contributions invariably would have been—gasp!—"philosophical"), but I continued to try to follow him in his journey and root for his success. Although I never really thought about how "success" in this context would be measured, I was quite pleased to find that his internet community of correspondents was increasing all the time and that many were taking an active interest in his work—some even advancing it within their own domain of interests. This volume itself is the sign of this community, which I hope continues to cohere and grow even in Ernest's absence.

I always admired Ernest's work ethic (a force of nature to the very end!) and his love of his work. This is a mark of a true scholar. He always challenged me and kept me interested. I still believe he was on the right track, but like most frontiersmen, he hewed a rough path. It's the work of those he's left behind to straighten it and smoothen it. I believe that there now are enough good people out there to do so, if they choose.

I'll end this recollection, meagre as it is, with the somewhat mawkish, but nonetheless sincere, entry that I contributed to the New York Times "Guest Book" connected to Ernest's obituary:

Goodbye, Ernest, and enjoy your journey into that pythagorean comma that brings one world to an end while bringing another world into existence. You were the archetype of the scholar until your last breath, and everyone in your orbit was a tone trying to harmonize with your search for the lost chord. Now our screens are blank and our ears are empty, but hopefully not for long. I hope the echoes of the past that you amplified for our hearing can find full articulation in the work of the colleagues that you've left behind. My sympathies to your family, and my blessings to the cosmos for having brought us together.